# Energy penalties for non-shortest paths in wireless sensor networks with link failures

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#### **ABSTRACT**

This paper addresses the additional energy consumption in wireless sensor networks where the communication between the sensor nodes and the sink nodes does not always make use of the shortest path, due to the presence of link failures. For simplicity, link failures are assumed to be stochastic and independent. The basis for the analysis is a planned topology, with multiple sinks deployed spatially apart, in order to optimise the probability of reporting sensed data over the shortest path. The extra amount of energy consumed by the occurrence of link failures and transmission errors is then analysed. In addition to analysing the steady-state availability of the shortest path, the time dependent behaviour is also found. Simulations also support these results.

# **Categories and Subject Descriptors**

C.2.1 [Computer-Communications Network ]: Network Architecture and Design—Wireless communication

#### **General Terms**

Performance, Reliability

#### Keywords

Wireless sensor network, energy consumption, multiple sinks.

#### 1. INTRODUCTION

A wireless sensor network (WSN) consists of a large number of intelligent sensor nodes distributed over a widely spread geographical area. The sensors detect and measure a certain (target) phenomenon via its changing parameters [12], e.g. to provide real-time information about environmental conditions. WSNs are typically applied for military operations, area surveillance, environmental monitoring, remote sensing, and global awareness. The nodes in a sensor network are typically battery-powered and thus energy constrained, so node failures caused by power outage are

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IWCMC'09 June 21-24, Leipzig, Germany. Copyright 2009 ACM 978-1-60558-569-7/09/06 ...\$5.00. common. Thus, a main design goal of a WSN is to reduce the energy drain of the sensors.

In wireless networks radio-links are vulnerable to failures caused by fading, signal attenuation, radio interference, background noise and other inherent characteristics of the wireless medium [13, 11]. In most cases this has a negative impact on the network reliability and availability. Link failures are typically of temporary nature, but have an effect on the energy usage in a WSN. This is because if a link failure breaks the shortest path between a sensor node and a sink, the data might have to be relayed via a longer path. This causes additional energy to be consumed, since more relaying nodes are involved in the forwarding. Also, packet loss caused by transmission errors might trigger a series of retransmissions, draining additional energy.

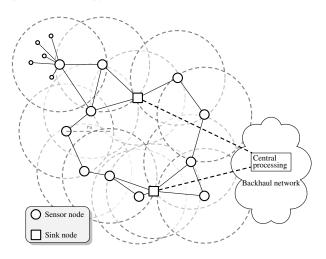


Figure 1: A sensor network with multiple sink nodes.

Figure 1 illustrates a wireless sensor network with multiple sink nodes. Typically, each sensor node is connected to a sink node, and the sensor node's traffic is forwarded along the shortest path between the sensor node and the sink node. However, if a link in the shortest path becomes unavailable, a routing protocol can ensure that a new path between the sensor node and a sink node is formed. The new path length is either equal to, or longer than the the shortest path.

The main contribution of this paper is an analysis of how link failures and packet transmissions over non-shortest paths result in increased energy consumption. Furthermore, a method to forecast how the introduction of multiple sink nodes affects the sensor energy consumption is also proposed. The usefulness of such a method is evident. Moreover, a method to estimate the Mean Time Between Failure (MTBF) of the shortest path is presented. Thus, it is possible to find the duration in which the shortest-path is unavailable, i.e. the Mean Down Time (MDT). Combining this with the sensor nodes' reporting rate might provide information about the lifetime of a WSN in terms of energy consumption.

The rest of the paper is organised as follows: Section 2 describes the assumptions of the analysis. Section 3 provides the wireless sensor network model in addition to presenting a measure for analysing the energy consumption when reporting sensed data over non-shortest path. The section also evaluates the effect of multiple sink nodes for an example topology. The time-dependency of the energy consumption is addressed in Section 4. This section describes how to obtain the mean time between failure and the mean down time for the shortest path availability in a WSN. Section 5 presents the most relevant related work, and finally, Section 6 concludes the paper.

# 2. ASSUMPTIONS

#### 2.1 The network infrastructure

The WSN sensors can either be organised in a flat architecture where the sensors relay each other's data in multiple hops towards a sink node, or in a hierarchical architecture where sensors form clusters and report their data to a cluster head which is responsible to send the aggregated data to a sink node. The sink node then forwards the sensed data to a central processing station (Figure 1). The WSN network architecture analysed in this paper is assumed to be flat.

The communication subsystem of a sensor normally has a major impact on the power consumption. In general, the power consumption of the transmitting circuitry is greater than the receiving circuitry, i.e. the Mica2 module requires 27mA transmitting at max power and 10mA to receive [16]. For simplicity, this paper assumes that the energy required for receiving is the same as for transmitting/forwarding. Furthermore, it is assumed that the energy used by the communication subsystem is much larger than the sum of the continuous energy required to sustain the sensor nodes, i.e. sensor sleeping energy, and the energy required by the sensor node to measure its phenomena. In other words, the wireless communication is the primary source of energy consumption. WSNs often implement a routing protocol that can dynamically form a multi-hop path between each sensor node and a sink. Exchanging routing protocol information and detecting 1-hop neighbours will consume energy, however, for simplicity the analysis assumes that the amount of routing traffic is negligible compared to the sensor reporting traffic. Also, we assume a perfect link failure detection mechanism, and that the routing protocol determines the path according to a shortest-path metric.

Since routes converge at the sink node, there is a strong likelihood that relaying nodes near the sink will forward more data than other intermediate nodes, thus consuming more energy. In order to reduce this problem, this paper assumes that *multiple* sinks located spatially apart from each other can be deployed. This leads to a more balanced traffic pattern and less burden on the sensors located close to a sink.

The data collection of a sink node is typically either on-demand (e.g. by a query-response cycle) or event-driven (e.g clock driven). In the analysis presented in this paper, data collection initiation is assumed to be clock- driven, where the sensor nodes collects and transmit data at predefined time intervals.

# 2.2 The reliability of the links in a WSN

Different layers of the networking stack introduce repair mechanisms that try to remedy and hide the effects of loss of signal or loss of packets due to various radio effects. Such repair mechanisms include modulation and coding techniques at the physical layer and the use of handshake at the MAC layer, as well as retransmission of lost MAC frames (until the retry counter expires). Furthermore, often a neighbour detection scheme is used in order to find 1-hop neighbours [10], which is required if a routing protocol is used. The detection scheme can be extended to also include short keep-alive messages that are exchanged periodically between 1-hop neighbours. Typically, some type of threshold is employed in order to determine whether a link should be marked as failed, i.e before a number of consecutive keep-alive packets are lost. If a routing protocol is used, it will try to conceal link failures by trying to find an alternative route.

#### 2.3 Link failures

A practical parameter to estimate when studying the energy consumption in a WSN, is the lifetime of the WSN [6], e.g. the time until the first sensor experiences power outage. Since this paper addresses the energy consumed by the sensor's reporting traffic, there are no sensor failures due to power outage in the time span the analysis cover. Thus, the state of the WSN is solely determined by the number of failed links. An underlying presumption of the analysis is that the frequent link failures seen in WSNs can be modelled by the link failure probability p and that p is determined independently for each link. The work in [9] demonstrates that this is a fair assumption. Thus, under these assumptions, a WSN consisting of n nodes can be analysed by the means of a random graph G(n, p). Employing the random graph analysis presented in this paper, the amount of energy required when the shortest path is unavailable can be found. In addition, a method for calculating the duration of the period the shortest path is unavailable, is presented.

In this paper we use the link failure probability, p, to describe links failing due to either loss of keep-alive messages, or that the two nodes forming the link fail to detect each other due to lack of node synchronisation. Furthermore,  $p_{txerr}$  is used to describe the loss of a single packet in the data reporting phase. The short term failure probability  $p_{txerr}$  and the long term failure probability p are related (i.e. the former is certainly one of the parameters that determines the latter e.g. through the loss of beacons or through the expiry of the retry limit). However, in our analyses both these probabilities are assumed given directly by the given by the physical conditions and network configuration, and their interrelation is not relevant for our analysis.

Using the same link model as in [9], the steady-state link failure probability is:

$$p = \frac{\lambda}{\mu + \lambda} \tag{1}$$

where  $\lambda$  and  $\mu$  are the rate parameters of the exponentially distributions of the failure rate and the repair rate, respectively.

# 3. RANDOM GRAPH ANALYSIS OF THE ENERGY CONSUMPTION IN WSN

## 3.1 The wireless sensor network model

The wireless sensor network in the analysis is modelled as an undirected graph G=(V,E), where the nodes in the network serve as the vertices  $v_j\in V(G)$ . Any two distinct nodes  $v_i$  and  $v_j$  create an edge  $\epsilon_{i,j}\in E(G)$  if there is a link between them. For

simplicity, we let  $\epsilon$  denote the size of the graph, i.e. the number of edges,  $\epsilon = |E(G)|$ . A minimal set of edges in the graph whose removal disconnects the graph is an *edge cutset*. The minimum cardinality of an edge cutset is the *edge connectivity* or *cohesion*  $\beta(G)$ .

Definition 1. The minimal set of edges in the graph whose removal increases the shortest path length between two vertices is defined as the edge shortest path cut-set (SPC).

THEOREM 1. The process of finding the shortest path cut-set is #P-hard.

PROOF. Suppose there exist an efficient algorithm for calculating the SPC between two nodes s and d. We transform (in polynomial time) the instance of computing the 2-terminal reliability of a wireless sensor network to SPC. This is achieved by weighting every operational link with 1 and failed links with 0. Clearly, a minimum aggregate path weight equal to the shortest path length when the network is fully connected is achieved if and only if there exist an operational path from s to d. However, computing the 2-terminal reliability has been shown in [1] to be #P-complete, which is a contradiction.  $\square$ 

## 3.2 The shortest path reliability of a WSN

We start the analysis of network reliability and availability by applying the k-terminal reliability [8] of a network. This measure is defined as the probability that a path, or route, exists and connects k nodes in a network. Substituting the *minimum cut-set* with the SPC, the k- terminal reliability,  $P_c^k(G,p)$ , becomes a measure for the probability of k-1 sensor nodes being connected to a sensor node over the shortest path. This is then expressed as:

$$P_c^k(G, p) = 1 - \sum_{i=\beta^k}^{\epsilon} \mathbf{SPC}_i^k p^i (1-p)^{\epsilon-i}$$
 (2)

where  $\mathbf{SPC}_i$  denotes the number of shortest path cutset of cardinality i. The corresponding network availability is found by inserting the steady state link failure probability found in Eq. (1) into Eq. (2) [9].

Under normal network operation, the traffic in the WSN is directed along the shortest path between a sensor node and one of the sink nodes. When a link fails, the routing protocol will provide a new path. If the length of the new path is greater than the length of the shortest path, the network is consuming more energy than necessary. Thus, the energy consumption is optimal if the path between the sensor nodes to one of the sink nodes has a length equal to the shortest path. This is true if, and only if, the sensor node and any of the sink nodes are connected through any path *not* in the SPC.

The additional energy consumed due to link failures, is proportional to the number of additional hops introduced by the paths of the SPC and can be computed using the same algorithm as for finding the SPC. This is shown in Algorithm 1.

Using Algorithm 1, we can now calculate the average path length  $\hat{h}$ , which is the average length of the paths in the SPC, and is given by:

$$\hat{h_i} = \frac{H_i}{SPA_i}, \quad \forall i \in \{0, \dots, \epsilon\}$$
 (3)

where i is the number of failed links and  $\epsilon$  is the total number of links.

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Algorithm 1 SPC(G), SPA(G), H(G)
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```
Require: A graph G(V, E).
1: SPC = SPA = H \leftarrow \{0, \dots, 0\}
2: \{|SPC| = |SPA| = |H| = |E(G)|\}
3: S \leftarrow states of G (all \{0,\ldots,2^{E}-1\} link combinations of G
    where a link is either up= 1 or down= 0)
4: U \leftarrow all sink/sensor pairs in G
5: SP \leftarrow \{\text{shortest paths of } U\}
6: for s \in S do
       for u \in U do
7:
8:
          i \leftarrow \text{failed links in } s
9:
          if NOT (u is connected with i failed links and |u| > SP_u)
             SPC_i \leftarrow \{SPC_i + 1\}
10:
             if u with i failed links is connected then
11:
                H_i \leftarrow \{H_i + |u|\}
12:
13:
                SPA_i \leftarrow \{SPA_i + 1\}
14:
15:
          end if
16:
       end for
17: end for
18: return SPC, SPA, H
```

# 3.3 Number of reporting packet transmissions

If a path consisting of  $\hat{h}$  hops exists between a sensor node and a sink, the sensed data will be received by the sink only if the data transmissions over  $all~\hat{h}$  hops are successful. The probability for successful delivery of a packet that is not allowed to be retransmitted on the same link is  $P_{ok} = (1-p_{txerr})^{\hat{h}}$ . Here  $p_{txerr}$  is the probability of a packet being lost due to transmission errors. The overall probability of successfully collecting sensed data over the shortest path is then expressed as:

$$P_{succ}^{k}(G, p, p_{txerr}) = P_{ok} \left[ 1 - \sum_{i=\beta^{k}}^{\epsilon} \mathbf{SPC}_{i}^{k} p^{i} (1-p)^{\epsilon-i} \right]$$
(4)

It can be shown [17] that when packets are lost due to transmission errors, the probability of successful packet delivery is:

$$P_{ok} = I(1 - p_{txerr}; \hat{h}, \hat{h}(M - 1) + 1)$$
 (5)

where I(x; a, b) is a *regularised Beta function* and M represents the retry limit, i.e. how many times a node can attempt to transmit a packet to the next hop in the forwarding path, before the uplink node gives up and discards the packet. (A value of M means that a maximum of M-1 re-transmissions are allowed).

In order to calculate the energy required for successful delivery of sensed data, we need the mean number of transmission attempts for every measured data that is transmitted. This can be found from [17] and is given by:

$$\overline{T} = \frac{\hat{h}}{1 - p_{txerr}} I(1 - p_{txerr}; \hat{h}, \hat{h}(M - 1) + 1) + \frac{\hat{h}(M - 1) + 1}{p_{txerr}} I(p_{txerr}; \hat{h}(M - 1) + 2, \hat{h})$$
(6)

Eq. (6) reduces to:

$$\overline{T} = 1 + \frac{1 - p_{txerr}}{p_{txerr}} (1 - (1 - p_{txerr})^{\hat{h} - 1})$$
 (7)

for M=1, i.e. for a scenario with no retransmissions on the links.

Eqs. (3)-(7) now enable us to calculate the excess energy required when sensed data is not reported over the shortest path.

# 3.4 Evaluation by example

#### 3.4.1 No link retransmissions (M = 1)

Consider a sensor network with a set of sink nodes  $z_i = \{z_1, ..., z_F\}$  and N sensor nodes. Without loss of generality, consider a 2-D regular grid (i.e. with equidistant edges of distance  $r_0$ ) consisting of F sinks and  $N \times N - F$  sensor nodes. When a sensor node  $s_i$  has sensed data to report, it will transmit this over the shortest path to one of the sink nodes. It is assumed that the procedure for discovering and selecting a sink node is already carried out. This is illustrated in Figure 2 for N=4 and F=4. The path from the sensor node  $s_{11}$  to one of the set of sink nodes  $Z_{i\in\{1,2,3,4\}} \in \{(z_1),(z_1,z_2),(z_1,z_2,z_3),(z_1,\ldots,z_4)\}$  is chosen as the basis for the analysis. A similar analysis can be done for other topologies and for other numbers of sensors and sink nodes.

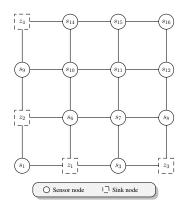


Figure 2: A  $4 \times 4$  grid network with multiple sinks.

Using Eq. (4), we first calculate the probability for successful delivery of sensed data over the shortest path. For these calculations, the mean hop length has the value of  $\hat{h} = 3$ , which can easily be deduced from Figure 2. The results are presented in Figure 3. Each graph shows how this probability is dependent on the link failure probability, p, or on the corresponding number of failed links. Indeed, the average number of failed links is equal to  $\epsilon \cdot p$ . By comparing the various graphs for a given packet error probability,  $p_{txerr}$ , it is also observered how the probability of reporting sensed data over the shortest path is increasing as additional sink nodes are deployed. This is not unexpected, but illustrates how the SPC can be used to investigate the performance for multiple sinks. Finally, by comparing the two sets of graphs, corresponding to the two different values of  $p_{txerr}$  it is also observed how the packet error probability influence the probability of successfully deliver sensed data over the shortest path.

When the shortest path is unavailable, the additional energy required to successful forward the data over longer paths to one of the sinks, can be calculated as:

$$E_{tx}(i) = \frac{\overline{T}(\hat{h} = H_i/SPA_i)}{P_{ok}(\hat{h} = H_i/SPA_i)} - \frac{\overline{T}(\hat{h} = 3)}{P_{ok}(\hat{h} = 3)}$$
(8)

where i is the number of failed links and  $H_i$  and  $SPA_i$  is calculated using Algorithm 1. This is shown in Figure 4 where it can be observed that the additional energy required decreases as the number of sinks increases. We can also note that the energy first increases, then decreases when the number of failed links is above

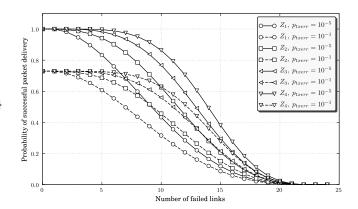


Figure 3: The probability for sensor node  $s_{11}$  to successful sensed data delivery over the shortest path to one of the sink nodes in the set  $Z_{i\in\{1,2,3,4\}}$  with a packet transmission error probability of  $p_{txerr} \in \{10^{-5},10^{-1}\}$  and M=1.

a certain level. This change in  $E_{tx}$  is caused by the variations in the average hop length from  $s_{11} \leftrightarrow Z_{i \in \{1,2,3,4\}}$ . When the number of failed links is relatively low, numerous short paths that connect the sinks and the sensor node  $(s_{11})$  can be found. These result in little contribution to the average path length. However, as more and more links fail, the length of the paths that connect the sinks and the sensor node increases, resulting in an increase in the average path length. When the number of failed links is larger than the number of operational links, the path lengths of the connecting paths decreases, in addition to that fewer connecting paths are available. Thus, the average path length decreases. Table 1 shows the calculated average path length using Eq. (3). Here we see that the average hop length decreases as more sinks are available  $(Z_2, Z_2, Z_4)$ .

Table 1: Average hop length for  $s_{11} \leftrightarrow Z_{i \in \{1,2,3,4\}}$  Failed links 4 6 8 10 12 14 16

Failed links	4	6	8	10	12	14	16	18
$\hat{h}, Z_1 = \{z_1\}$	5.03	5.21	5.43	5.48	5.35	5.19	5.06	5.0
$\hat{h},\; Z_{2}=\{z_{1},z_{2}\}$	5.02	5.13	5.25	5.31	5.25	5.14	5.05	5.0
$\hat{h}, Z_3 = \{z_1,, z_3\}$	5.01	5.11	5.21	5.27	5.24	5.13	5.04	5.0
$\hat{h}, Z_4 = \{z_1,, z_4\}$	5.0	5.07	5.14	5.19	5.19	5.12	5.04	5.0

When a sensor node have no paths to any of the sinks, we assume that it will listen continuously for beacons from neighbour nodes through which it can forward its data. This will require  $E_{rx}$  energy and is defined as:

$$E_{rx} = a \cdot \sum_{i=\beta^k}^{\epsilon} \left[ \mathbf{SPC}_i^k - \mathbf{SPA}_i^k \right] p^i (1-p)^{\epsilon-i}$$
 (9)

where the first term, a, is a given constant and the second is the probability of not having a path to any of the sink nodes.

The sum of the energy  $E_{rx}$  and  $E_{tx}$  is shown in Figure 5. This illustrates how energy consumption caused by the lack of connectivity might influence the total energy requirement. The figure shows that depending on the value of a, the energy  $E_{rx}$  might be the main source of energy consumption as the number of failed links increases.

#### 3.4.2 Link retransmissions (M > 1)

Retransmission of packets that are lost due to transmission errors, is an effective mechanism when it comes to increasing the

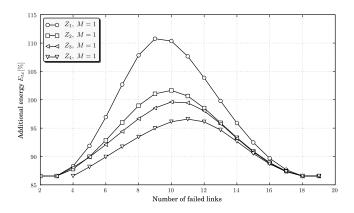


Figure 4: The percentage increase in  $E_{tx}$  for node  $s_{11}$  to successful deliver sensed data to one of the sink nodes in the set  $Z_{i\in\{1,2,3,4\}}$  over a non-shortest path. The packet transmission error probability  $p_{txerr}=10^{-1}$  and a=1.

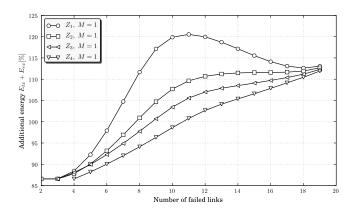


Figure 5: The percentage increase in  $E_{tx}+E_{rx}$  for node  $s_{11}$  to successful deliver sensed data to one of the sink nodes in the set  $Z_{i\in\{1,2,3,4\}}$  over a non-shortest path. The packet transmission error probability  $p_{txerr}=10^{-1}$  and a=1.

probability for successful packet delivery. We investigated the probability to successfully deliver data over the shortest path similar to Figure 3 and found that the calculations for  $p_{txerr}=10^{-1}$  and M=3 gave approximately the same results as for  $p_{txerr}=10^{-5}$  and M=1. This clearly shows the benefit of using retransmissions.

Figure 6 shows the probability for sensor node  $s_{11}$  to successfully deliver data to  $Z_1$  over a non-shortest path as the transmission error  $p_{txerr}$  varies. The figure demonstrates the incremental benefit from no retransmissions, i.e. M=1, to retransmitting once where M=2. The number of failed links in the figure is either 4 or 8, where the average hop length is 5.03 and 5.43 respectively (Table 1). As the packet transmission error increases, one can observe that the difference in probability for success between 4 and 8 failed links decreases. This is because the difference in average path length becomes negligible as the transmission error increases, since the probability of a packet failing on the first hop, thus not being forwarded, becomes the dominating factor.

In Figure 7 we show the additional energy required for every sensed data that is successfully delivered to one of the sinks when the shortest path is unavailable. The figure shows the difference

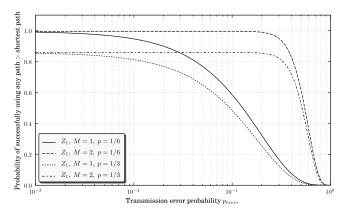


Figure 6: The probability for sensor node  $s_{11}$  to successful deliver sensed data to  $Z_1$  over a non-shortest path. The link failure probability is  $p = \{\frac{1}{6}, \frac{1}{3}\}$ , i.e. 4 and 8 failed links.

in energy for  $[E_{tx}+E_{rx}]_{M=1}-[E_{tx}+E_{rx}]_{M=3}$ . The probability of having a path  $(P_c)$  is equal for M=1 and M=3, so the difference in energy is just  $\Delta E_{tx}$ . The variation in energy as the number of failed links increases, is a result of the average path length between the sensor node and the sink nodes and can be explained in a similar fashion as for Figure 4. The figure illustrates how retransmissions enable less energy consumption per successfully data delivery. However, a routing protocol may fail to detect absent links, or at least there will be a time period needed from the link has failed until it is detected. In these cases the energy required will be proportional to the number of M-1 retransmissions. This is however out of scope for this paper, since a perfect link failure detection mechanism is assumed.

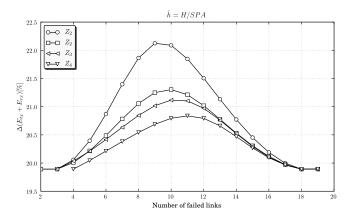


Figure 7: The percentage difference in energy  $(E_{tx})$  for M=1 and M=3 that is required for sensor node  $s_{11}$  to successfully report sensed data to one of the sink nodes in the set  $Z_{i\in\{1,2,3,4\}}$  over a non-shortest path. The packet transmission error probability is  $p_{txerr}=10^{-1}$ .

# 4. MEAN TIME BETWEEN FAILURE FOR THE SHORTEST PATH AVAILABILITY

While the shortest path availability is a useful reliability measure, it does not provide every aspect of network reliability, since it

is a steady- state measure. For example, it describes the portion of the time the shortest path between a sensor and a sink node is available, but does not provide insight about the duration the shortest path can be used. This is however an important parameter for both on-demand and event driven data collection schemes. The mean time between failures (MTBF) and the mean down time (MDT) can provide a measure of the frequency the shortest path is unavailable and the duration of this, respectively.

In order to find the MTBF, the state of the sensor network can be modelled using a Markov model with a link failure rate of  $\lambda$  and a repair rate  $\mu$  were the state of the sensor network is either connected by the shortest path, or disconnected. For this analysis, we use a Markov model that can be found in [9].

Using Korolyuk's theorem, the MTBF can be expressed as  $1/\Lambda$ , where:

$$\Lambda = \sum_{i=\beta}^{\alpha} p_{i,d} \cdot \mu \cdot i \cdot \frac{\binom{\epsilon}{i-1} - \mathbf{SPC}_{i-1}}{\binom{\epsilon}{i-1}}$$
 (10)

where  $p_{i,d}$  is the state probability being disconnected with i link failures and  $\alpha$  is the SPC where any link removal result in a path length greater than the shortest path, and  $\beta$  is the minimum cardinality of the SPC.

The Mean Down Time (MDT) can easily be determined using Little's formula, and is given by:

$$MDT = \sum_{i=\beta}^{\alpha} p_{i,d} \times MTBF \tag{11}$$

In Figure 8 shows the calculated MTBF for the grid topology in Figure 2 while Figure 9 shows the MDT. The figures also include results based on simulations. The simulation results are obtained using a discrete event WSN simulator applying the grid topology as an input. The links in the WSN simulator fail and are repaired at events decided by  $\lambda$  and  $\mu$ , where the link failure probability is given by Eq. (1). The simulation results in Figure 8 and Figure 9 are shown with 95% confidence intervals. As the figures show, the calculations for the MTBF and MDT correspond well with the results from the WSN simulator.

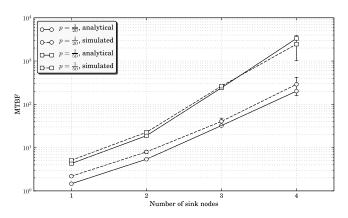


Figure 8: Mean time between when the shortest path is available as the number of sinks is increased

### 5. RELATED WORK

There exist much work on WSN, especially in the area of energy efficient routing [3, 14], low power constrained medium access protocols [15, 20] and topology control [18, 19].

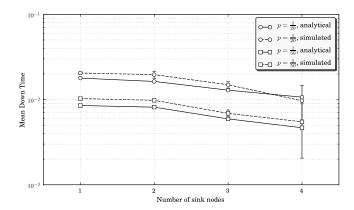


Figure 9: Mean time a shortest path is unavailable as the number of sinks is increased

In [2] the reliability of a WSN, in terms of being connected, is studied. The work focuses on reliability, and defines a reliability measure that considers the aggregated flow of sensor data in to a sink node. Exact and approximate methods for computing the reliability is presented.

Another study on the reliability of WSN is [5], where coverage and connectivity is used as constraints for a reliability model for a WSN. This model can then be utilised by a topology control algorithm.

The work in [6] studies the lifetime of a WSN, and a general formula for the lifetime is provided, showing the relationship between wasted energy, the reporting rate and the reporting energy. Based on this a medium access protocol is proposed. An upper bound for the lifetime of a WSN is also presented in [4], where energy consumption is modelled and constraint by power required to transmit a given distance.

There are, however, only a few studies on how energy consumption is related to link failure probability. The work in [17] deals with the problems of computing a measure for the energy consumption in a distributed sensor networks, when transmission errors are present. This work however, assumes that the links do not fail.

### 6. CONCLUSIONS AND FUTURE WORK

This analysis might be pertinent when planning wireless sensor network structures. The wireless sensor network is modelled as a random graph, where the inherent characteristics of the wireless medium are assumed to result in stochastic link failures with a link failure probability p. The results show how multiple sink nodes reduces the energy consumption required to report sensed data, when the shortest path connecting sensor nodes and sink nodes is unavailable, and packet transmission errors are present. Although analyses and results depend on the actual topology, the same analyses as presented here can be applied to any specific sensor network topology of interest.

In addition to finding the steady-state required energy, the timedependent behaviour of the energy consumption is also analysed, demonstrating a method for determining the frequency and the duration for when the shortest path is unavailable.

The paper demonstrates that when the shortest path is unavailable, the additional energy required in order to report sensed data is a key performance metric. However, the problem of computing the *shortest path cut-set* is shown to be #P-hard. This is a definitive limitation, since these problems have exponential time complexity.

A potential source of inaccuracies for the work presented is the modelling of the link failure probabilities. Although the accuracy might be improved by addressing link dependencies (e.g. as in [7]), a shortcoming is that the link failure probabilities are assumed to be equal for all links. With the grid topology used as an example in Figure 2, this might be a fair assumption. For less regular topologies, these probabilities will vary between different links, since the link failure probability will depend on the distance between each pair of nodes. However, the work in [9] where different link failure models are compared, suggest that a distance-independent link model with a fixed link failure probability might yield useful information about the network behaviour in terms of network availability.

The link failures are in this paper assumed to be a result of either loss of keep-alive messages, or due to the two nodes forming a link failing to detect each other because of poor time synchronisation. An interesting topic for future research is to study how poor performance of various time synchronisation schemes results in failure to establish links between nodes. Furthermore, loss of keep-alive messages is often a result of transmission errors caused by fading or radio interference. In a multi-hop network, interference caused by overlapping transmissions from neighbouring and hidden nodes will also contribute to the loss of keep-alive messages. In future work, we want to study the relationship between links identified as failed as a result of a consecutive loss of keep-alive messages, and the resulting link failure probability and its impact on the overall reliability of a WSN in terms of energy constraints.

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