Energy Minimization Approach for Optimal Cooperative Spectrum Sensing in Sensor-Aided Cognitive Radio Networks

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***Abstract*—In a sensor-aided cognitive radio network, collaborating battery-powered sensors are deployed to aid the network in cooperative spectrum sensing. These sensors consume energy for spectrum sensing and therefore deplete their life-time, thus we study the key issue in minimizing the sensing energy consumed by such group of collaborating sensors. The IEEE P802.22 standard specifies spectrum sensing accuracy by the detection and false alarm probabilities, hence we address the energy minimization problem under this detection accuracy constraint. Firstly, we derive the bounds for the number of sensors to simultaneously guarantee the thresholds for high detection probability and low false alarm probability. With these bounds, we then formulate the optimization problem to find the optimal sensing interval and the optimal number of sensor that minimize the energy consumption. Thirdly, the approximated analytical solutions are derived to solve the optimization accurately and efficiently in polynomial time. Finally, numerical results show that the minimized energy is significantly lower than the energy consumed by a group of randomly selected sensors. The mean absolute error of the approximated optimal sensing interval compared with the exact value is less than 4% and 8% under good and bad SNR conditions, respectively. The approximated optimal number of sensors is shown to be very close to the exact number.**

I. Introduction

Cognitive Radio (CR) envisioned by J. Mitola in [1] has emerged as the innovative dynamic spectrum access technology [2] to improve the current utilization of assigned spectrum. It is reported by The Federal Communications Commission (FCC) in [3] that the spectrum is only 15% - 85% utilized depending on geographical and temporal variations. In a cognitive radio network, the unlicensed (secondary) devices can utilize the licensed spectrum when it is unused by the licensed (primary) devices. However, the occupied spectrum will need to be vacated instantly when a primary device starts using it in order to avoid interfering with the primary transmission. Thus, spectrum sensing is specified as a mandatory feature within the IEEE P802.22 standard [4] to enable a CR device to detect and adapt to the primary usage of a spectrum band. The sensing performance metric is summarized in IEEE P802.22 in terms of sensing receiver sensitivity, channel detection time (sensing interval), detection probability, and false alarm probability. Hence, improving the sensing performance has emerged as one of the most important issues in spectrum sensing recently.

Collaborative spectrum sensing by multiple collaborating sensing devices is studied in [5], [6], [7] to increase the detection probability. The cooperative spectrum sensing is also considered in [8] to minimize the total error rate given the number of sensing nodes and their Signal-to-Noise-Ratio (SNR). In [9], the problem of maximizing the ratio of the transmission duration over the entire sensing cycle is studied. *However, for practical purpose of using energy-constrained sensor network for spectrum sensing in cognitive radio networks* [10], *it is critical to use fewer sensing sensors performing in a shorter sensing interval in order to preserve as much energy as possible while still satisfying the requirement for spectrum detection accuracy*. The present paper investigates this issue in terms of finding the optimal sensing interval and the optimal number of sensors in order to minimize the total energy consumption for cooperative spectrum sensing.

This paper studies cooperative spectrum sensing by a power-constrained sensor network in sensor-aided cognitive radio networks [10]. These sensors can sense the spectrum band continuously and reports the detection results to a fusion center as demonstrated in Fig. 1. In the considered cooperative

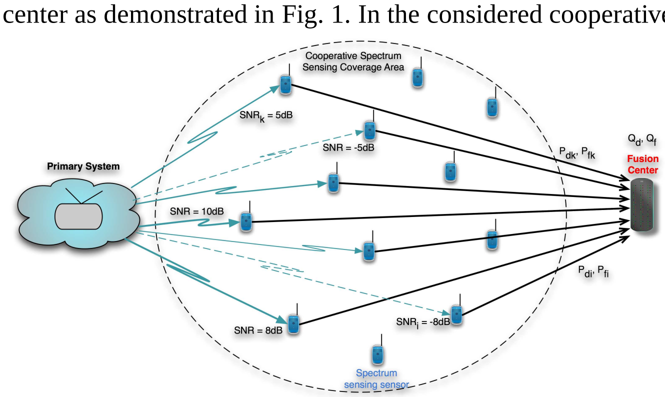

Fig. 1. Cooperative spectrum sensing model

spectrum sensing scheme, the fusion center invites a specific number of sensors in the network to participate in a sensing group, say $\mathbf{S}$. Then, the invited sensors independently start sensing the spectrum and report their observations back to the fusion center who performs the “OR-rule” fusion mechanism [11] to make a decision on the availability of the monitored spectrum.

In this scenario, each sensor uses the energy detection scheme [12], [5] for spectrum sensing, whose performance is evaluated by the detection and false alarm probabilities. A high detection probability means a high accuracy of detecting the activity of a primary user. A low false alarm probability indicates a high usage of available spectrum by the secondary users, due to a low chance that the spectrum is mistakenly believed to be occupied when it is actually available. There is a tradeoff in keeping high detection probability and low false alarm probability at the same time in the “OR-rule” fusion mechanism. The more sensors the higher detection and false alarm probabilities and vise versa. Hence, this paper first finds the lower bound and upper bound for the number of sensors under a given requirement for the spectrum sensing accuracy.

Spectrum sensing consumes energy and therefore depletes the life-time of the power-constrained sensors. Hence, energy minimization is critical to prolong the life-time of the sensor network. This paper formulates an optimization problem to minimize the total energy consumption. It is desirable to gain a high detection accuracy by using a cooperative group of many sensors performing a long channel sensing interval, which in turn consumes more energy. On the other hand, it is also highly desirable to save as much energy as possible by using fewer sensors and sensing for a shorter time. This tradeoff is addressed in the proposed optimization.

Finally, this paper proposes an efficient approximation approach to analytically and accurately solve the optimization in polynomial time, since the optimization is shown to be extremely difficult to solve directly. The approximated analytical solutions for the optimal sensing interval and the optimal number of sensors are derived accurately. We find that under good SNR conditions, the mean absolute error of the approximated optimal sensing interval is less than 4% compared to the exact optimal one. In the worst SNR conditions, this error is around 8%.

The rest of the paper is organized as follows. The related work is presented in Section II. Then Section III presents the system model. Next, the energy minimization problem is addressed in Section IV. Then, Section V proposes an approximation approach to analytically solve the optimization problem in polynomial time, which is proved in Appendices A, B $\&$ C. Numerical results are presented in Section VI to explore the optimization and validate the accuracy of the approximated optimal solutions. Finally, conclusions and future direction are stated in Section VII.

II. Related Work

There have been some recent studies on improving the performance in cooperative spectrum sensing. Peh and Liang show in [13] that an optimum performance can be achieved by the cooperation of only a certain number of secondary users, i.e. users who sense the highest SNR of the primary users transmission. They study the optimization of the detection probability and false alarm probability separately with regard to (w.r.t) the number of cooperation users. The tradeoff between keeping high detection probability and low false alarm probability at the same time w.r.t the optimal number of cooperation users is not addressed yet.

In [8], Zhang et al. focus on finding an optimal fusion rule to minimize the summation of false alarm and miss-detection probabilities by assuming that the number of cognitive radios and their SNR are known. However, the SNR received by all cognitive radios changes over the time due to the changing communication environment. The present paper shows that in order to find the optimal number of sensors under the constraint of detection accuracy, knowing in advanced the number of all sensors in the network is not required.

In [9], Lee and Akyildiz study the problem of maximizing the ratio of the transmission duration over the entire sensing cycle. They report that the optimal sensing parameters will need to be adapted to the number of cooperative sensing users, which varies over time. Liang et al. also study the sensing duration problem in [14] as a sensing-throughput tradeoff to minimize the false alarm probability given the detection probability threshold. The present paper, on the other hand, studies the tradeoff in deriving the optimal sensing duration and the optimal number of sensors while preserving as much energy as possible under a given detection accuracy constraint. Optimal cooperative spectrum sensing by minimizing the energy consumption is also studied in [15]. However, the tradeoff in keeping a high detection probability and a low false alarm probability simultaneously in their optimization is not studied. In addition, the approach in [15] yields a fairly high error in the approximated results.

The present paper differs from the previous work in terms of comprehensively studying and formulating the energy minimization problem for cooperative spectrum sensing while satisfying a given threshold for detection accuracy. The tradeoff between the optimal number of sensors and the optimal sensing interval as well as the tradeoff in keeping a high detection probability and a low false alarm probability simultaneously are considered and formulated in the proposed optimization problem.

III. System Model and Problem Formulation

Table I lists the main notations used in this paper. The system model for cooperative spectrum sensing is illustrated in Fig. 1. The sensor network $\mathcal{N}$ is deployed to detect the activity of a primary system on a given spectrum band. Each sensor i in $\mathcal{N}$ receives the primary signal with an instant SNR γ_i and this signal-to-noise-ratio varies from sensor to sensor depending on the surrounding wireless communication environment. The details of the studied cooperative spectrum sensing scheme is presented as follows.

TABLE I

Symbol	Definition
$\mathcal{N}$	The set all sensors in the network
$\mathbf{S}$	The group of sensors for cooperative spectrum sensing
γ_i	Signal-to-noise ratio (SNR) at sensor i (dB)
γ^{min}	The minimum SNR among the sensors (dB)
σ_n	The ground noise (dB)
$\mathbf{N}(\mu_i, \sigma_i^2)$	Chi-square distribution with mean μ_i and variance σ_i^2
λ	Energy threshold used by the energy detector (dB)
W	The spectrum bandwidth (Hz)
$\hat{P}_{di}$	Single-node detection probability of sensor i
$\hat{P}_{fi}$	Single-node false alarm probability of sensor i
$\hat{P}_d^{min}$	The minimum single-node detection probability
$\hat{P}_f^{max}$	The maximum single-node false alarm probability
Q_d	Cooperative detection probability of the sensing group $\mathbf{S}$
Q_f	Cooperative false alarm probability of the sensing group
$\bar{Q}_d$	Threshold for cooperative detection probability
$\bar{Q}_f$	Threshold for cooperative false alarm probability
t_s	The spectrum sensing interval (sec)
t_s^*	The optimal spectrum sensing interval (sec)
n	The number of sensors included in $\mathbf{S}$
n^*	The optimal number of sensors included in $\mathbf{S}$
$\mathbb{Q}(z)$	The Gaussian Q-function of a random variable z [16]

A. Maximum A Posteriori (MAP) Energy Detection for Spectrum Sensing

In this paper, we follow the approach of MAP energy detection scheme describing in [9] as follows. By adopting the energy detection scheme [12], [5] for the spectrum sensing, each sensor i detects the presence of the primary user by the single-node detection and false alarm probabilities P_{di} and P_{fi}, respectively. This sensor receives the primary signal $r_i(t)$ in the following form [12]:

$$r_i(t) = \begin{cases} n_i(t) & \text{hyphothesis } H_0 \\ s_i(t) + n_i(t) & \text{hyphothesis } H_1 \end{cases} \quad (1)$$

where, H_0 and H_1 are the hypotheses corresponding to "no signal transmitted" and "signal transmitted", respectively. $s_i(t)$ is the received signal waveform and $n_i(t)$ is a zero-mean additive white Gaussian noise (AWGN). Hence P_{di} and P_{fi} are derived as follows [12]:

$$\begin{aligned} P_{di} &= P_r[Y_i > \lambda | H_1] \\ P_{fi} &= P_r[Y_i > \lambda | H_0] \end{aligned} \quad (2)$$

where λ is the energy detection threshold for every sensor. The test or decision statistic $Y_i \sim \mathbf{N}(\mu_i, \sigma_i^2)$ is the Chi-square distribution and can be approximated as a Gaussian distribution as [12], [9, Ref. 13]:

$$Y_i \sim \begin{cases} \mathbf{N}\left(u\sigma_{ni}^2, 2u\sigma_{ni}^4\right), & H_0 \\ \mathbf{N}\left(u(\sigma_{ni}^2 + \sigma_{si}^2), 2u(\sigma_{ni}^2 + \sigma_{si}^2)^2\right), & H_1 \end{cases}$$

where $u = 2t_sW$ is the number of samples. t_s is assumed to be the same for every sensor. σ_{ni}^2 and σ_{si}^2 are the variance of the noise $n_i(t)$ and the received signal $s_i(t)$, respectively. The SNR is derived as: $\gamma_i = \sigma_{si}^2/\sigma_{ni}^2$. Without loss of generality, the variance of the noise is assumed to be the same at every sensor and is simply denoted by σ_n. Thus, the tail probability of the Gaussian distribution, $P_r[Y > \lambda]$, can be derived in terms of the Gaussian $\mathbb{Q}$-function [16] as:

$$\begin{aligned} P_r[Y > \lambda] &= P_r\left[\frac{Y-\mu}{\sigma} > \frac{\lambda-\mu}{\sigma}\right] = \mathbb{Q}\left(\frac{\lambda-\mu}{\sigma}\right) \\ &\triangleq \mathbb{Q}(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-\frac{x^2}{2}} dx \end{aligned} \quad (3)$$

Then, P_{di} and P_{fi} can be easily derived as follows:

$$\begin{cases} P_{di} = \mathbb{Q}\left(\dfrac{\lambda - 2t_sW(\gamma_i+1)\sigma_n^2}{2\sqrt{t_sW}(\gamma_i+1)\sigma_n^2}\right) & (4) \\ P_{fi} = \mathbb{Q}\left(\dfrac{\lambda - 2t_sW\sigma_n^2}{2\sqrt{t_sW}\sigma_n^2}\right) & (5) \end{cases}$$

In addition, the traffic pattern of the primary user can be modeled as a two state independent and identically distributed (i.i.d) ON-OFF random process [17], whose ON and OFF periods are exponentially distributed with the means in terms of time as T_{on} and T_{off}, respectively. Hence, sensor i detects the monitored spectrum availability by the following single-node detection and false alarm probabilities [9]:

$$\begin{aligned} \hat{P}_{di} &= P_{on}.P_{di} = \frac{T_{on}}{T_{on}+T_{off}}.P_{di} \\ \hat{P}_{fi} &= P_{off}.P_{fi} = \frac{T_{off}}{T_{on}+T_{off}}.P_{fi} \end{aligned} \quad (6)$$

Since, P_{di} is monotonically increasing w.r.t the sensing interval and the SNR, the sensors that experience the lowest SNR will yield the lowest detection probabilities or the least accurate detection. Thus, by excluding these weak-SNR sensors from the spectrum sensing group, the total energy consumption might be reduced while the total detection probability is still kept high.

B. A Cooperative Scheme for Spectrum Sensing

As described earlier in Section I and Fig. 1, the fusion center performs the "OR-rule" [11] to derive the cooperative detection and false alarm probabilities Q_d and Q_f from aggregating the single-node detection and false alarm probabilities, which are estimated from the test or decision statistic (2) provided by the sensors in the cooperative sensing group $\mathbf{S}$, respectively. The decision on the occupancy of the monitoring spectrum will then be concluded by comparing Q_d and Q_f with the given thresholds for detection accuracy $\bar{Q}_d$ and $\bar{Q}_f$, respectively. By performing the "OR-rule", Q_d and Q_f can be derived as follows [5]:

$$\begin{aligned} Q_d &= 1 - \prod_{i=1}^{n}(1 - \hat{P}_{di}) \\ Q_f &= 1 - \prod_{i=1}^{n}(1 - \hat{P}_{fi}) \end{aligned} \quad (7)$$

where n is the number of the sensors in $\mathbf{S}$. The single-node probabilities $\hat{P}_{di}$ and $\hat{P}_{fi}$ derived by (6) are reported to the fusion center by each individual sensor i in the sensing group $\mathbf{S}$. This scheme shows that when n increases, Q_d will increase and as a consequence the accuracy of the primary user being detected also increases. However, the higher the value of n, the higher the cooperative false alarm probability Q_f which in turn causes a higher chance that a spectrum opportunity will be missed. In addition, the more sensors included in $\mathbf{S}$, the more energy is consumed for spectrum sensing, which is undesirable since the sensors have limited power resource. Hence, finding an optimal size of the group $\mathbf{S}$ is an important issue to be solved in this paper.

Furthermore, energy-efficient selection of the appropriate sensors to be included in $\mathbf{S}$ is also an important problem. For example, how to efficiently coordinate and select the sensors that experience the highest SNR and that are well separated from each other in order to avoid correlation shadowing in the cooperative spectrum sensing is an essential question. This issue is raised as the future work of this paper.

IV. Energy Minimizing in Cooperative Spectrum Sensing

A. Bound for the Number of Sensors

Given the thresholds $\bar{Q}_d$ and $\bar{Q}_f$ for cooperative detection and false alarm probabilities, respectively, the conditions $Q_d \geq \bar{Q}_d$ and $Q_f \leq \bar{Q}_f$ are needed to satisfy the detection accuracy and to be confident that a spectrum opportunity is not missed. Thus, the cooperative scheme (7) yields:

$$1 - \prod_{i=1}^{n}(1 - \hat{P}_{di}) \geq \bar{Q}_d \Leftrightarrow 1 - \bar{Q}_d \geq \prod_{i=1}^{n}(1 - \hat{P}_{di}) \quad (8)$$

$$1 - \prod_{i=1}^{n}(1 - \hat{P}_{fi}) \leq \bar{Q}_f \Leftrightarrow 1 - \bar{Q}_f \leq \prod_{i=1}^{n}(1 - \hat{P}_{fi}) \quad (9)$$

As denoted in Table I, $\hat{P}_d^{min}$ and $\hat{P}_f^{max}$ can be derived as:

$$\begin{aligned} \hat{P}_d^{min} &= P_{on}.\mathbb{Q}\left(\frac{\lambda - 2t_s W(\gamma^{min}+1)\sigma_n^2}{2\sqrt{t_s W}(\gamma^{min}+1)\sigma_n^2}\right) \\ \hat{P}_f^{max} &= \max\{\hat{P}_{fi}, \quad i = [1 \ldots n]\} \end{aligned} \quad (10)$$

where the minimum SNR: $\gamma^{min} = \min\{\gamma_i\}$. Then:

$$\begin{cases} (1 - \hat{P}_d^{min})^n \geq \prod_{i=1}^{n}(1 - \hat{P}_{di}) \\ (1 - \hat{P}_f^{max})^n \leq \prod_{i=1}^{n}(1 - \hat{P}_{fi}) \end{cases}$$

Hence, the conditions (8) and (9) will be satisfied if the following inequalities are kept:

$$\begin{cases} 1 - \bar{Q}_d \geq (1 - \hat{P}_d^{min})^n \\ 1 - \bar{Q}_f \leq (1 - \hat{P}_f^{max})^n \end{cases}$$

which require the bounds for n as follows:

$$\left\lceil \frac{\log(1 - \bar{Q}_d)}{\log(1 - \hat{P}_d^{min})} \right\rceil \leq n \leq \left\lfloor \frac{\log(1 - \bar{Q}_f)}{\log(1 - \hat{P}_f^{max})} \right\rfloor \quad (11)$$

where $\lceil . \rceil$ and $\lfloor . \rfloor$ denote the ceiling and flooring functions for the rounding of a real number to an integer, respectively.

The lower bound shows that the higher single-node detection probability, the fewer sensors are needed to guarantee a given threshold. More importantly, the higher the minimum SNR among the sensors, the fewer sensors are required. Thus, the fusion center should only invite the sufficiently high SNR sensors. Furthermore, the upper bound indicates an invaluable physical meaning on the specification of the threshold $\bar{Q}_f$. The threshold $\bar{Q}_f$ cannot be as low as possible, since the low $\bar{Q}_f$ requires the small number of sensors, which might break the detection accuracy by violating (11). Hence, the tradeoff in keeping $\bar{Q}_d$ high and $\bar{Q}_f$ low simultaneously is addressed in formulating the optimization problem in this paper.

B. Optimal Sensing Interval & Optimal Number of Sensors to Minimize the Energy Consumption

For energy efficiency, the lower bound for n in (11) is used as the minimum number of sensors included in the sensing group $\mathbf{S}$. However, it does not mean that n is optimal in terms of minimizing the total energy consumed by group $\mathbf{S}$ for cooperative spectrum sensing. Equation (4) shows that the longer the sensing interval t_s, the higher the detection accuracy, hence fewer sensors are needed and consequently less energy will be spent. On the other hand, the higher t_s, the more energy is consumed for spectrum sensing. This paper addresses that important tradeoff in formulating the energy minimization problem as follows.

Let δE^{ss} denote the sensing energy consumption per time unit during the spectrum sensing interval. δE^{ss} is assumed to be the same for every sensor in the network. Hence, during t_s, each sensor i consumes a sensing energy $\Delta E_i^{ss} = t_s \delta E^{ss}$. The minimization of the total sensing energy consumed by group $\mathbf{S}$ is then formulated as:

$$\begin{aligned} & \underset{t_s}{\text{Minimize:}} \quad \sum_{i=1}^{n} \Delta E_i^{ss} \triangleq n t_s \delta E^{ss} \\ \Leftrightarrow \quad & \underset{t_s}{\text{Minimize:}} \quad \frac{\log(1 - \bar{Q}_d)}{\log(1 - \hat{P}_d^{min})} t_s \delta E^{ss} \end{aligned} \quad (12)$$

Equation (12) can be further refined by the observation that the absolute function $|\log(1 - \hat{P}_d^{min})|$ is monotonically increasing w.r.t $\hat{P}_d^{min}$ as:

$$\begin{aligned} & \underset{t_s}{\text{Minimize:}} \quad |\log(1 - \bar{Q}_d)|.\delta E^{ss}.\frac{t_s}{|\log(1 - \hat{P}_d^{min})|} \\ \Leftrightarrow & \underset{t_s}{\text{Maximize:}} \quad \frac{1}{|\log(1 - \bar{Q}_d)|.\delta E^{ss}}.\frac{\hat{P}_d^{min}}{t_s} \\ \Leftrightarrow & \underset{t_s}{\text{Maximize:}} \quad \frac{P_{on}}{|\log(1 - \bar{Q}_d)|.\delta E^{ss}}.\frac{\mathbb{Q}\left(\frac{\lambda - 2t_s W(\gamma^{min}+1)\sigma_n^2}{2\sqrt{t_s W}(\gamma^{min}+1)\sigma_n^2}\right)}{t_s} \end{aligned}$$

Without loss of generality, it is assumed that δE^{ss}, T_{on}, and T_{off} are known and independent of the sensing interval and that $\bar{Q}_d$ is given. Thus, the optimal sensing interval t_s^* that minimizes the total sensing energy consumed by the cooperative spectrum sensing group $\mathbf{S}$ can be solved by the following maximization problem:

$$t_s^* = \arg\max_{t_s} \frac{\mathbb{Q}\left(\frac{\lambda - 2t_s W(\gamma^{min}+1)\sigma_n^2}{2\sqrt{t_s W}(\gamma^{min}+1)\sigma_n^2}\right)}{t_s} \quad (13)$$

subject to:

$$c_1: \quad t_s \geq 0 \quad (13a)$$

$$c_2: \quad n \leq \frac{\log(1-\bar{Q}_f)}{\log(1-\hat{P}_f^{max})} \quad (13b)$$

where:

$$n = \frac{\log(1-\bar{Q}_d)}{\log(1-\hat{P}_d^{min})}$$

$$\hat{P}_d^{min} = P_{on}.\mathbb{Q}\left(\frac{\lambda - 2t_s W(\gamma^{min}+1)\sigma_n^2}{2\sqrt{t_s W}(\gamma^{min}+1)\sigma_n^2}\right)$$

$$\hat{P}_f^{max} = P_{off}.\mathbb{Q}\left(\frac{\lambda - 2t_s W\sigma_n^2}{2\sqrt{t_s W\sigma_n^4}}\right)$$

Obviously, solving (13) directly and analytically is extremely difficult due to the exponential characteristic of the $\mathbb{Q}$-function. Hence, Section V presents an approximation approach to efficiently solve this optimization problem.

C. Discussion on the Optimization's Constraint

As discussed earlier in subsection IV-A, there is a tradeoff in satisfying a high threshold $\bar{Q}_d$ for the cooperative detection probability and a low threshold $\bar{Q}_f$ for the cooperative false alarm probability at the same time. The meaning of this tradeoff indicates in keeping the upper bound of n (w.r.t a given threshold $\bar{Q}_f$) satisfying the constraint (13)b of the optimization problem (13). The detailed discussion on this issue is presented as follows. Recall the constraint (13)b as:

$$n^* = \frac{\log(1-\bar{Q}_d)}{\log(1-\hat{P}_d^{*min})} \leq \frac{\log(1-\bar{Q}_f)}{\log(1-\hat{P}_f^{*max})}$$

The following transformations are then derived to reason about the specific requirement of the threshold $\bar{Q}_f$:

$$\log(1-\bar{Q}_f) \leq \frac{\log(1-\hat{P}_f^{*max})}{\log(1-\hat{P}_d^{*min})}.\log(1-\bar{Q}_d)$$

$$\Leftrightarrow \quad e^{\log(1-\bar{Q}_f)} \leq e^{\frac{\log(1-\hat{P}_f^{*max})}{\log(1-\hat{P}_d^{*min})}.\log(1-\bar{Q}_d)}$$

$$\Leftrightarrow \quad \bar{Q}_f \geq \tilde{Q}_f^* = 1-(1-\bar{Q}_d)^{\frac{\log(1-\hat{P}_f^{*max})}{\log(1-\hat{P}_d^{*min})}} \quad (14)$$

where t_s^* is the optimal sensing interval of (13) and:

$$\begin{cases} \hat{P}_d^{*min} = P_{on}.\mathbb{Q}\left(\frac{\lambda - 2t_s^* W(\gamma^{min}+1)\sigma_n^2}{2\sqrt{t_s^* W}(\gamma^{min}+1)\sigma_n^2}\right) \\ \hat{P}_f^{*max} = P_{off}.\mathbb{Q}\left(\frac{\lambda - 2t_s^* W\sigma_n^2}{2\sqrt{t_s^* W}\sigma_n^2}\right) \end{cases}$$

The inequality (14) indicates that with a given threshold $\bar{Q}_d$ for the cooperative detection probability, the requirement threshold $\bar{Q}_f$ for the cooperative false alarm probability must be lower-bounded by $\tilde{Q}_f^*$ in order to hold the optimality of the proposed optimization problem.

V. Analytical Solutions for the Energy Minimization Problem

This section presents an approximation approach to accurately solve the optimization (13) in polynomial time. Recall the formulation of the Gaussian $\mathbb{Q}$-function as [16]:

$$\mathbb{Q}(z) \triangleq \frac{1}{\sqrt{2\pi}}\int_z^\infty e^{-\frac{x^2}{2}}dx \quad (15)$$

$$\text{where: } z = \frac{\lambda - 2t_s W(\gamma^{min}+1)\sigma_n^2}{2\sqrt{t_s W}(\gamma^{min}+1)\sigma_n^2}$$

The exponential characteristic of $\mathbb{Q}(z)$ implies that solving (13) analytically is extremely difficult. Hence, approximation approaches can be proposed to make (13) solvable. For example, [15] considers these approximations:

$$\mathbb{Q}(z) \approx \begin{cases} \frac{1}{2}e^{-z^2/2} & \text{if } z \geq 0 \quad (16) \\ 1-\frac{1}{2}e^{-z^2/2} & \text{if } z < 0 \quad (17) \end{cases}$$

and shows that when z is positive, t_s^* can be found as:

$$t_s^* = \frac{1}{W}\left[\sqrt{\frac{\lambda^2}{4(\gamma^{min}+1)^2\sigma_n^4}+1}-1\right]$$

However, this approximation produces around 20% error compared to the exact result of the original optimization, which is mainly due to the high inaccuracy of the approximation (16) for $\mathbb{Q}(z>0)$. In the following, more accurate and polynomial time analytical solutions are derived.

A. Linearization when $0.5 \geq z \geq -0.5$

It is observed from the $\mathbb{Q}$-function that its curvature is close to linear when z varies from -0.5 to 0.5. The following linearization is proposed for that variation of z to accurately and analytically solve the optimization (13). Recall the partial derivative (*Slope*) of the $\mathbb{Q}$-function as:

$$Slope \triangleq \frac{\partial(\mathbb{Q}(z))}{\partial z} = -\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$$

The curvature of the linearization can be approximated as $Slope(z=0) = -\frac{1}{\sqrt{2\pi}}$, hence the linearization is derived:

$$\mathbb{Q}(0.5 \geq z \geq -0.5) \approx \frac{1}{2} - \frac{z}{\sqrt{2\pi}} \quad (18)$$

Substituting (18) into (13) and following the transformations in Appendix A, t_s^* can be found analytically as:

$$t_s^* = \frac{1}{W}\left[-\pi\sqrt{1+\frac{3\lambda}{\pi(\gamma^{min}+1)\sigma_n^2}}+\pi+\frac{3\lambda}{2(\gamma^{min}+1)\sigma_n^2}\right] \quad (19)$$

B. Approximation when $z > 0.5$

This approximation is derived similar to (16) as:

$$\mathbb{Q}(z > 0.5) \approx \frac{1}{2} e^{-\frac{(z+0.5)^2}{2}} \tag{20}$$

Then, following the derivations in Appendix B in solving the optimization (13), $t_s^* = \frac{1}{W} u^*$ can be found as the root of the following polynomial of degree four:

$$p_4.u^4 + p_3.u^3 + p_2.u^2 + p_1.u + p_0 = 0 \tag{21}$$

where:

$$u = t_s W, \text{ and } A = 2(\gamma^{min} + 1)\sigma_n^2$$
$$p_4 = 4A^4$$
$$p_3 = 15A^4$$
$$p_2 = 2A^2(8A^2 - 4\lambda^2 - \lambda A)$$
$$p_1 = 17\lambda^2 A^2$$
$$p_0 = 4\lambda^4$$

C. Approximation when $z < -0.5$

When $z < -0.5$, (17) cannot be used in solving (13). Thus, this subsection focuses on finding an approximation that has the similar *Slope* as that of the original $\mathbb{Q}$-function. Since $\mathbb{Q}(z < 0)$ is monotonically increasing when z is decreasing below zero, then the following approximation is proposed when $z < -0.5$:

$$\mathbb{Q}(z < 0) \approx z.Slope = -\frac{z}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \tag{22}$$

Hence, following the derivations in Appendix C, the approximated optimal sensing interval can be found as the root of the following cubic function:

$$u^3 + a.u^2 + b.u + c = 0 \tag{23}$$

where:

$$u = t_s W$$
$$a = \frac{2(\gamma^{min} + 1)\sigma_n^2 - \lambda}{2(\gamma^{min} + 1)\sigma_n^2}$$
$$b = -\frac{\lambda^2 + 6\lambda(\gamma^{min} + 1)\sigma_n^2}{(2(\gamma^{min} + 1)\sigma_n^2)^2}$$
$$c = \left(\frac{\lambda}{2(\gamma^{min} + 1)\sigma_n^2}\right)^3$$

VI. Numerical Results

This section presents numerical calculation to validate the *approximated optimal results* solved by the proposed approach compared with the *exact optimal results* of the optimization. The minimum energy consumption is also validated comparing with the *random energy* consumed by the random group formed by a random number of sensors. In all comparisions, the exact optimal results are numerically estimated from (13) in Matlab. *Mean absolute error (MAE)* is used to validate of the accuracy of the approximated optimal results.

In all the calculations, the following setting are used: $\delta E^{ss} = 0.05$ J; the ON-OFF period of the primary user is modeled as $T_{on} = 1$ s, $T_{off} = 2$ s [9]. The monitored spectrum bandwidth is $W = 10$ kHz. The energy detection threshold and the ground noise are chosen as $\lambda = 4.5$ dB and $\sigma_n = -10$ dB, respectively. The detection accuracy thresholds $\bar{Q}_d = 0.9$, and $\bar{Q}_f = 0.1$ are followed the IEEE P802.22 standard. The performance of the proposed optimization and approximation approach is validated through a wide range of the minimum SNR from -30 dB to 50 dB.

Firstly, the minimum energy consumption is validated comparing with the energy consumed by a random group formed by randomly selecting the number of sensors from the range $[1, 60]$. This range is similar to that of the approximated optimal number of sensor solved by the proposed approximated solutions. Even that the sensing interval yielded by the random case (the thick-dashed curves in Fig. 3) is sometimes smaller than the optimal value, the optimization always produces the minimum energy consumption as shown in Fig. 2. The total

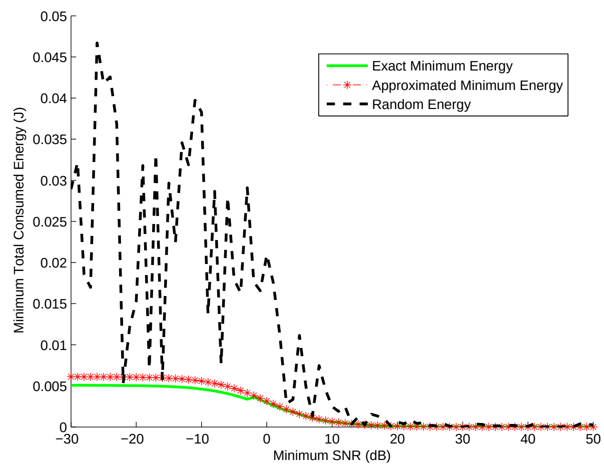

Fig. 2. Comparing the total energy consumption.

energy consumed by the optimal group of sensors is around 333.97% less than the total energy consumed by the random group. Huge energy saving under very high SNR condition is yielded since the proposed optimization produces much shorter optimal sensing interval.

Fig. 2 also validates the accuracy of the approximated solutions compared with the exact results as shown in the dash-dot-asterisk curve and the solid curve, respectively. The MAE between these curves is around 8.82%, which is caused mainly under very low SNR conditions. The results also show that when the minimum SNR increases, less energy is consumed, which confirms the observation discussed earlier on the influence of the SNR condition to the cooperative spectrum sensing problem. In particular, the high energy consumption under very low SNR condition implies the weakness of the energy detector scheme at low SNR.

The validation of the accuracy of the proposed solutions is also presented in Fig. 3 where the approximated optimal sensing interval (the dash-dot-asterisk curve) is very close to the exact optimal results (the solid curve). It shows here again that the higher the minimum SNR, the shorter time the sensing group performs spectrum sensing while still satisfying

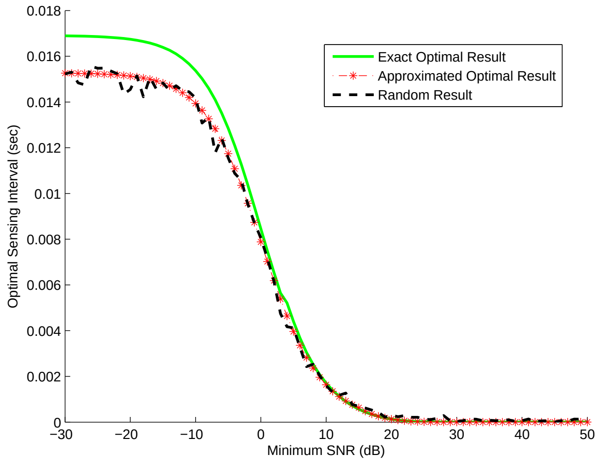

Fig. 3. Comparing the sensing interval.

the required detection accuracy. In addition, in the range -30 dB to 50 dB for the minimum SNR, the MAE produced by the proposed solutions (23), (19), and (21) are around 8.21%, 4.0%, and 2.13%, respectively. The highest error caused under the lowest SNR condition.

Fig. 4 presents the approximated optimal number of sensors (the dashed bar) and the exact optimal result (the solid bar). It shows that the difference between the approximated and the

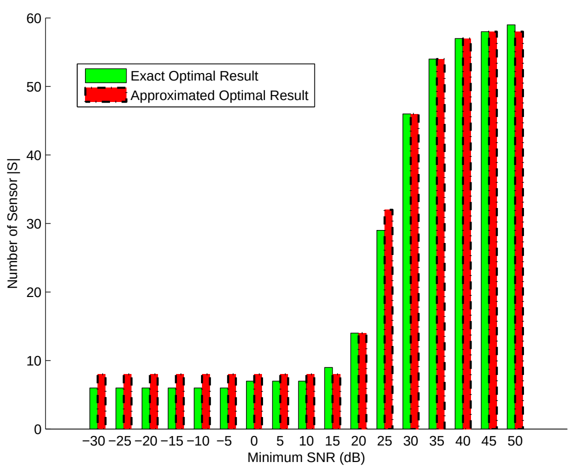

Fig. 4. The optimal number of sensor.

exact results is very small. It is also observed that the better the SNR condition the higher optimal number of sensors. The reason is that the proposed energy minimization produces much shorter optimal sensing interval under the good SNR condition. However, it needs to guarantee the given detection accuracy, hence the sufficient number of sensors will need to be included during the optimization as shown in Fig. 4.

Finally, figures 5 and 6 show the validations of the minimum cooperative detection probability and the maximum false alarm probability yielded by the optimization. The result in Fig. 5 indicates that the proposed solutions produce accurate detection by keeping the minimum cooperative detection probability above a given threshold. Fig. 6, on the other hand, restates the tradeoff in keeping very low cooperative false alarm

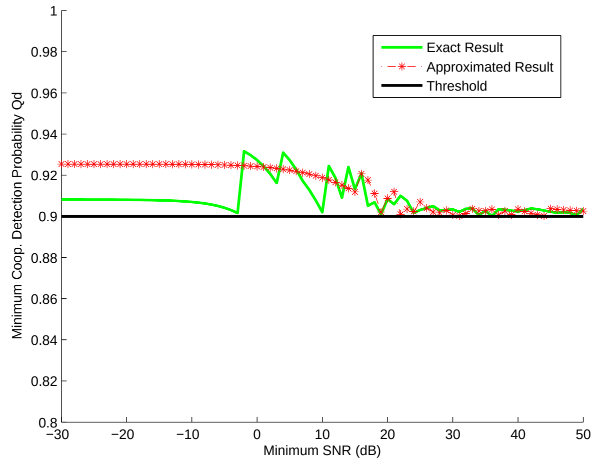

Fig. 5. The minimum cooperative detection probability Q_d^*.

probability when the SNR condition is very bad. It suggests

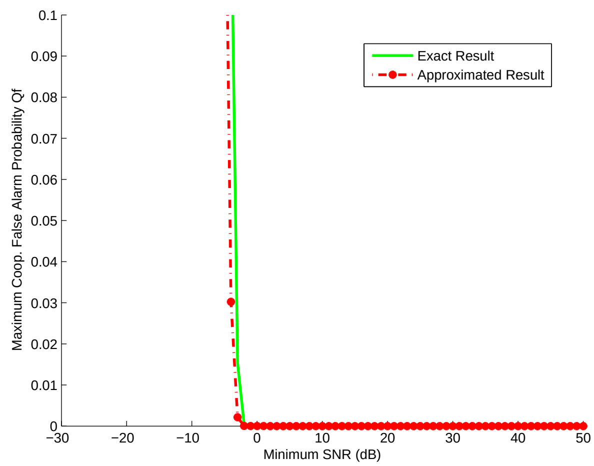

Fig. 6. The maximum cooperative false alarm probability.

that the fusion center should either only invite the high SNR sensors for cooperative spectrum sensing or accept higher tolerance for the cooperative false alarm probability under very low SNR situations.

VII. Conclusion

This paper has studied the minimization of the total energy consumed by a group of power-constraint sensors for cooperative spectrum sensing in sensor-aided cognitive radio networks. Firstly, the lower bound and upper bound for the number of sensors are found under the detection accuracy thresholds. Then, with the derived bounds, the optimization problem to minimize the total sensing energy consumption is formulated. Next, the approximated analytical solutions are found to solve the optimization accurately and efficiently in polynomial time. Finally, numerical calculations show that the minimized energy is significantly lower than the energy consumed by a group of randomly selected sensors. The approximated optimal number of sensors is shown to be very close to the exact number. Under good SNR conditions, the mean absolute error of the approximated optimal sensing interval is less than 4% compared to the exact value. In the worst SNR conditions, this error is around 8%. It has also observed that the energy detector scheme does not perform well at very low SNR conditions, which cause higher energy consumption for cooperative spectrum sensing.

This paper has also observed and discussed that energy-efficient selection of the appropriate sensors to be included in the cooperative spectrum sensing group is an important problem. The question of how to efficiently coordinate and select the sensors that experience the highest SNR and that are well separated from each other in order to avoid correlation shadowing in cooperative spectrum sensing is addressed as the future direction of this paper.

APPENDIX A
PROOF OF THE OPTIMAL RESULT (19)

Proof: The first derivative of (13) can be derived as:

$$\frac{\partial\left(\frac{\mathbb{Q}(z)}{t_s}\right)}{\partial t_s}=0 \quad \Leftrightarrow \quad \frac{\frac{\partial(\mathbb{Q}(z))}{\partial z}.\frac{\partial z}{\partial t_s}.t_s-\mathbb{Q}(z)}{t_s^2}=0$$
$$\Leftrightarrow \quad \frac{\partial(\mathbb{Q}(z))}{\partial z}.\frac{\partial z}{\partial t_s}.t_s=\mathbb{Q}(z) \qquad \text{(A.1)}$$

where $z=\frac{\lambda-2t_sW(\gamma^{min}+1)\sigma_n^2}{2\sqrt{t_sW(\gamma^{min}+1)\sigma_n^2}}$. Then this is derived:

$$\frac{\partial z}{\partial t_s}t_s=\frac{\partial\left(\frac{\lambda-2t_sW(\gamma^{min}+1)\sigma_n^2}{2\sqrt{t_sW(\gamma^{min}+1)\sigma_n^2}}\right)}{\partial t_s}t_s$$
$$=\frac{\frac{\partial\left(\lambda-2t_sW(\gamma^{min}+1)\sigma_n^2\right)}{\partial t_s}.2\sqrt{t_sW}(\gamma^{min}+1)\sigma_n^2}{\left(2\sqrt{t_sW}(\gamma^{min}+1)\sigma_n^2\right)^2}t_s-\frac{\left(\lambda-2t_sW(\gamma^{min}+1)\sigma_n^2\right).\frac{\partial\left(2\sqrt{t_sW}(\gamma^{min}+1)\sigma_n^2\right)}{\partial t_s}}{\left(2\sqrt{t_sW}(\gamma^{min}+1)\sigma_n^2\right)^2}t_s$$
$$=\frac{-2W(\gamma^{min}+1)\sigma_n^2.2\sqrt{t_sW}(\gamma^{min}+1)\sigma_n^2}{4W(\gamma^{min}+1)^2\sigma_n^4}-\frac{\left(\lambda-2t_sW(\gamma^{min}+1)\sigma_n^2\right).\frac{W}{\sqrt{t_sW}}(\gamma^{min}+1)\sigma_n^2}{4W(\gamma^{min}+1)^2\sigma_n^4}$$
$$=-\frac{\lambda+2t_sW(\gamma^{min}+1)\sigma_n^2}{4\sqrt{t_sW}.(\gamma^{min}+1)\sigma_n^2} \qquad \text{(A.2)}$$

Using the approximation (18) yields the following:

$$\frac{\partial(\frac{1}{2}-\frac{z}{\sqrt{2\pi}})}{\partial z}.\frac{\partial z}{\partial t_s}.t_s=\frac{1}{2}-\frac{z}{\sqrt{2\pi}}$$
$$\Leftrightarrow \quad z-\frac{\partial z}{\partial t_s}.t_s=\frac{\sqrt{2\pi}}{2} \qquad \text{(A.3)}$$

Denote $A\triangleq 2(\gamma^{min}+1)\sigma_n^2$, then (A.3) is further derived by substituting (A.2) as:

$$\frac{\lambda-t_sWA}{\sqrt{t_sWA}}+\frac{\lambda+t_sWA}{2\sqrt{t_sWA}}=\frac{\sqrt{2\pi}}{2}$$
$$\Leftrightarrow \quad \frac{3\lambda-t_sWA}{\sqrt{t_sWA}}=\sqrt{2\pi}$$
$$\Leftrightarrow \quad (3\lambda-t_sWA)^2=2\pi t_sWA^2$$
$$\Leftrightarrow \quad [t_sWA-(3\lambda+\pi A)]^2=6\lambda\pi A+\pi^2A^2$$

Finally, the optimal sensing interval is found as:

$$t_s^*=\frac{1}{W}\left[-\pi\sqrt{1+\frac{3\lambda}{\pi(\gamma^{min}+1)\sigma_n^2}}+\pi+\frac{3\lambda}{2(\gamma^{min}+1)\sigma_n^2}\right] \qquad \text{(A.4)}$$

■

APPENDIX B
PROOF OF THE OPTIMAL RESULT (21)

Proof: Substituting (20) into (A.1) yields:

$$\frac{\partial(\frac{1}{2}e^{-\frac{(z+0.5)^2}{2}})}{\partial z}.\frac{\partial z}{\partial t_s}.t_s=\frac{1}{2}e^{-\frac{(z+0.5)^2}{2}}$$
$$\Leftrightarrow \quad -(z+\frac{1}{2}).\frac{\partial z}{\partial t_s}.t_s=1 \qquad \text{(B.1)}$$

By denoting $A\triangleq 2(\gamma^{min}+1)\sigma_n^2$ and substituting (A.2) into (B.1), t_s^* can be found as follows:

$$\left(\frac{\lambda-t_sWA}{\sqrt{t_sWA}}+\frac{1}{2}\right)\left(\frac{\lambda+t_sWA}{2\sqrt{t_sWA}}\right)=1$$
$$\Leftrightarrow \quad \frac{\lambda+t_sWA}{4\sqrt{t_sWA}}=1-\frac{\lambda^2-(t_sWA)^2}{2t_sWA^2}$$
$$\Leftrightarrow \quad \sqrt{t_sWA}(\lambda+t_sWA)=2(t_sWA)^2+4t_sWA^2-2\lambda^2$$
$$\Leftrightarrow \quad t_sWA^2(\lambda+t_sWA)^2=\left[2(t_sWA)^2+4t_sWA^2-2\lambda^2\right]^2$$
$$\Leftrightarrow \quad p_4(t_sW)^4+p_3(t_sW)^3+p_2(t_sW)^2+p_1t_sW+p_0=0$$

where:

$$\begin{array}{llll} p_4= & 4A^4 & p_3= & 15A^4 \\ p_2= & 2A^2(8A^2-4\lambda^2-\lambda A) & & \\ p_1= & 17\lambda^2A^2 & p_0= & 4\lambda^4 \end{array} \qquad \text{(B.2)}$$

■

APPENDIX C
PROOF OF THE OPTIMAL RESULT (23)

Proof: When $z<-0.5$, (22) is used, then:

$$\frac{\partial(-\frac{z}{\sqrt{2\pi}}e^{-\frac{z^2}{2}})}{\partial z}.\frac{\partial z}{\partial t_s}.t_s=-\frac{z}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$$
$$\Leftrightarrow \quad \left[-\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}(1-z^2)\right].\frac{\partial z}{\partial t_s}.t_s=-\frac{z}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$$
$$\Leftrightarrow \quad \left[1-z^2\right].\frac{\partial z}{\partial t_s}.t_s=z \qquad \text{(C.1)}$$

Substituting (A.2) into (C.1) leads to the following result:

$$\left[(\frac{\lambda - 2t_sW(\gamma^{min}+1)\sigma_n^2}{2\sqrt{t_sW}(\gamma^{min}+1)\sigma_n^2})^2 - 1 \right] . \left[\frac{\lambda + 2t_sW(\gamma^{min}+1)\sigma_n^2}{4\sqrt{t_sW}.(\gamma^{min}+1)\sigma_n^2} \right]$$
$$= \frac{\lambda - 2t_sW(\gamma^{min}+1)\sigma_n^2}{2\sqrt{t_sW}(\gamma^{min}+1)\sigma_n^2}$$

Denoting $A \triangleq 2(\gamma^{min}+1)\sigma_n^2$, then:

$$\left[(\frac{\lambda - t_sWA}{\sqrt{t_sW}A})^2 - 1 \right] . \left[\frac{\lambda + t_sWA}{2\sqrt{t_sW}A} \right] = \frac{\lambda - t_sWA}{\sqrt{t_sW}A}$$
$$\Leftrightarrow \quad \frac{\lambda^2 - t_s(2\lambda WA + WA^2) + ts^2W^2A^2}{t_sWA^2} . \frac{\lambda + t_sWA}{2}$$
$$= \lambda - t_sWA$$
$$\Leftrightarrow \quad \left(\lambda^2 - t_s(2\lambda WA + WA^2) + ts^2W^2A^2\right).(\lambda + t_sWA)$$
$$= 2t_sWA^2(\lambda - t_sWA)$$
$$\Leftrightarrow \quad t_s^3W^3A^3 + t_s^2(W^2A^3 - \lambda W^2A^2) - t_s(\lambda^2WA + 3\lambda WA^2)$$
$$+ \lambda^3 = 0$$
$$\Leftrightarrow \quad u^3 + u^2\left(\frac{A-\lambda}{A}\right) - u\left(\frac{\lambda^2 + 3\lambda A}{A^2}\right) + \left(\frac{\lambda}{A}\right)^3 = 0$$
$$\Leftrightarrow \quad u^3 + au^2 + bu + c = 0 \quad \text{, where: } u = t_sW \qquad \text{(C.2)}$$

Thus, t_s^* is solved as a root of (C.2) as follows:

$$t_s^* = \frac{1}{W}\left[\max(x_1, x_2, x_3)\right] \quad \text{, where:}$$
$$\begin{cases} x_1 = s + t - \frac{b}{3a} \\ x_2 = -\frac{1}{2}(s+t) - \frac{b}{3a} + \frac{\sqrt{3}}{2}(s-t)j \\ x_3 = -\frac{1}{2}(s+t) - \frac{b}{3a} - \frac{\sqrt{3}}{2}(s-t)j \end{cases}$$
$$\begin{cases} s = (r + \sqrt{q^3 + r^2})^{\frac{1}{3}} \\ t = (r - \sqrt{q^3 + r^2})^{\frac{1}{3}} \end{cases} \quad \text{and} \quad \begin{cases} q = \frac{3ac - b^2}{9a^2} \\ r = \frac{9abc - 27a^2d - 2b^3}{54a^3} \end{cases}$$

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