# A Robust Model-based Approach to Indoor Positioning Using Signal Strength

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Abstract — A simple technique to estimate the position of a mobile node inside a building is based on the Received Signal Strength (RSS). In previous publications, we investigated the effectiveness of using circular array antennas and beamforming in order to enable an access point to estimate the position of a mobile inside a building. We also discussed the feasibility of using model-based radio maps to reduce the need for extensive offline measurements. In this paper, a positioning algorithm based on the relative order of the received signal strengths is discussed. This algorithm in conjunction with the ray-tracing propagation model can have promising performance for indoor environments and essentially eliminates the needs for an extensive set of a priori measurement, training or intricate calibration.

Keywords - localization; signal processing; ray-tracing; indoor; propagation; beamforming;

## I. INTRODUCTION

The ability to locate and track mobile users is a key service in pervasive information systems. Technologies that find the location of mobile sources inside buildings are an attractive area of research and development. Currently, a significant application of such technologies is in emergency situations but more commercial and public safety applications are emerging every day.

A simple technique to estimate the position of a given source is based on the Received Signal Strength (RSS). RSS is attractive because it is widely applicable to wireless sensor or local area networks and does not require sophisticated localization hardware. The general philosophy in this approach is to establish a one-to-one correspondence between a given position and the average received signal strength from at least three access points with known locations. One such system that has been implemented on the existing wireless local area network infrastructure is RADAR [1]. There are two main phases in the operation of this system: an off-line phase (i.e. data collection or training phase) and an online phase (i.e. mobile position estimation). In the off-line phase, a "Radio-Map" of the environment is created. A "Radio-Map" is a database of locations throughout the environment and their corresponding received signal strengths from several access points. In the on-line phase, access points measures the received signal strength from the mobile, and then search through the Radio-Map database to determine the best signal strength vector that matches the one observed. The system estimates the location associated with the best-matching signal strength vector (i.e. nearest neighbor) to be the location of the mobile. This technique essentially calculates the  $L_2$  distance (i.e. Euclidean distance) between the observed RSS vector and the entries in the set defined by the radio-map. It then picks the vector that minimizes this distance and declares the corresponding physical coordinate as the estimate of the mobile's location.

The main drawback of the RSS-based techniques such as RADAR is the need for a measurement-based training phase, during which the radio map of the environment is created. This map essentially contains the received signal strengths from all reference nodes throughout the environment. The process to generate a radio map is not only labor-intensive and costly but also very sensitive to changes in the environment and possible sources of interference in the building.

A simple alternative to generate the radio map for RSSbased positioning system is using an appropriate propagation model instead of the actual measurements. For example, deterministic channel models such as ones based on ray-tracing are a good candidate for this problem. However, in these models, only simple high-level building information such as layout is used and other detail information about the environment such as the exact radio properties of the walls, and other obstacles affecting the RSS such as furniture are often ignored. The accuracy of the predicted signal strengths can be highly dependent on this detailed information which is almost impossible to capture in the model. Therefore, the performance of the positioning system will depend on the model's detail. For example, Figure 1 illustrates the dependency of the positioning accuracy on different wall-types used in the ray tracing model. The observed variation (i.e. as much as 65%) in performance is mainly due to the fact that the algorithm is based on the numeric values of the received signal strengths; therefore any error in the predicted RSS values (i.e. caused by the ray-tracing model) will directly reflect on the accuracy of the positioning system. This means that the system performance could potentially be improved if there are positioning algorithms that do not rely on the exact values of the predicted RSS.

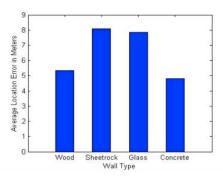


Figure 1. Average location error for various wall types

In [11], we discussed a positioning algorithm based on the relative order of the received signal strengths. That algorithm in conjunction with the ray-tracing propagation model showed promising performance for indoor environments without any needs for an extensive set of a priori training. However, the drawback of that approach was the need for high access point density (i.e. number of access points per square meter). This high density requirement essentially provided the robustness of the methodology against the inherent inaccuracies that exist in the model-based radio-map. At the same time, a high number of access points could create obstacles in practical implementation and deployment of such systems.

In [2], we proved that information pertaining to the angular distribution of power can be used to increase the accuracy of an RSS-based localization methodology. We showed that by using access points capable of measuring the spatial power spectrum the system would be able to estimate the mobile position with fewer access points and higher accuracy. However, a more generalized and sophisticated measurement-based radio-map that contained received signal strength information from various directions was required in order to implement that approach.

In [10], we proposed a novel positioning methodology that extended our work in [2] and exhibited acceptable performance without the need for an extensive set of measurements in the off-line mode. The only drawback in [10] was the need for calibration of the ray-tracing tool that generates the radio-map of the environment. This was needed to ensure that the mean of the average received signal strength (when measured in decibels) between the ray-tracing prediction and actual measurement is zero. Any bias on the average predicted RSS could directly influence the accuracy of the estimated positions. Therefore, in practice, some a priori work to adjust the parameters of the ray-tracing tool is needed. This might not be feasible for large size environments.

In this paper, we extend our work in [2], [10] and [11]; and, combine the strength of each previously proposed approach to obtain a positioning methodology that 1) totally removes the dependence of the system on the existence of an accurate unbiased radio map 2) requires low access points density 3) offers acceptable level of accuracy for most commercial applications.

The rest of this paper is as follows. Section 2 will describe our proposed positioning approach. Modeling the error in the estimated RSS by the ray-tracing tool is discussed in Section 3. System performance is provided in Section 4 and finally concluding remarks and future plans are expressed in Section 5.

## II. APPROACH

Consider a simple system consisting of  $M \ge 1$  Access Points (AP) and a mobile node inside a building. Assume that the grid of points  $(x_i, y_i)$  i = 1, 2, ...., N cover the entire building layout with a known resolution. If the mobile's coordinate is  $(x^*, y^*)$  then the objective of the positioning system is to estimate the mobile's location by finding the closest grid point to the mobile's coordinate. In other words, find 'k' where

$$k = \arg\min_{i} \|(x_{i}, y_{i}) - (x^{*}, y^{*})\|.$$

Assume that the mobile node is a simple transmitter with an omnidirectional antenna and known transmit power 'P'. The AP is a receiver that is equipped with a circular array antenna with beamforming capability. In this way, the RSS in any given direction can be measured by electronically rotating the main lobe of the antenna pattern to the desired direction. Define  $RSS_{AP_m}(\theta)$  to be the RSS from the mobile at the access point m (i.e.  $AP_m$  m=1,2,...,M & M is the total number of access points) when the main lobe of its antenna is pointing at azimuth direction  $\theta$  (Fig. 2a). Also, as shown in Fig. 2b, define  $R_{(x_i,y_i)}^{AP_m}(\theta)$  to be the received signal strength from  $(x_i,y_i)$ 

the  $AP_m$  at the grid point  $(x_i, y_i)$  if the access point transmits a signal identical to the mobile (i.e. same power and frequency). If  $(x_j, y_j)$  happens to be the same as  $(x^*, y^*)$  (i.e. actual coordinate of the mobile node), then due to symmetry in the propagation of radio waves, we should have:

$$R^{AP_m}_{(x_j,y_j)}(\theta) = R^{AP_m}_{(x_j^*,y_j^*)}(\theta) = RSS_{AP_m}(\theta)$$

Now define  $\hat{R}^{AP_m}_{(x_i,y_i)}(\theta)$  to be the received signal strength

from the  $AP_m$  at the grid point  $(x_i, y_i)$  estimated by a deterministic model such as ray-tracing. There will be a difference between this *estimate* and the actual value of the RSS. Define the "error" in the estimated RSS at grid point  $(x_i, y_i)$  to be the absolute value of this difference. In

other words, if  $E_{(x_i,y_i)}^{AP_m}(\theta)$  denotes this error, then:

$$E_{(x_{i},y_{i})}^{AP_{m}}(\theta) = R_{(x_{i},y_{i})}^{AP_{m}}(\theta) - \hat{R}_{(x_{i},y_{i})}^{AP_{m}}(\theta)$$

The value of this error in dB can be modeled as a *Normal* Random Variable with intensity  $\sigma$  and mean  $\mu$ . Derivation of this modeling has been provided in [10].

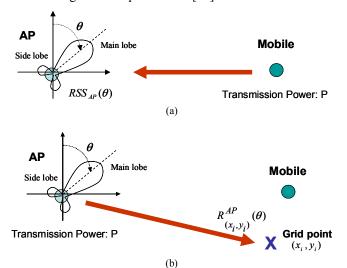


Figure 2. RSS from (a) Mobile to AP, and (b) from a grid point to AP

$$\hat{R}^{AP_m}_{(x_i,y_i)}(\theta)$$
  $i=1,2,...,N$  can be calculated by the ray-

tracing model and the results can be saved in a database to form an *estimated radio map* of the environment. N is the total number of grid points on the estimated radio map.

Our approach in estimating the mobile's coordinate is to build a likelihood map for every access point, and then combine the information on all such maps to estimate the position of the mobile. A likelihood map, as the name implies, should highlight the likely positions of the mobile as seen by the corresponding access point. Quantitatively, a likelihood map is basically a collection of Likelihood Scores (LS) for each grid point. Likelihood score at the grid point  $(x_i, y_i)$  as seen by access point m (i.e.  $AP_m$ ) is defined as follows:

$$LS_{AP_{m}}(x_{i}, y_{i}) = \frac{1}{\sum_{kl} f_{kl}((x_{i}, y_{i}), (x^{*}, y^{*}))}$$

where

$$f_{kl}((x_i, y_i), (x_j, y_j)) = \begin{cases} 1 & c_{kl}(x_i, y_i) \neq c_{kl}(x_j, y_j) \\ 0 & c_{kl}(x_i, y_i) = c_{kl}(x_j, y_j) \end{cases}$$

and

$$c_{kl}(x^*, y^*) = \begin{cases} +1 & RSS_{AP_m}(\theta_k) > RSS_{AP_m}(\theta_l) \\ -1 & RSS_{AP_m}(\theta_k) < RSS_{AP_m}(\theta_l) \\ 0 & RSS_{AP_m}(\theta_k) = RSS_{AP_m}(\theta_l) \end{cases} \quad \forall k \neq k$$

$$c_{kl}(x^*, y^*) = 0 \quad \forall k = l$$

and

$$c_{kl}(x_i, y_i) = \begin{cases} +1 & \hat{R}_{(x_i, y_i)}^{AP_m}(\theta_k) > \hat{R}_{(x_i, y_i)}^{AP_m}(\theta_l) \\ -1 & \hat{R}_{(x_i, y_i)}^{AP_m}(\theta_k) < \hat{R}_{(x_i, y_i)}^{AP_m}(\theta_l) \end{cases} \quad \forall \, k \neq l \\ 0 & \hat{R}_{(x_i, y_i)}^{AP_m}(\theta_k) = \hat{R}_{(x_i, y_i)}^{AP_m}(\theta_l) \end{cases}$$

 $c_{kl}(x_i, y_i) = 0 \quad \forall k = l$ 

The  $f_{kl}((x_i,y_i),(x_j,y_j))$  basically calculates the hamming distance between the two matrices of  $c_{kl}(x_i,y_i)$  and  $c_{kl}(x_j,y_j)$ . The value of this function is independent of the average error (i.e. mean) in the estimated radio map. This property does not exist in any methodology that directly uses the RSS values in the mobile position estimation process. Therefore any bias in the estimated radio map will not impact the performance of the system.

The mobile is likely to be close to grid points that have a high likelihood score. So, for each likelihood map, the set of grid points with the highest likelihood scores determine possible position of the mobile inside the building. Recall that each likelihood map is associated to and generated by an individual access point. By judiciously combining information for all likelihood maps (i.e. access points), a single grid point can be found that represents the estimated position of the mobile. We propose the following approach to estimate the mobile's position. Define the Total Likelihood Score ( $TLS_A$ ) at grid point ( $x_i$ ,  $y_i$ ) as:

$$TLS_A(x_i, y_i) = \frac{1}{\sum_{m} LS_{AP_m}(x_i, y_i)}$$

The grid point with the highest TLS is the mobile's estimated position i.e.  $(x_k, y_k)$  where 'k' is

$$k = \arg\max_{i} TLS_{A}(x_{i}, y_{i})$$

Figure 3 displays an example of a 2D likelihood map for the building layout shown in Figure 5.

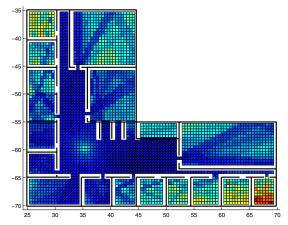


Figure 3. Example likelihood map showing possible positions of the mobile

The total likelihood map is the result of integration of total likelihood scores for all grid points across all access points. An example of a total likelihood map (in 3D) for the same layout is shown in Figure 4.

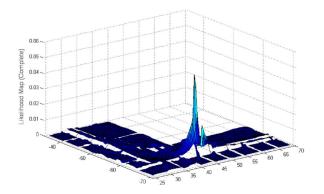


Figure 4. Example of the total likelihood map showing the estimated position of the mobile

### III. MODELING THE ERROR IN ESTIMATED RSS

At indoor environments, the radio wave may travel through various obstructions such as walls, doors and furniture before reaching the mobile. Since, it is almost impossible to create a precise replica of the environment for the ray-tracing tool (including radio properties of all obstructions); it is valuable to have a model that can account for the difference between the average RSS estimates made by ray-tracing and actual measurement.

The difference in the average received signal strength (when measured in decibels) between the ray-tracing prediction and actual measurement (i.e.  $E^{AP_m}_{(x_i,y_i)}(\theta)$ ) can be modeled by

a *Normal* random variable. The advantage of our proposed positioning strategy is that the mean of the corresponding distribution does not have any effect on the system performance. So, we only need to consider the impact of the variance of this normal distribution on the achievable accuracy. We refer to this variance as "*error intensity*". We will investigate the performance of the positioning algorithm for various error intensities.

In order to do so, we need to generate sample realizations of  $E^{AP_m}_{(x_i,y_i)}(\theta)$  for different values of error intensity.

However, these realizations should be correlated for different  $\theta$ . Consequently, we need a method to generate angle-dependent correlated Gaussian random variables. To do this, we follow the methodology outlined in [5,6]. If  $X = [x_1, x_2, ...., x_N]$  is a random vector containing uncorrelated Gaussian random variables  $X_i$ , then random vector  $Y = [y_1, y_2, ...., y_N]$  (with correlated Gaussian random variables  $y_i$ ) can be generated by using a matrix C as

$$Y = CX$$
.

C is a matrix of weight coefficient satisfying the following relation:

$$\Gamma = CC^T$$

In other words, C is the *Cholesky factorization* of  $\Gamma$ , where  $\Gamma$  is the correlation matrix of random vector Y i.e.

$$\Gamma = \begin{bmatrix} 1 & \rho_{12} & \dots & \rho_{1N} \\ \rho_{21} & 1 & \dots & \rho_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N1} & \rho_{N2} & \dots & 1 \end{bmatrix}$$

 $\rho_{ij}$  is the angular cross-correlation function of the error in the average RSS estimates. We have observed that the following exponential decaying function accurately models this function.

$$\rho_{ij} = e^{-\frac{\left|\theta_i - \theta_j\right|}{\theta_{cor}} \ln 2}$$

 $\theta_i$  i=1,2,....,N is the direction of the main lobe of the antenna pattern at the AP. Given an angular rotation step-size of  $\delta$ , we will have  $N=2\pi/\delta$ .  $\theta_{cor}$  is called the decorrelation angle and corresponds to the angle at which the correlation drops to 50%. Through experiment with a directional antenna on a rotating platform, (details of which have been omitted for brevity), we observed a range of 10° to 25° for  $\theta_{cor}$ . Interestingly, similar distance dependent correlation functions have been reported for outdoor systems [7,8].

# IV. SYSTEM PERFORMANCE

To evaluate the performance of our proposed positioning algorithm, we used WiSE. WiSE (i.e. Wireless System Engineering) is a ray-tracing tool that has been developed and verified by Bell Laboratories [3,4]. For each  $\theta$ , we simply used WiSE to estimate values of RSS for all grid points in order to build an estimated radio map of the environment. We made sure that the antenna pattern at the AP is the same pattern that is generated by beamforming with a circular array antenna.

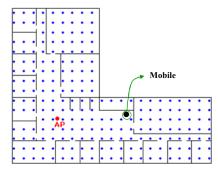


Figure 5. Building layout with the location of the mobile, AP and grid points

For the building layout shown in Fig. 5, we conducted extensive simulations to obtain the system performance.

Transmission frequency was set at 2.4 GHz. We have studied the effect of many parameters such as grid resolution, decorrelation angle ( $\theta_{cor}$ ), rotation step-size ( $\delta$ ), number of APs, number of the array elements at the AP and error intensity; however, for brevity, we only present the main performance result in the following.

Fig. 6 demonstrates the average error in the estimated mobile position versus error intensity for various number of access points. For error intensities up to 4 dB, it seems that using only two access points can lead to reasonable accuracy for most applications. For higher error intensities, more number of access points can greatly help in reducing the average error in the estimate position. As observed, using 5 access points for the layout in Fig. 5 leads to a very good performance even in the presence of 8dB modeling error. Considering that the size of the area under test is  $1575 \text{ m}^2$ , this corresponds to an access point density of  $0.0032 \text{ AP/m}^2$  with an average error of less than 2 meters. This level of accuracy has been shown to be the best achievable for most RSS-based positioning approaches [12].

We also observed that higher number of antenna elements at the AP will increase the resolution of the beamformer; which along with an appropriately chosen  $\delta$  can enhance the positioning accuracy. For brevity, we have omitted these results.

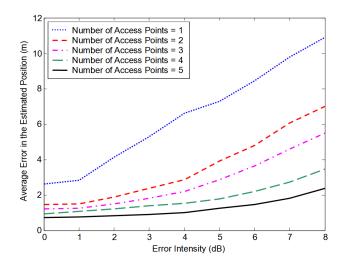


Figure 6. Performance vs. error intensity for various number of access points ( $\theta_{cor}$  =10°,  $\delta$  =5°, Grid Resolution=1 m, Array Elements=8)

## V. CONCLUSION

The focus in this research was to provide robust methodologies that essentially eliminate the need for extensive a priori measurement in RSS-based positioning systems. By exploiting the information in the angular distribution of RF

energy around a receiver along with appropriate positioning algorithms, one can build reasonably accurate systems that do not require any offline measurement, training or intricate calibration. The inherent robustness in the algorithm provided here creates the possibility to build systems that are quickly deployable in any given environment. By integrating any simple ray-tracing program with the proposed positioning algorithm, a complete system can be designed on recent MIMO-enabled 802.11-based systems. Further studies need to be done before such systems can have widespread applications in our daily life.

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