Mesh Motion in Fluid-Structure Interaction with DeepONets

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MESH MOTION

Mesh motion is a sub-problem in a number of applications involving a change-of-coordinates between reference and target domains. The change-in-coordinates in mesh motion methods is often given by the solution of PDEs, with Dirichlet boundary conditions representing the deformation of the domain boundary [3].

A mesh motion operator is a mapping $g \mapsto u$, where $g : \partial\Omega \to \mathbb{R}^d$ represents the deformation of the boundary $\partial\Omega$ of the reference domain and the resulting $u : \Omega \to \mathbb{R}^d$ defines the change-of-coordinates $\chi : \Omega \to \mathbb{R}^d$, $\chi(x) = x + u(x)$. For this change-of-coordinates to be well defined and suitable for computations, some requirements are that it is bijective, that $u|_{\partial\Omega} = g$, and that $J = \det(\nabla\chi) > 0$ everywhere in Ω.

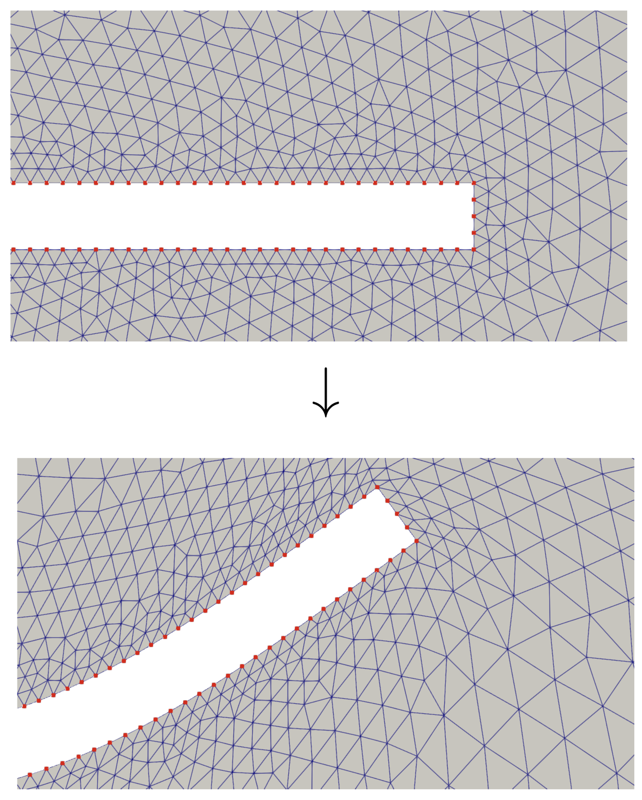


Figure 1: Illustration of mesh motion using biharmonic model. All mesh vertices are moved by a deformation field $u : \Omega \to \mathbb{R}^d$ computed from boundary deformation (red), such that deformed mesh is non-degenerate.

Mesh motion operators used

Harmonic

- fast, easy to compute
- can only handle relatively small domain deformations

$$-\Delta u = 0 \text{ in } \Omega, \quad u = g \text{ on } \partial\Omega$$

Biharmonic

- hard to solve (mixed formulation, H^2-conforming elements, DG-formulation)
- can robustly handle large deformations

$$\Delta^2 u = 0 \text{ in } \Omega, \quad u = g \text{ on } \partial\Omega, \quad \nabla u \cdot \mathbf{n} = 0 \text{ on } \partial\Omega$$

DeepONet-corrected harmonic [1]

- harmonic mesh motion plus DeepONet correction
- exact satisfaction of boundary condition $u|_{\partial\Omega} = g$ by modifying trunk output

$$u = \mathrm{harm}(g) + l \cdot \mathcal{D}(g), \quad l = 0 \text{ on } \partial\Omega, \quad l > 0 \text{ in } \Omega$$

Harmonic with DeepONet source term

- a DeepONet defines a source term for a Poisson equation
- equivalent to DeepONet-corrected harmonic

$$-\Delta u = \operatorname{div} \mathcal{D}(g) \text{ in } \Omega, \quad u = g \text{ on } \partial\Omega$$

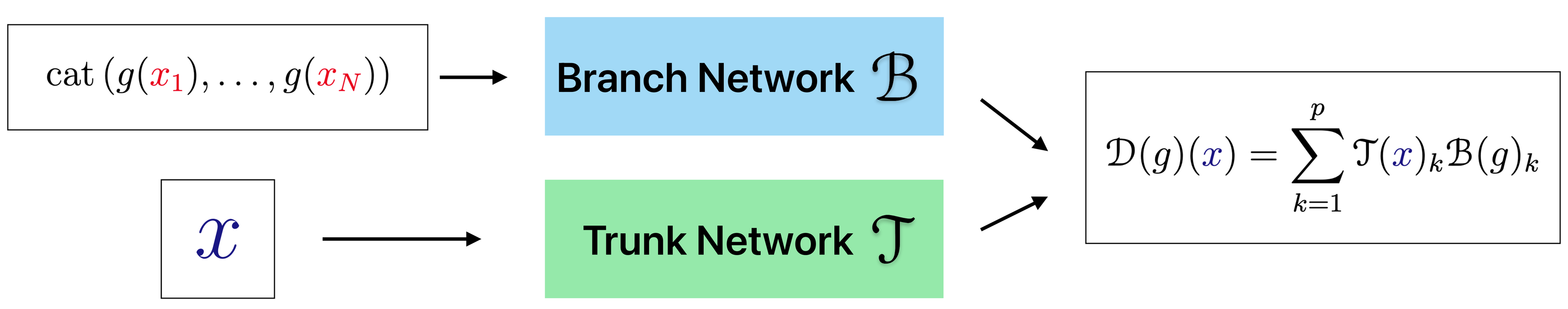


Figure 2: Illustration of DeepONet model.

FLUID-STRUCTURE INTERACTION TEST PROBLEM

We train on data from and evaluate against biharmonic mesh motion on common fluid-structure interaction (FSI) benchmark problem [2]. In FSI problems, the movement of the solid domain changes the geometry of the fluid domain over time. In certain formulations of the FSI equations, mesh motion is used to solve the fluid equations over the time-varying domain by constantly updating a reference mesh of the initial domain.

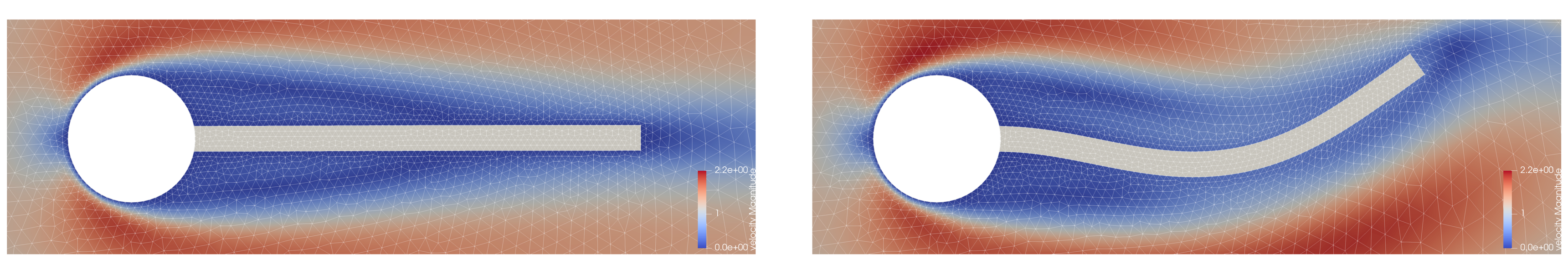
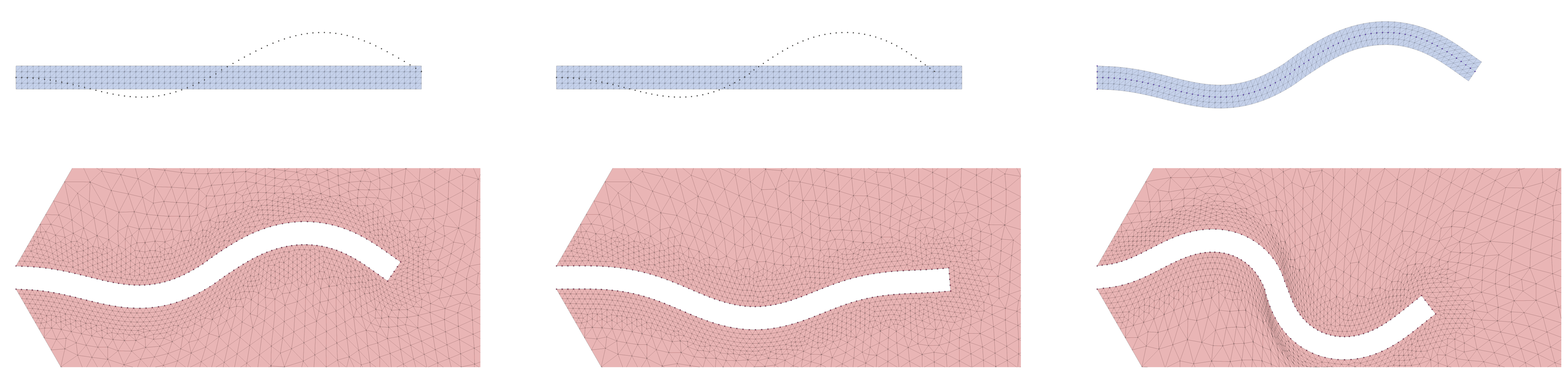


Figure 3: Two snapshots of fluid velocity magnitude in FSI benchmark problem before (left) and after (right) oscillations have developed. The solid domain (white and gray) is slightly off-center and a parabolic fluid inflow from the left causes periodic oscillations.

Randomly generated training set

A training set of plausible solid deformations is made using Gaussian processes. A nonlinear elasticity problem with sampled Dirichlet boundary conditions for the center line of the solid gives the fluid domain boundary deformations to create the training set.



MESH QUALITY COMPARABLE WITH BIHARMONIC

Mesh quality of DeepONet-corrected harmonic mesh motion is on par with biharmonic mesh motion on the FSI benchmark test data

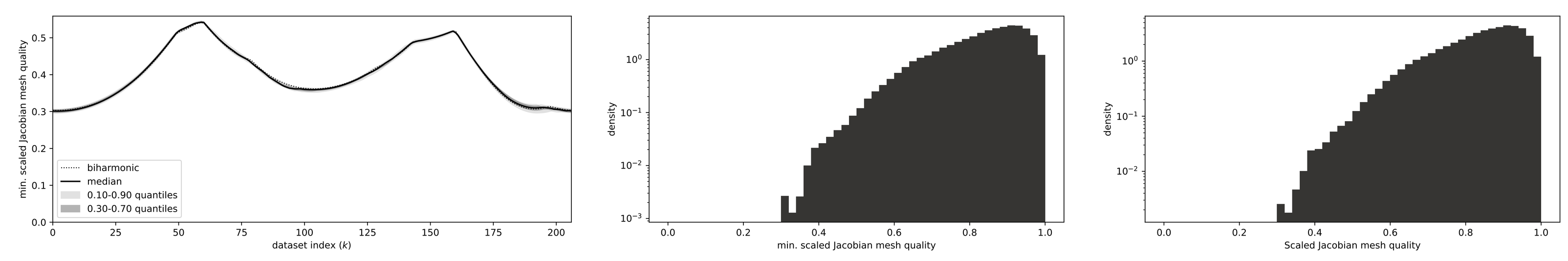


Figure 4: Quantiles of scaled Jacobian mesh quality over training set from 20 initializations of chosen DeepONet architecture (left) and histogram of scaled Jacobian mesh quality over training set for biharmonic (middle) and best performing DeepONet (right) mesh motion.

IMPLEMENTATION FOR FLUID-STRUCTURE INTERACTION

For FSI and other problems, g is an unknown and solved for simultaneously with the mesh motion. Thus we need to solve the nonlinear problem

$$-\Delta u = \operatorname{div} \mathcal{D}(u) \text{ in } \Omega, \quad u = g \text{ on } \partial\Omega.$$

The Jacobian of this problem is dense, due to the non-local basis of the trunk network, and a standard Newton solver is therefore inefficient. We use an approximated Newton solver where the DeepONet's contribution to the Jacobian is neglected and observe fast convergence.

QUANTITIES OF INTEREST VALIDATED ON BENCHMARK

Quantities of interest for the FSI2 benchmark problem, such as drag and lift forces, agree with the reference values and those computed using biharmonic mesh motion, validating the DeepONet mesh motion models.

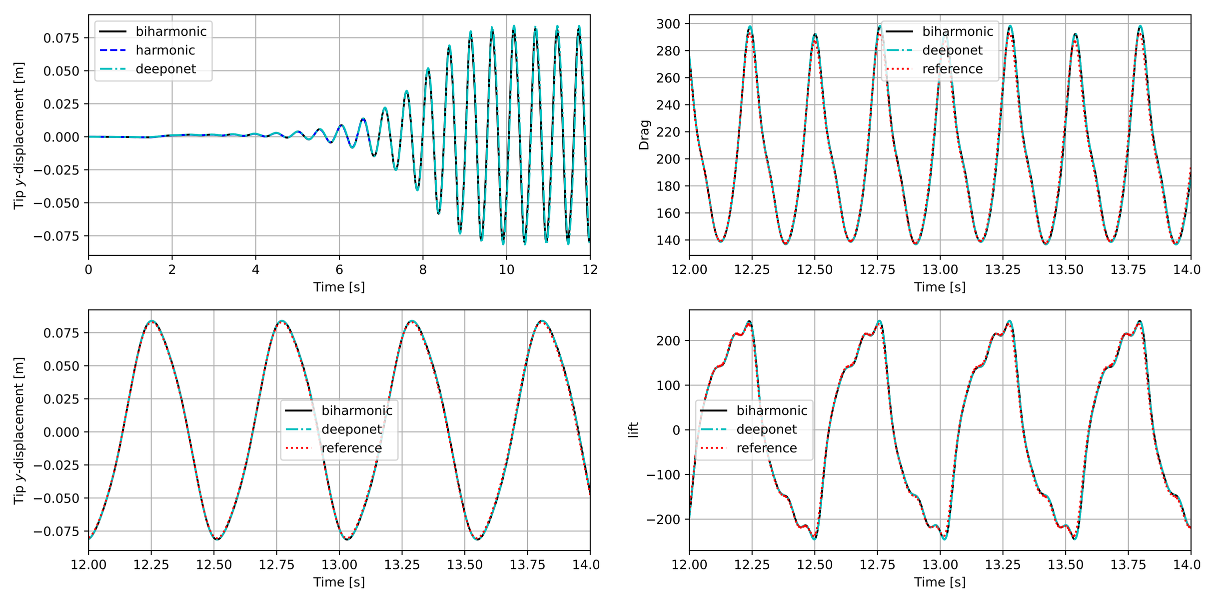


Figure 5: y-displacement of flag in warming up (top left) and reference window (bottom left), drag (top right) and lift (bottom right) from FSI2 benchmark computed with different mesh motion models.

COMPUTATIONAL COST COMPARABLE WITH HARMONIC

In a monolithic ALE-formulation FSI solver, the DeepONet mesh motion model is faster than biharmonic and competitive with harmonic mesh motion. This also holds with increasing number of parallel processes.

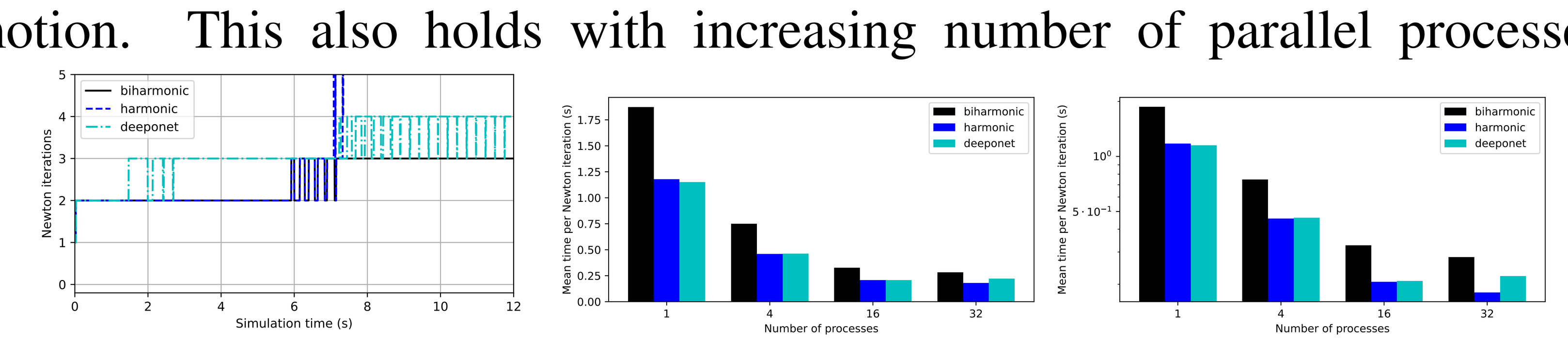


Figure 6: Newton iterations per time step in FSI2 benchmark (left) and mean time per Newton iteration in linear (middle) and log (right) scale. Simulation run with 50458 degrees of freedom for harmonic and DeepONet mesh motion and 71658 degrees of freedom for biharmonic mesh motion. Total simulation time: 2714 s for biharmonic and 2423 s on 20 processes.

References

[1] O. Hellan. Mesh motion in fluid-structure interaction with deep operator networks, Feb. 2024. arXiv:2402.00774 [cs, math].

[2] S. Turek and J. Hron. Proposal for Numerical Benchmarking of Fluid-Structure Interaction between an Elastic Object and Laminar Incompressible Flow. In *Fluid-Structure Interaction*. Springer Berlin Heidelberg, 2006.

[3] T. Wick. Fluid-structure interactions using different mesh motion techniques. *Computers & Structures*, 2011.