Real-time flow simulation

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Real-time flow simulation

Goal

Reduce computation time from hours/days to seconds/minutes.

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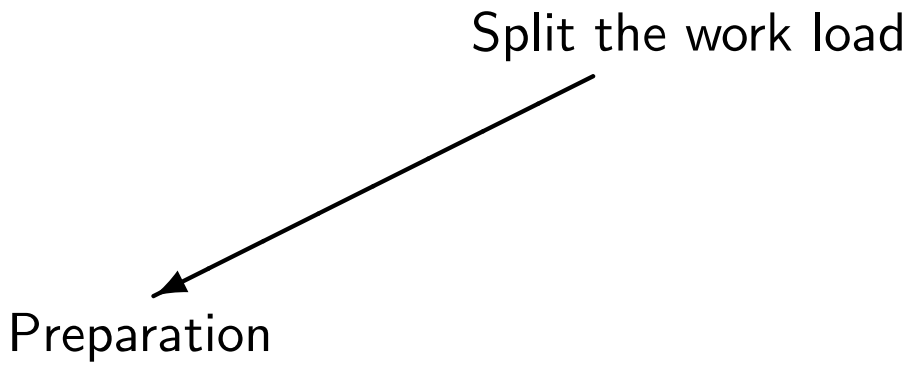
Main idea



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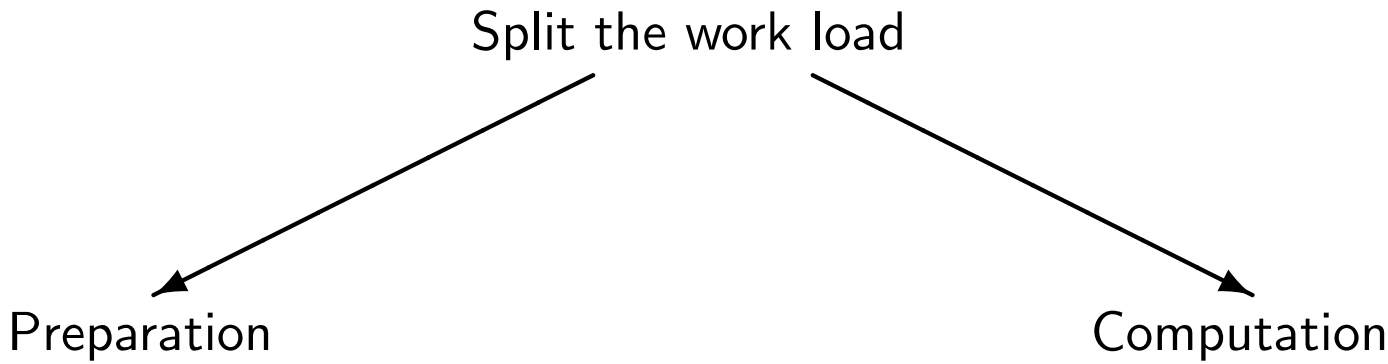
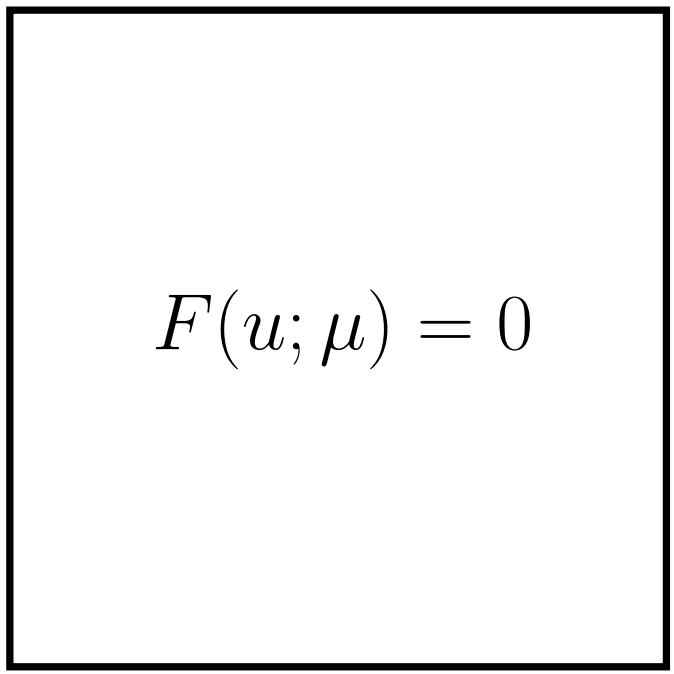


The reduced basis method



For a parameter dependent problem in a given domain:

$$F(u;\mu) = 0 \quad \text{in } \Omega,$$

we assume that for small changes in the parameter μ, the corresponding solution u varies in a smooth fashion.

The reduced basis method

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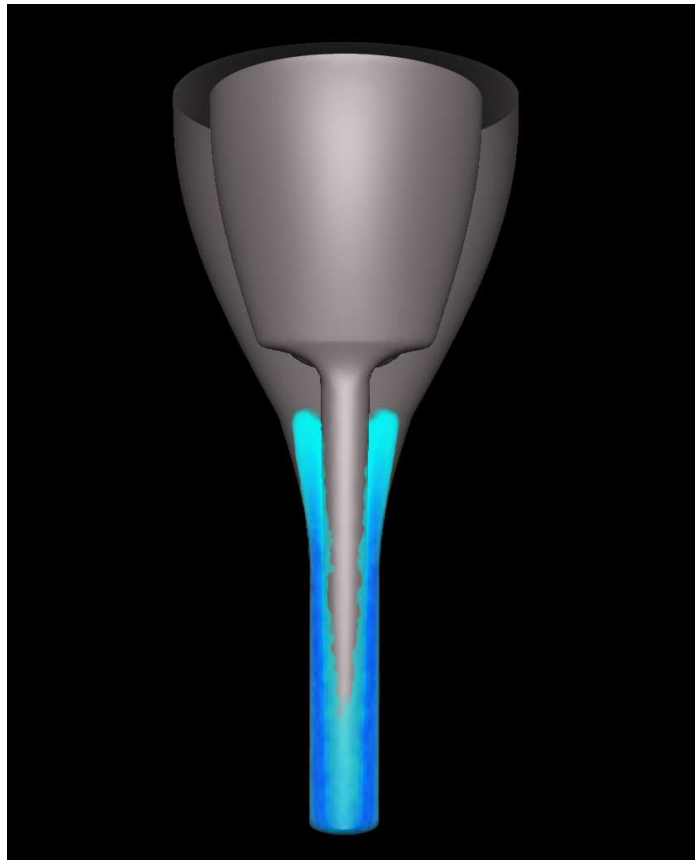

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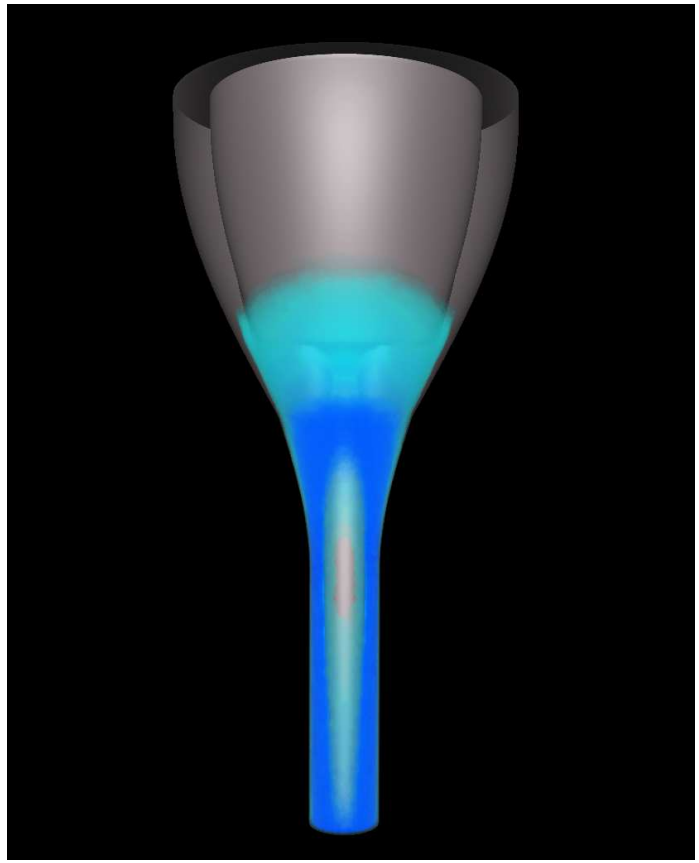

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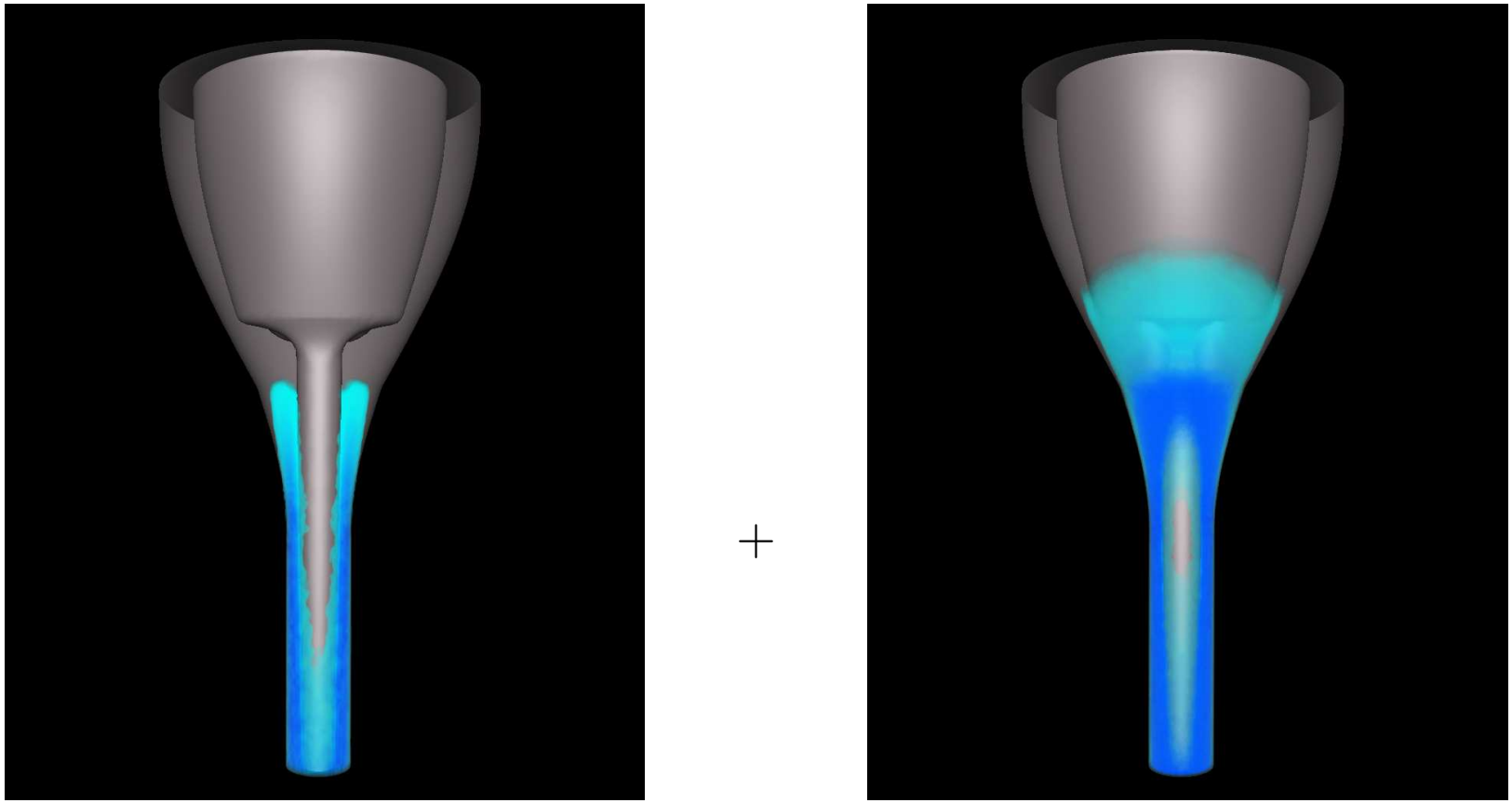
The reduced basis method

Store the solutions found for different values of μ.

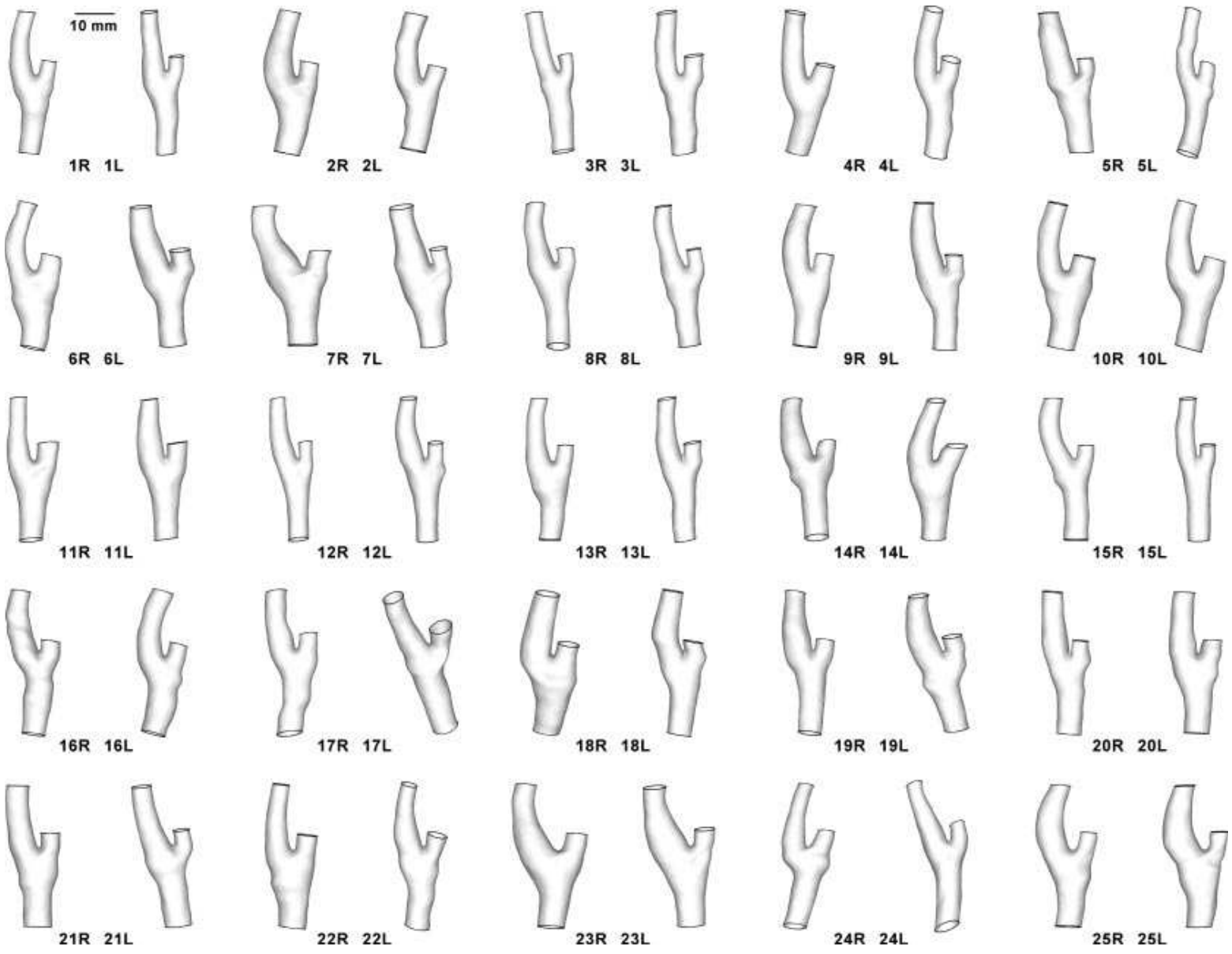


For a new μ, find the reduced basis approximation

$$u_N(\mu) = \sum_{i=1}^{N} \alpha_i(\mu) u_i \quad \text{such that } F(u_N; \mu) = 0 \text{ in } \Omega,$$

by determining appropriate coefficients α_i.

The reduced basis method: Sample geometries



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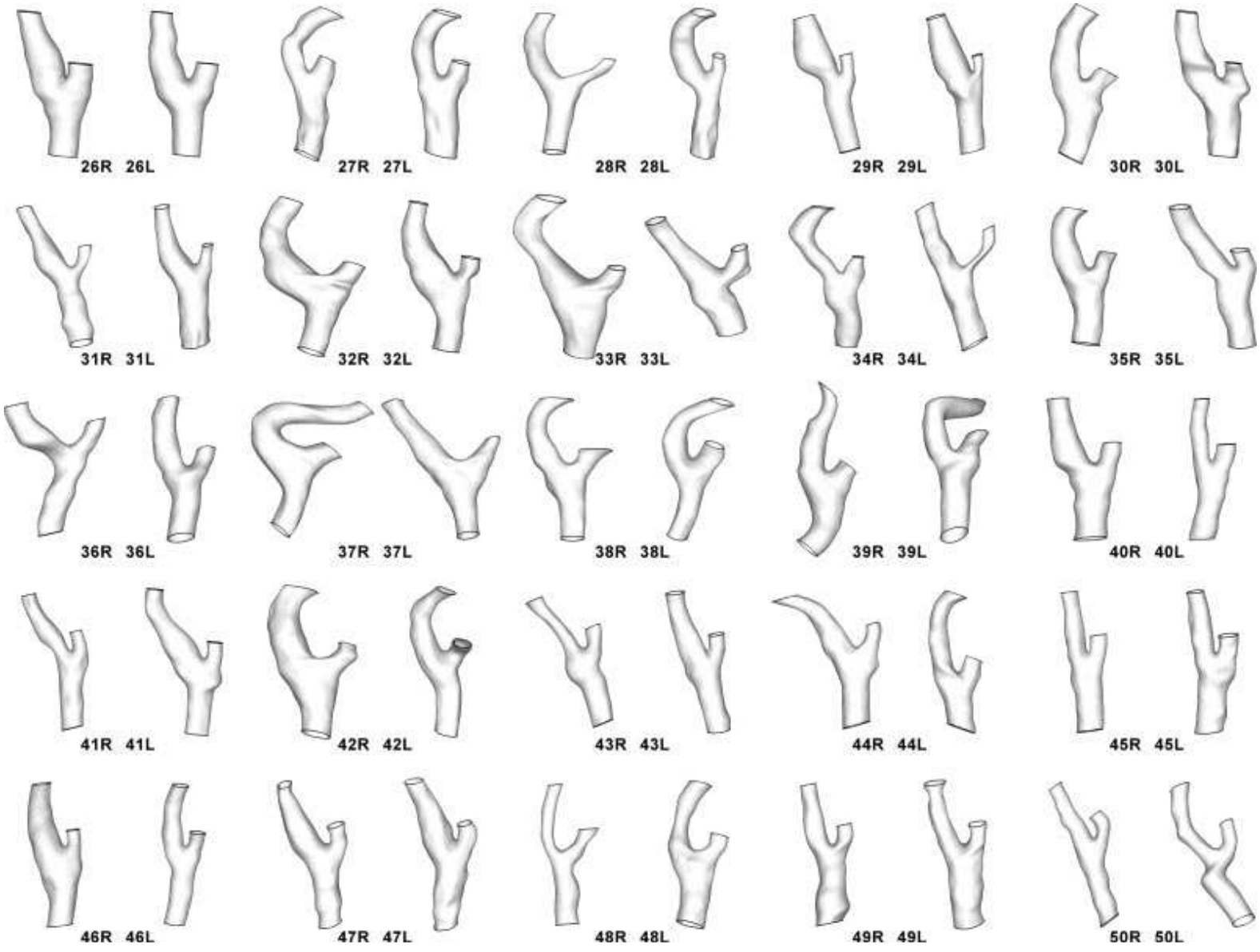

The reduced basis method

Main idea

Preparation stage:

- Choose an appropriate parameter space, $\mu_i \in S_N$.
- Compute and store the corresponding basis functions, u_i.
- Do a little magic. (Orthonormalization, inner products, etc.)

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Computation stage:

- Identify the parameter μ.
- Load and assemble the stored components.
- Solve the reduced system.
- Evaluate the error.

Potential benefits:

- Improved understanding of CSF flow, pressure and forces in different geometries.
- Assist examination.
- Assist surgery planning: optimize the geometry.
- Assist surgery: real-time simulation.

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Our interest:

- Measurements of geometries, before and after surgery.
- Measurements of velocities, both for boundary conditions and for verification of computations.
- Measurements of pressures.
- Color Doppler Ultrasonography.
- Real life problems and challenges.

Thank you!