Game Theoretic Approaches for Next Generation Networks

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Abstract

Energy and radio are the two most important and expensive resources in the world, for which the demands have been constantly increasing. We address the economic research problems for these resources in the communications-centric or communications-enabled next generation networks such as cognitive radio networks and smart grids. The next generation networks are complex systems formed by multiple entities where the benefits of every single entity is coupled with that of others. Thereby, we employ economic approaches such as game theory, to solve the resource management and optimization problems in the next generation networks.

A cognitive radio network is a next generation technology with high potentials to accommodate the communication needs of emerging wireless communication networks. The central resource in a cognitive radio network is the electromagnetic spectrum, which, is limited. Spectrum sensing, a key mechanism to provide radio awareness necessary for opportunistic spectrum access in cognitive radio networks, demands resources such as time and power. We formulate resource allocation problems for cooperative spectrum sensing and propose efficient solutions to improve the quality of service and the payoffs for the cognitive radio users. In a next generation energy-information network like a smart grid, the resources are not limited to radio. The resource management in the smart grid involves new dimensions such as power, energy, time, scheduling, etc. Moreover, the interacting players for the economic problems in smart grids are not limited to data transmitting units, e.g., electricity providers or utility companies, distributors, consumers, etc. The smart grid, being a complex heterogeneous system combining the traditional electric power grid with the information and communication network, brings into surface numerous unique challenges and new research problems. We study demand response problems for efficient generation, trade and consumption of electric power, and address communication problems such as improving the quality of communication, and dependability, in the smart grid.

There are three major connections between our work on cognitive radio networks and smart grids: (i)- Energy and radio are the two most limited and expensive resources in the world, for which the demands have been constantly increasing. We study the resource management and optimization problems for these two most important resources (ii)- Communication is the central or the enabling technology in both cognitive radio networks and smart grids. (iii)- We employ economic approaches such as game theory, to solve the economic problems in both cognitive radio networks and smart grids.

Our three main contributions in this thesis can be summarized as follows.

The first contribution is the development of cooperative spectrum sensing mechanisms among the secondary users of a cognitive radio network utilizing the hetero-

geneity in traffic dynamics of the secondary users. We employ queuing theory for packet delay analysis, and apply game theory to enforce cooperation among the selfish secondary users. The proposed scheme on cooperative spectrum sensing improves the quality of service for the real-time services significantly by reasonably exploiting the non-real time traffic. Our cooperative spectrum sensing mechanism can dynamically enforce cooperation among the secondary users for spectrum sensing, utilizing the diversity in the traffic density of the secondary users, in either fairness aspect or efficiency aspect, as required.

The second contribution is the design of efficient and more practical demand response management schemes in the smart grid, integrating entities at different levels of the power generation, distribution and consumption processes. Our analyses involve multiple players, including renewable energy sources, and we have extended our demand response management framework to consider both spatial and temporal dimensions. We have mainly used game theory in our studies on demand response management, and have derived explicit closed-form and analytical solutions characterizing the equilibria of the games for both the supply and the demand sides. The proposed demand response management schemes either reduce the costs for the users, or they maximize the payoffs of both the supply and the demand sides given others' parameters. We also provide some insights on the equilibrium behavior of the involved players in a large population regime.

The third contribution lies in establishing a connection between the control unit, smart grid communications and demand response management, in the smart grid. We mainly use optimization based and game theoretical tools for this analysis. We study the dependability of the coupling between demand response management and smart grid communications in the presence of an attacker who can manipulate the price information, and propose common reserve power based mechanism to maintain the dependability of the grid. Our cognitive radio enabled smart grid framework based on opportunistic spectrum access for smart grid communications, sheds light on the existing tradeoff between the control performance i.e., the demand response management, and the communication performance i.e., the spectrum sensing time.

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Paper I:
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Paper V:
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Paper VI:
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The seven papers are self contained and thus some information might be redundant in the papers. Different abbreviations or symbols may have been used in the papers.

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Part I: Summary

1 Introduction

Communication technologies have been an integral part of our daily lives, directly or indirectly, for many important activities. For instance, multimedia transmissions, video streaming, telepresence systems etc occupy the biggest proportion of the data being communicated over the internet. The development of smart phones and tablets have further added the penetration of the services such as location based identification and tracking, mobile social networks, etc. The advancement in communication technologies are anticipated to grow further, facilitating the deployment of ubiquitous systems such as cyber physical systems (CPSs) and Internet of Things (IoTs).

Most of the systems, today, are dependent on communication technologies. Consequently, the advancement in communication technologies is a means to enhance many existing and upcoming systems such as logistics, transportation, automation and industrial manufacturing, etc. Accompanied with the growth and development of such systems, however, are the concerns towards an increased need of the resources. Among others, energy and radio are the two most important and expensive or limited resources in the world. The demands and reliability requirements for both energy and radio, have been constantly increasing, due to the extensive development in communication and networking technologies, followed by the emergence of advanced new systems, and an increased number of appliances, equipments, and industries. While radio is limited, energy is very expensive. Consequently, the *economics* of these resources is a component of utmost importance. Next generation networks (NGNs) such as cognitive radio networks (CRNs) and smart grids (SGs), respectively, have been envisioned as the enablers of the reliable and efficient management of radio, and energy. The resource management and optimization aspects in the NGNs bring into existence numerous interesting issues such as the trade of the resources among sellers and buyers, allocation of the available resources within a system through appropriate scheduling, etc. The problems addressed in this thesis can be classified into two main categories as shown in Figure 1: trade based and allocation based resource optimization and management. In the trade based scenarios, resources may be limited or can be produced according to the consumers' demands, and are sold and purchased with some form of payment (usually price). In the resource allocation based approaches. the available resource is limited or given, and the given amount of resources are allocated among the units in a system or a network. Resource allocation here, may refer to communication resources or other resources such as electric power.

In an NGN like a cognitive radio network (CRN), the major resources are the wireless channels (spectrum), the bandwidth, the transmission power etc. The medium of data transmission for wireless communications, i.e., the electromagnetic radio spectrum, is limited. On the contrary, the development of various new communication sys-

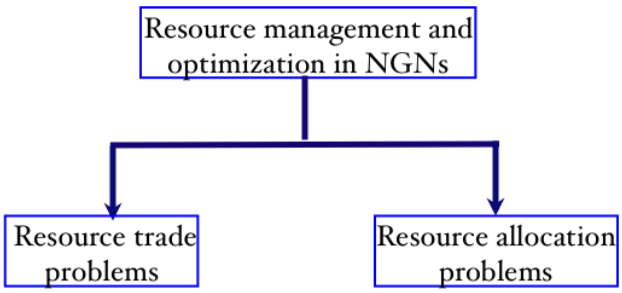

Figure 1: Classification of research problems addressed in this thesis.

tems and networks, indicates the need of more spectrum. Since the spectrum licenses are assigned to specific existing wireless communication networks, the competition for the spectrum increases in the unlicensed bands, thus creating and increasing interference to each other. Cognitive radio (CR) [1], [2], [3], [4] is a technology proposed for spectrum sharing between the licensed and the unlicensed users. The licensed users are called the primary users (PUs) and the unlicensed users are known as the secondary users (SUs). Spectrum sensing [5] is necessary for spectrum sharing to detect the PUs, and thus to protect them sufficiently from possible interference due to the transmission of the SUs. The economic research challenges in CRNs that we address in this thesis include, the development of cooperation among the SUs for spectrum sensing, and optimal allocaton of the PU spectrum among the SUs.

In an NGN like the smart grid (SG), the resources are not limited to radio. While radio resource management is of significant importance for smart grid communications, electric power is the central resource for trade in the SG. Electric power is necessary for all electric appliances to operate. The world's consumption of all forms of energy, including electric power, continues to increase [6]. Accompanied with the growth of power consumptions and the ever-increasing demands of electricity, are concerns about the environmental impacts and the reliability of power supplies. Traditional power grids, however, are not able to meet these demand and reliability requirements because of their inflexible designs and the lack of prompt and efficient communications between the supply and the demand sides. The SG [7] is the next generation grid system

that integrates advanced information and communication technologies (ICT) with the grid for bidirectional communications (smart grid communications) between the supply side and the demand side through the market. Smart grid communications facilitate the transformation of the current electric grid into the SG, which will (i)-enable active participation of the electricity consumers in efficient and effective supply-demand management, (ii)- enhance the control capability of the whole system and (iii)- improve the reliability and transparency of the grid.

Demand response management (DRM), a key component of the SG, is the response system of the end-users to adjust their power demands according to the supply conditions. The smart grid communications are central to enhancing the DR capabilities of the whole system in the SG. E.g., consumers will be able to control and manage their electricity usage, and the providers can plan and manage the generation and supply of power in a more structured and efficient manner. Thus, power generation, distribution and consumption are envisaged to be more efficient, more economical and more reliable in the SG, with the help of the smart grid communications. While smart grid communications can enhance the DRM capability, and can improve the reliability and transparency of the grid, many uniques research challenges need to be addressed for integrating the communications network with the electricity grid network before the benefits can be fully realized. The economic research challenges in the SG, that we cover in this thesis, include the efficient (and reliable) generation, distribution and consumption of electric power, and optimal allocation of the radio resources such as spectrum, for smart grid communications.

1.1 Motivation

Although spectrum sensing facilitates the sharing of the spectrum between the primary and the secondary users, the process of spectrum sensing is resource demanding. The SUs who perform spectrum sensing, need to invest resources such as power and time to detect the PU(s). These investments in spectrum sensing may incur extra cost (e.g., power) or may degrade the quality of service (QoS) of SU transmission. For instance, for SUs, contributing in sensing is, in many cases, synonymous to sacrificing the transmission time, thus resulting an increase in the incurred packet delay or packet drop rate, and thus, decreasing the obtained throughput of the SUs. Consequently, despite the improved detection performance that can be obtained by cooperative spectrum sensing (CSS), the altruistic and cooperative behavior from the SUs may not be practically reasonable. Thus, proper incentives or punishments (for the lack of cooperation) are necessary for the SUs to cooperate in spectrum sensing. Despite the availability of a broad range of literature on different spectrum sensing techniques in CRNs, there are very few studies which consider the selfish behavior of the SUs towards contributing

for CSS. Moreover, heterogeneity among the SUs, which can offer either incentives or punishment mechanisms for cooperation among the SUs, has not been explored enough. For instance, the SUs can be heterogeneous with respect to their traffic dynamics, resource availability, etc. In this thesis we exploit the heterogeneity among the SUs to develop priority based and enforced CSS techniques. We propose efficient schemes and algorithms to minimize the use of resources for spectrum sensing, and to utilize the benefits of spectrum sharing effectively, by improving the QoS of SU transmission.

In the case of the SG, the research area is relatively new, the existing literature mainly focusses on different DRM programs, the integration of the electric vehicles (EVs) and renewable energy resources into the DRM framework, scalable communication architectures, and the upcoming security issues associated with the possible use of wireless communications in the SG. Nonetheless, many of them are based on simulations or on the electricity usage data of a certain city or a town. Hence, the studies lack analytical elements. In addition, the existing DRM studies are mostly focused either on the supply side or on the demand side of the interactions. Moreover, there are very few studies where suitable technologies discussed for smart grid communications are properly assessed and the connection between the communication and the control aspects are explored. In this thesis, we develop extensive analytical and closed form results for efficient DRM in the SG considering the entities at different levels from customers to suppliers, to market players. We provide novel insights into the reliability and dependability analysis of smart grid communications, and introduce benchmark studies such as large population approximations for DRM.

1.2 Approaches

Resource management and optimization (trade based and allocation based) in NGNs involves the interaction among multiple players such as PUs, SUs, spectrum brokers, electricity providers, electricity consumers, etc. The key to addressing the economic research problems in the NGNs lies in modeling the behavior of and interaction among these players, for example, as competition, and/or cooperation. When all players in a network have a common goal, e.g., to maximize the system payoff or social welfare, the whole system can be treated as one player. Thereby, optimization techniques can be implemented to solve the economic problems. On the other hand, when the players' behaviors are interdependent and their strategies are coupled, game theory is an appealing tool to understand and to model the economic problems in the NGNs. Consequently, we employ game theory for modeling and analysis in most of our work included in this thesis. Depending on the scenario of interest, however, we also deploy tools such as queuing theory and optimization techniques in some of our studies, to capture the

behavior of various entities in the NGNs. E.g., in our work on collaborative spectrum sensing where all SUs contribute to improve the QoS of the real time traffic in a CRN, as all SUs are controlled by the the secondary base station (BS), we employ queuing theory to derive an analytical form for the average packet delay of the transmission from SUs. On the other hand, when each SU is treated as an individual entity who is concerned about maximizing its own payoff, we use a game-theoretical framework to enforce cooperation among them for spectrum sensing, and improve the average payoff of each user by introducing randomization in their strategies. In most of our studies on DRM in the SG, we formulate the economic problems as specific game models, in particular, non-cooperative games and Stackelberg games. In our CR enabled smart grid communications model, the goal is to maximize the social welfare, thus we use an optimization based framework for that study.

In summary, in this thesis, we address various economic research problems in two NGNs: CRNs and SGs. There are three major connections between our work in these two areas: (i)- Energy and radio are the two most limited and expensive resources, with constantly increasing demands, all over the world. We formulate the resource management and optimization problems for these two most important resources, and provide optimal solutions for efficient allocation or payoff maximization of the associated entities. (ii)- Communication is the central or the enabling component in both CRNs and SGs, and we develop schemes and algorithms for optimal radio resource management in both the NGNs. (iii)- We employ game theoretical frameworks and approaches to solve the economic problems in both the networks.

The rest of this thesis is organized as follows. This thesis consists of two parts:
Part I: Summary: The rest of the summary consists of the following sections: Sections 2, 3 and 4 provide the background information required to understand the summary of the thesis. In Section 2, we describe about the spectrum scarcity problem, cognitive radio networks, and spectrum sensing. In Section 3, we present an overview of the SG concept, discuss the potential of, and challenges associated with DRM problems, and describe the role of smart grid communications in enabling the timely and accurate demand response (DR) programs. Section 4 presents a brief introduction of game theory, discusses the basic concepts of game theory that we have used in our papers, and points out the applications of game theory in modeling the behavior dynamics of the players in the NGNs such as CRNs and SGs. Section 5 presents an overview of the research methodology that was followed for the work during my PhD. Section 6 provides a brief summary of the contributions and the results of the papers included in this thesis. Section 7 concludes my contributions in this thesis, and Section 8 outlines possible extensions and future research directions based on my work presented in this thesis. The list of my publications during Ph.D. studies is presented in Section 9.
Part II: Papers: The second part of the thesis presents the published or submitted

research papers. which are included in this thesis.

2 Cognitive Radio Networks and Spectrum Sensing

The radio spectrum ranging from 3 KHz to 300 GHz is the basic resource for data transmission, propagation and reception in wireless networks. In each country, spectrum is regulated by its radio regulatory agency, such as Federal Communications Commission (FCC) in USA [8], Electronic Communications Committee (ECC) in Europe [9], The Norwegian Post and Telecommunications Authority (NPT) in Norway [10], and Ofcom in UK [11]. Traditionally, spectrum has been assigned via a fixed frequency allocation policy, where each band of spectrum is exclusively allocated to a specific wireless communication system or network. Moreover, these fixed licenses over spectrum bands are extremely expensive. E.g., for getting the fixed licenses to deploy 4G cellular networks, TeliaSonera pays 563 million SEK for the 15-year's license in Sweden on four frequency blocks totaling 2×20 MHz in the 2.6 GHz band [12], 336.3 million DKK for the 20-year's license in Denmark on 2×20 MHz paired spectrum and 10 MHz unpaired spectrum in the 2.5 GHz frequency band [13], 819000 EUR for the 20-year's license in Finland on five 2×5 MHz frequency band pairs in the 2.6 GHz band [14].

This spectrum assignment policy is static, and it does not allow reuse or dynamic use of the spectrum allocated to the licensed users by unlicensed users. On the other hand, rapid development of new wireless systems and applications and growth of the existing wireless systems in the recent years demands more spectrum. The Industrial, Scientific and Medical (ISM) spectrum band, reserved internationally for the use of radio frequency energy for industrial, scientific and medical purposes, is the only spectrum that can be shared by the unlicensed users from different networks. This band too, is, already crowded by many systems such as wireless local area networks (WLANs), wireless personal area networks (WPANs), cordless phones, microwave ovens, etc., which are suffering interference from each other. Hence, in order to accommodate the new wireless systems and applications, a more flexible spectrum assignment policy is required such that the available spectrum can be effectively exploited and dynamically accessed and (re)used.

Interestingly, the licensed spectrum utilization is highly dependent on the geographic location and time of its use. For instance, during some time periods in certain geographic areas, the allocated spectrum bands may be rarely used by the assigned system, as illustrated in Figure 2. A report published by FCC in November 2002, indicates that for 90% of the time considerable number of licensed frequency bands remain underutilized or unused [15]. Similarly, Shared Spectrum Company (SCC) has

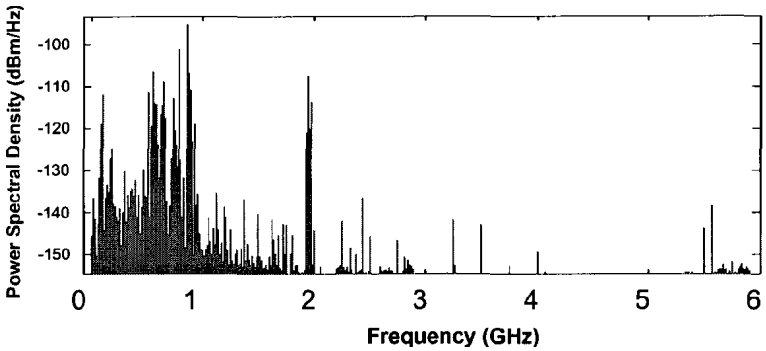

Figure 2: Spectrum utilization measurements ($0-6$ GHz) in Downtown Berkeley [20].

published a bunch of spectrum measurement results of US and some European Countries since 2004 [16], and the reports point to similar facts. Their spectrum reports in [17], [18], [19], indicate that the utilization of many licensed frequency bands in many cities is under 25%. This implies that the actual problem is not the spectrum scarcity but the static and inefficient spectrum allocation and usage.

Consequently, since 2004, FCC has recommended to consider authorizing new devices in the TV broadcast spectrum at locations where TV channels are not being used for authorized services [21], [22]. The IEEE 802.22 Working Group on Wireless Regional Area Networks (WRANs) [23],[24], the first worldwide wireless standard based on CRs, was formed in October 2004. Motivated by the idea of exploiting the unused or not fully utilized parts of the licensed TV broadcast spectrum, it has been working on the standardization for the rural broadband wireless access using CR technologies for opportunistic access of the TV broadcast spectrum [25].

2.1 Cognitive Radio Networks

The original concept of CR was proposed by Joseph Mitola III at Royal Institute of Technology (KTH), Sweden, in 1999 [1], [2]. Cognitive radio is an enabling technology to allow unlicensed users to exploit the spectrum originally assigned to licensed users. A typical implementation scenario of the coexistence of PUs and SUs is shown in Figure 3. The primary system has an exclusively allocated spectrum, which is vacant at times, as indicated by the white time slots. The CRN opportunistically accesses the spectrum when the spectrum is vacant.

The research issues in wireless communications in general, are inherent in CRNs. In addition, some distinct characteristics of CR introduce new issues to implementing

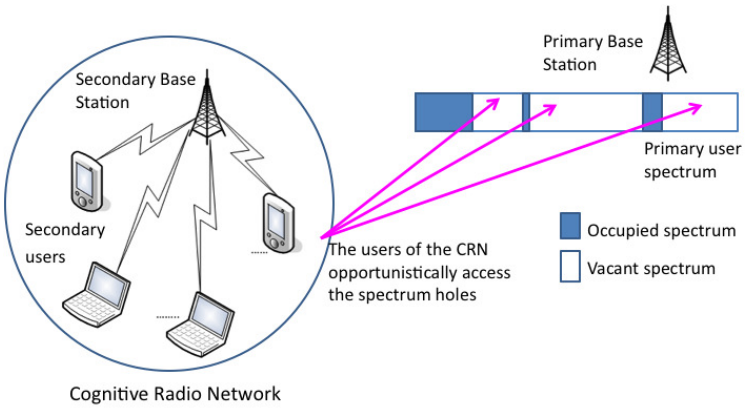

Figure 3: Coexistence of primary and secondary users based on opportunistic spectrum access

CRNs. The changing spectrum environment and the need of protecting the transmission of the PUs from the possible interference from the SUs, mainly differentiate CRNs from traditional wireless networks. CR technology possesses a huge potential for efficient use of spectrum through opportunistic access of the licensed spectrum by the SUs. However, in order to gain opportunistic access to the PU spectrum, the SUs should ensure that the PUs are sufficiently protected from the possible interference caused by their transmission. Therefore, the SUs must be aware of the activity of the PU in the target band. They should spot the spectrum holes, i.e., the vacant spectra, and the idle state of the PUs in order to exploit the free bands, and also promptly vacate the band as soon as the PU becomes active. CR encompasses this awareness by dynamically interacting with the environment through spectrum sensing and adaptively choosing its transmission parameters according to sensing outcomes.

2.2 Spectrum Sensing

Spectrum sensing refers to the phase during which the SUs must sense the radio frequencies in order to make a decision on whether to transmit or not, depending on the state of the PU. SUs need to regularly sense the allocated channels and reliably detect the presence of the PU signal with a small delay. In the IEEE 802.22 standard, for ex-

ample, the SUs need to detect the TV and wireless microphone signals and upon their detection, they are required to vacate the channel within two seconds [26]. For TV primary signals, a probability of detection of 90% and a probability of false alarm of 10% should be maintained [26]. By using local measurements and local observations from spectrum sensing, a secondary user can detect the transmitted signal from a PU. Based on how much information about the primary signal is available to the SUs, spectrum sensing techniques can classified as follows:

- Matched filtering
- Cyclostationary feature detection
- Energy detection

2.2.1 Matched Filtering

Matched filter detection is generally used to detect a signal by comparing a known signal (i.e., template) and the input signal. It is well known that the optimal method for signal detection is through a matched filter as they maximize the received signal-to-noise ratio (SNR) [5],[27]. In addition, by using a matched filter detector, the PU signal can be detected in a short time [20], which is one of the main advantages of matched filter detection. However, utilizing a matched filter in spectrum sensing requires demodulation of a PU signal, which implies that the CR must have a priori knowledge of different PHY and MAC characteristics of the PU signal such as pulse shaping, packet format, and so on [5], [27], [28], [29]. In CRs, however, such knowledge is not readily available to SUs, and the implementation cost and complexity of this detector is high, especially as the number of primary bands increases. Therefore, this method may not be practical for CR technology.

2.2.2 Cyclostationary feature detection

Cyclostationary feature detectors can distinguish between modulated signals and noise [5], [27], [28], [29], [30]. This detector exploits the fact that the transmitted signal from a PU generally possesses some periodic pattern, i.e., the signal from the licensed PU is cyclostationary with spectral correlation due to the built-in redundancy of signal periodicity. On the other hand, the noise is a wide-sense stationary signal with no correlation [30], [31]. Thus, using the cyclostationarity, the transmitted signal from a licensed PU can be distinguished from noise. In general, cyclostationary feature detectors can provide a more accurate sensing result and they are robust to the uncertainty introduced by the variation in the noise power. However, these advantages are obtained at the cost of a higher complexity for implementation and longer observation times.

2.2.3 Energy Detection

When the information on the PU signal is unavailable to the SUs, energy detection can be applied to detect the existence of the PU signal. This simple scheme accumulates the energy of the received signal during the sensing interval and declares the band to be occupied if the energy surpasses a certain threshold, which is set based on the desired probability of false alarm [32][33]. The received signal $x(t)$ will have either of the following two forms:

$$\begin{cases} H_0 : x(t) = w(t), & \text{when PU is absent,} \\ H_1 : x(t) = gs(t) + w(t), & \text{when PU is present,} \end{cases} \tag{1}$$

where g is the gain of the channel between PU transmitter and the SU receiver, $s(t)$ is the signal from the PU, which is assumed to be an independent and identically distributed (i.i.d.) random process, $w(t)$ is an additive white Gaussian noise, $t = 1, 2, ..., T$ is the discrete time index, and T is the total number of samples collected during sensing period. H_0 and H_1 represent, respectively, the hypotheses of the signal from a licensed PU being absent or present in the target frequency band. Consequently, the spectrum sensing phase boils down to a decision between two hypotheses H_0 or H_1, depending on the received signal at the SU.

For measuring the performance of spectrum sensing, three key metrics are usually explored: the probability of detection, the probability of miss detection and the probability of false alarm. The probability of detection is defined as $P_{\mathrm{D}} = \mathrm{Prob}(\text{decision} = H_1|H_1)$, which is the probability of correctly detecting the transmission of the PU when it is active. The probability of miss detection is defined as $P_{\mathrm{MD}} = \mathrm{Prob}(\text{decision} = H_0|H_1)$, which is the probability of not detecting the PU transmission while the PU is active, i.e., $P_{\mathrm{MD}} = 1 - P_{\mathrm{D}}$. The probability of false alarm is defined as $P_{\mathrm{FA}} = \mathrm{Prob}(\text{decision} = H_1|H_0)$ which is the probability of deciding that the PU is transmitting while the PU is, in fact, idle.

Energy detection, unlike the other schemes, does not require any information about the PU signal and channel gains, and is robust to unknown fading channel. Compared to matched filter detection and cyclostationary feature detection, it has simpler implementation, and hence, is less expensive. Therefore, energy detection has been widely adopted in many spectrum sensing scenarios [5], [27], [28], [29], [34], [35], [36]. Despite its practicality and appeal, energy detection suffers from the following drawbacks: (i)- It is susceptible to the uncertainty of noise, (ii)- It can only detect the presence of the signal, but cannot differentiate the type of the signal. As a result, energy detection can confuse signals resulting, for example, from other SUs with the PU signal, (iii)- Energy detectors do not work for spread spectrum signals.

In addition to these methods, recent work has also investigated the use of advanced

detection techniques, such as wavelet detectors [37], for performing spectrum sensing. Moreover, one can also integrate, in a single secondary system, different detection methods. For instance, energy detection can be used to perform a fast scan of a wide range of spectrum bands. Subsequently, the results from energy detection can be used to eliminate the spectrum bands with high energy densities (e.g., due to the transmission of PUs). Then, feature detection can be applied to a few candidate bands with low energy densities to search for a unique feature of signals pertaining to PUs.

2.3 Cooperative Spectrum Sensing

The performance of spectrum sensing is significantly affected by the degradation of the PU signal due to path loss or shadowing, which lowers the received SNR. Added to the issue of low SNR is the hidden terminal problem that arises because of shadowing. SUs may be shadowed away from the PU transmitter but there may be primary receivers close to the SUs that are not shadowed from the PU transmitter. Thus, if the SU transmits, it may interfere with the primary receiver's reception. This problem is called the hidden terminal problem. Consequently, advanced methods for improving spectrum sensing are being sought. In particular, as different users in distant locations undergo independent channel fading, it has been shown that, through cooperation among the SUs, i.e., CSS, the effects of the hidden terminal problem can be mitigated by utilizing the inherent diversity associated with the SUs. Consequently, the reliability of detecting the PU will be improved [34], [35], [36]. In this manner, the individual SUs require less detection resources while increasing the overall detection reliability. A typical CSS scenario in the presence of hidden terminal problem, is illustrated in Figure 4. In the figure, SU1 is a shadowed SU. So, it cannot detect the PU signal. But SU2 and SU3 can detect the PU signal and report their sensing results to the secondary BS, thus preventing SU1 from transmitting when the PU is active, although its local decision implies that the PU is not active. These advantages, however, come at the price of added communication overhead due to cooperation.

FCC has also acknowledged the need for CSS for TV bands [23], [25], [26]. CSS is mainly composed of two steps. In the first step, each SU performs its individual spectrum sensing. In the second stage, the SUs provide locally sensed information on the PUs activity status to a decision-making fusion center (FC) which can be an access point or a BS or another SU. The FC analyzes the information using specific decision fusion rules and determines the activity status of the PU.

In our studies, we mostly deploy energy detector, and use *OR* fusion rule at the BS. In our CSS scheme in IEEE 802.22 WRAN, we employ feature detection for fine sensing, as specified by the standard.

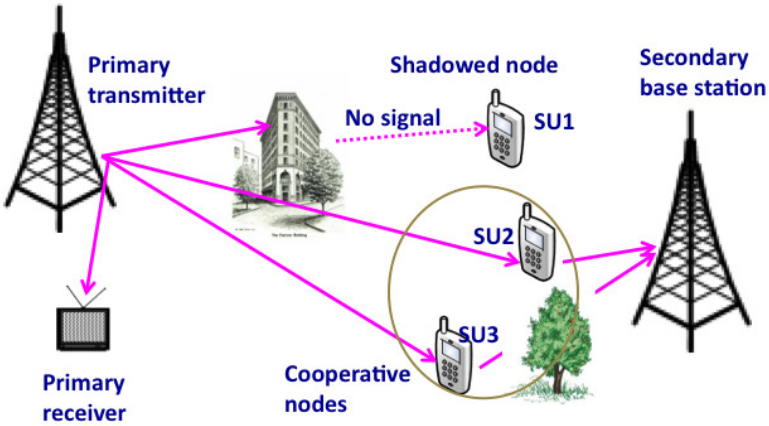

Figure 4: Cooperative spectrum sensing

3 Smart Grid Communications and Demand Response Management

Over the last century the human race has been consuming energy on a scale that continues to break records decade after decade. By 2050 the world's population is estimated to grow from 6.7 billion to 9.2 billion as reported by The United Nations [38]. Naturally, the demand for electricity has been growing, and is anticipated to grow further due to the increased use of machines and new types of appliances such as EVs, Plug-in hybrid electric vehicles (PHEVs), etc. A typical deployment of a conventional electric power grid, is illustrated in Figure 5 [39]. The power is generated by the energy sources, transmitted through the transmission lines and then distributed to the end-users through the distribution lines. A detailed description about the various entities acting at different levels of the current power grid, such as generation companies, transmission companies, market structure, distribution companies, etc., can be found in [40].

While the electric power demands and the reliability requirements are increasing, the current electric power grid is aging. A series of severe blackouts in the United States [41]: Northeast blackout on November 9, 1965, New York city blackout on July 13, 1977, West coast blackout on December 22, 1982, West Coast blackout on July

$2-3, 1996$, West coast blackout on August $10, 1996$, Upper midwest blackout on June $25, 1998$, Northeast blackout on August $14, 2003$, etc., affecting several millions of people, have indicated the inability of the current electric grid to meet the demands and to maintain the reliability of power supply. The recent power cuts in India in July 2012 left 700 million people without power, as energy suppliers failed to meet the growing demands [42], which further verifies the vulnerabilities associated with the current power grid. In addition to its inability to meet the scale and reliability requirements of the growing demands, the conventional power grid plays a significant role in environmental degradation, as electricity generation plants are one of the major contributors to greenhouse gas emission. Integrating renewable energy resources into the electricity grid, is one of the approaches to reduce greenhouse gas emission. However, incorporating renewable energy sources implies inherent intermittency and uncertainties in the supply side. To mitigate or to reduce the impact of such uncertainties, to maintain the reliability of power supply, and to enable DRM to be effectively explored and exploited, it is essential to transform the traditional power grid into a more responsive, efficient and reliable system.

3.1 Smart Grid

The smart grid [7] is an energy information network that incorporates a smart metering infrastructure capable of sensing, measuring and shifting the power consumption of consumers using advanced ICT, i.e. smart grid communications. Bi-directional smart grid communications between the supply and the demand sides will enable consumers to control and manage their electricity usage, and it will help the providers or the utility companies to plan and manage the generation and supply of power in a more structured and efficient manner. Thus the power generation, distribution and consumption is expected to be more efficient, more economical and more reliable in the SG. A typical SG scenario is depicted in Figure 6, where the the upper part of the figure represents the power flow and the lower part represents the smart grid communications network. The power flow is unidirectional, i.e., from the energy sources to the end-users, but the information flow is bi-directional.

Smart grid communications play a central role for effective implementation of the DR programs or for maintaining and enhancing the reliability of the power supply. In the SG, each end-user (a household, a commercial user or an industrial user, with a number of electrical appliances) will have a smart meter. The smart meters are programmed to determine their strategies that minimize the cost for the users or the cost of power generation, as needed. Groups of end-users form a neighborhood area network (NAN). The data from each NAN is supplied to the control unit (CU) through an aggregator. Several NANs form a wide area network (WAN). The data from a WAN is

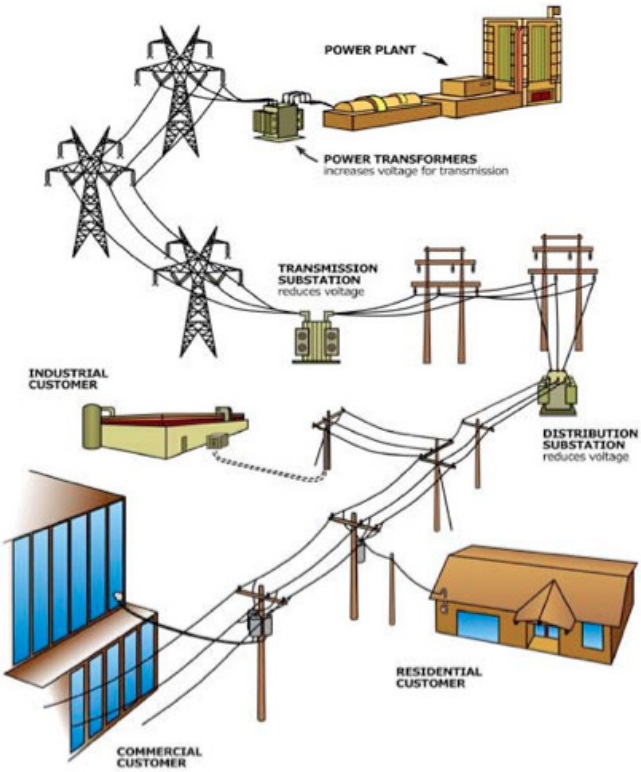

Figure 5: The processes of electric power generation, transmission and distribution in conventional electric power grid [39] (included with the permission of the auhors)

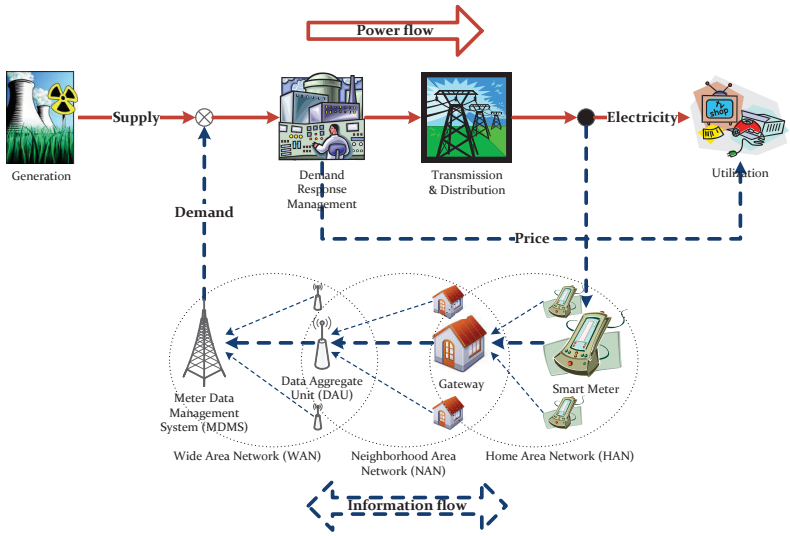

Figure 6: The smart grid: an intelligent grid system incorporating ICT into the electricity grid for communication and control.

communicated to the meter data management systems BS, which further submits this data to the control center. The control center, based on the electricity usage patterns or demands or preferences of the users, will in turn, adjust the supply parameters such as the amount of power to generate, the unit price etc.

The U.S. Government has announced the largest power grid modernization investment in the U.S. history, i.e., 3.4 billion USD in grant awards, funding a broad range of smart grid technologies [43]. Local Distribution Companies (LDCs) are integrating advanced metering and two-way communication, automation technologies to their distribution systems [44].

3.2 Demand Response Management

In [45], DR is defined as *Changes in electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized.*

DR from electricity consumers facilitated at higher time resolution is important for

deregulated competitive electricity markets to function properly. An active demand side that responds when wholesale prices vary, may contribute to efficiency and reliability, reduced price volatility, mitigation of exercise of market power, as well as several other advantages in the electricity market. Despite this importance, the traditional electricity markets are characterized by either no response or a rather low response from the demand side. One reason for the low response in the traditional power grid is that most consumers do not obtain the information about the time-differentiated prices that reflect the wholesale price variation. Instead, they have prices that are fixed for longer periods of time (weeks, months, years). Currently, many of these consumers choose the fixed contracts voluntarily, in order to ensure more stable and predictable prices. Consumers can not afford the possibility of making different choices since most households are equipped with an electric metering system which can only measure their accumulated electricity consumption. This implies a disconnection between the wholesale market and the retail market in which households purchase their power. The rapidly changing costs of electricity are not communicated to the consumers, and consumers' willingness to pay for electricity is not reflected in the market. As a result, the consumers lack incentives to respond to immediate market price fluctuations.

Lack of demand response may have adverse implications. Electric generators with high costs may be used to cover demand during short-term peak price periods, even though many consumers would have preferred to use less electricity if they had obtained the actual cost of their consumption. Similarly, because consumers increase consumption during short-term off-peak price periods, some generators are not utilized even though they may offer electricity at costs below what many consumers would be willing to pay if they had the opportunity to do so. This inefficiency in the allocation of resources may also have long-term impacts through inefficient investments in generation capacity. Moreover, low demand response, accompanied with the special properties of electricity as a commodity such as non-storability, capacity constraints and long lead times for new capacity expansions, also contributes to volatile prices. Less participation of the consumers in demand response may make it easier and more profitable to exercise market power, which exacerbates price volatility. Increased price volatility increases uncertainties regarding long-run average rate of return on capacity investments which in turn, may reduce the investment level and thus the reliability of power supply.

There are two basic types of DR options, price-based demand response and incentive-based demand response. Price-based demand response includes real-time pricing (RTP), critical-peak pricing (CPP), and time-of-use (TOU) rates. In price based demand response programs, customers can respond to the price structure with changes in energy use, reducing their electricity bills if they adjust the timing of their electricity usage to take advantage of lower-priced periods and avoid consuming when prices are

higher [45]. Incentive-based demand response schemes pay participants to reduce their loads at times requested by the program sponsor, triggered either by a grid reliability problem or high electricity prices [45]. In the current electricity grid, basic types of DR programs have been implemented such as CPP, TOU, and incentive-based demand response schemes. However, the available information to the consumers for these programs are not real-time. E.g., in CPP schemes, consumers are informed of a very high unit price during peak time slots, which is the same for days, weeks or even months, years, and the consumers try to avoid or minimize consuming energy in these slots. In TOU, the unit prices are different at different hours in a day (e.g., peak hours vs. off peak hours), which is informed to the consumers beforehand. The consumers schedule their appliances at cheaper time slots. Although these schemes have fundamental sense of DRM, they are essentially non-real time schemes and they fail to take into consideration the response from the consumers if the prices should be adjusted in real-time.

In the SG, DRM is a much wider concept where the users have options in scheduling their appliances in time, choosing providers, and the feedback from the demand side to the CU is provided at a much finer granularity, e.g., every 10 minutes. When consumers know the prices that are closer to the marginal costs of supply through real-time-differentiated tariffs, and are metered automatically, they have incentives to adjust their demand to the varying prices. Consequently, the generators can be operated near their optimal operating points, and a reliable power supply can be maintained to the customers. Thus, in the SG, the significance of DRM goes beyond reducing the power generation costs and the costs for the users.

DR programs typically specify a method for establishing customers' baseline energy consumption level below which demand reductions are not allowed. The energy requests that customers send to utility/providers consist of two parts: non-flexible load request and flexible load request. The non-flexible part is the minimum amount of energy that the utility/providers needs to provide at a specific time. The flexible part can be reallocated over time according to a certain load management strategy. For any load management strategy there are two common primary goals: peak load saving, and load profile flattening, also called peak-to-average-ratio (PAR) minimization, [46], [47], [48].

Buildings (residential, commercial and/or industrial) are one of the greatest consumers of electric power. For instance, in The United States, at a macroscale, buildings use approximately 70% of total electricity usage and emit approximately 40% of greenhouse gases annually [49]. Thereby, we solve DRM problems for residential, commercial and industrial users (buildings), and propose efficient and reliable solutions in the context of multiple providers and multiple consumers. We develop provider selection schemes as well as schemes for load scheduling in time.

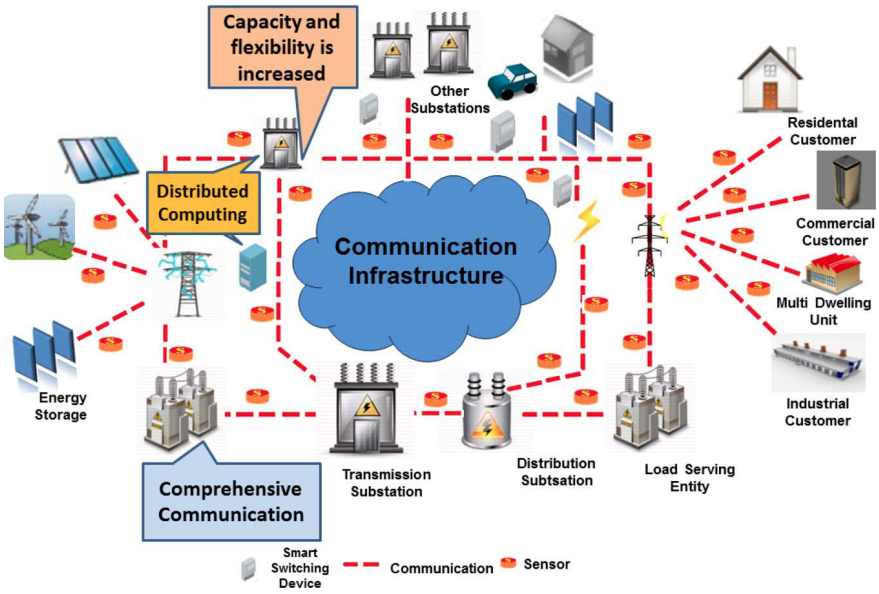

Figure 7: Advanced sensing and control through modern communications technologies in the smart grid [50] (included with the permission of the auhors)

3.3 Smart Grid Communications

The interactions between the providers and the consumers are enabled by the smart grid communications infrastructure between the supply and the demand sides. An advanced metering infrastructure is a communication infrastructure that enables meters and providers to exchange information such as power consumption, price update, or outage awareness. Different components of the smart grid system are linked together with communication paths as shown in Figure 7 [50]. In the SG, real-time and reliable information becomes the key factor for reliable power generation, and distribution from the generation units to the end-users. The impact of equipment failures, capacity constraints, and natural accidents and catastrophes, which cause power disturbances and outages, can be largely avoided by online power system condition monitoring, diagnostics and protection [51]. To this end, the intelligent monitoring and control enabled by modern information and communication technologies, i.e., smart grid communications, have become essential to realize the envisioned SG [50].

Currently, in addition to research and development activities, many electric utilities

are taking incremental steps to make the smart grid technology a reality. Most of them are signing agreements with telecom operators or smart meter vendors to carry out smart grid projects. All these agreements define the main requirements and features of the necessary communications infrastructure to provide real-time communications between smart meters and the utilities'/providers' back-haul system, i.e., the advanced metering infrastructure [50].

With the integration of advanced technologies and applications for achieving a smarter electricity grid infrastructure, a huge amount of data will be generated and needs to be communicated for further analysis, control and real-time pricing mechanisms. Hence, it is of critical importance for electric utilities to define the communications requirements and find the best communications infrastructure to handle the output data and deliver a reliable, secure and cost-effective service throughout the system.

Communication technologies supported by two main communications media, i.e., wired and wireless, can be used for data transmission between smart meters and providers. In some cases, wireless communications have some advantages over wired counterparts, such as low-cost infrastructure and ease of connection to difficult or unreachable areas. However, energy constraints, signal attenuation and interference may degrade the performance of communications. On the other hand, wired solutions do not have these problems. However, they are more expensive. Different technologies have been proposed for smart grid communications, such as GSM, GPRS, 3G, WiMAX, PLC, ZigBee, etc., [50]. Based on the specific parameters of each technology e.g., the coverage range, data rate offered, sensitivity towards noise and interference, etc., their applications vary, e.g., advanced metering, DR, home area networks, fraud detection etc.

While SG communications facilitate DRM and enhance the capability of DRM, the SG needs resources for communications too. Wireless communications are cost effective, and are likely to be the technology for communications in the SG. However, due to the spectrum scarcity issue discussed in Section 2, SG communications should be based on the ISM band, or on opportunistic access of the spectrum licensed to the PUs. Thus, radio resource management and optimization for smart grid communications, is an important research problem, and is another problem we address in this thesis.

4 Game Theory

For resource management and optimization in NGNs such as CRNs and SGs, we need to address several research challenges. Since the economic problems involve multiple players whose decisions are coupled, we employ game-theoretical approaches in many of our studies, to solve the economic problems in CRNs and SGs.

Game theory is a formal analytical framework with a set of mathematical tools to study the complex interactions among independent rational players. It aims to help us understand the situations in which decision-makers interact. A game in the everyday sense is *a competitive activity in which players contend with each other according to a set of rules* [52]. But, the scope of game theory is much larger. Throughout the past decades, game theory has made a revolutionary impact on a wide number of disciplines ranging from economics, politics, philosophy, or even psychology, [52], [53]. Some examples from our day-to-day lives, which signify the range of situations to which game theory can be applied are, firms competing for business, political candidates competing for votes, jury members deciding on a verdict, animals fighting over a prey, bidders competing in an auction, and the role of threats and punishment in the long-term relationships etc.

Game theoretic approaches can be distinguished into two main categories: Non-cooperative [54], [55] and cooperative game theory [53], [56]. While non-cooperative game theory mainly deals with modeling competitive behavior, cooperative game theory is dedicated to the study of cooperation among a number of players.

In the emerging NGNs such as a CRN or a SG, the players (PUs and SUs for a CRN, utilities/providers and consumers for a SG) are autonomous decision makers. Thus, the emergence of such networks and the recent interest in resource management and optimization in such networks has brought to surface many interesting game theoretic problems that arise from the competitive and cooperative interplay of the different players. Furthermore, the need for self-organizing, self-configuring, and self-optimizing networks eventually led to the use of many game theoretic concepts in wireless communication networks [57], [58]. Cooperative game theory models are mainly useful when the entities involved are concerned about *the greater good of the system* than the payoff of each individual player. For instance, when all the players belong to one single network where all players are controlled by a single centralized entity such as a BS or a CU, or when all the players are homogeneous meaning that maximizing the social welfare also leads to optimal solution for each player, or when every single player is programmed to act towards optimizing the total payoff. This is, however, often not the case in complex and heterogeneous networks like CRNs and SGs where distributed implementations are preferred/required. When a central entity controls all the players, optimal centralized solutions can be obtained by using optimization techniques [59]. The essence of game theoretical models lies in modeling and designing schemes and algorithms for multi-player optimizations.

Consequently, in NGNs like CRNs and SGs, game theory proves to be a very powerful and appealing tool to model and solve various economic research problems. Next in the following subsection, we explain the basic concepts of game theory.

4.1 Basic Concepts

Actions and Strategies. Each decision-maker in a game is associated with two components: a set **A** consisting of all the actions that, under some circumstances, are available to the decision-maker, and a specification of the decision maker's preferences [52]. In any given situation, the decision maker must choose a single element from a subset of **A**. The choices available to the player are also called its strategies. The decision-maker knows this subset of available choices, and takes it as given. The subset is not influenced by the decision maker's preferences. The set **A** could, for example, be the set of the amount of electric power that the decision maker can possibly buy/consume, given its budget at any time, or it can be the set of time-slots that the decision maker can possibly schedule to consume a given amount of total power, etc.

Preferences and Payoff functions. Let us assume that the decision maker, when presented with any pair of actions, knows which of the pair it prefers, or knows that it regards both actions as equally desirable [52]. Further, let these preferences be consistent, i.e., if the decision maker prefers action a to action b, and action b to action c, then it prefers action a to action c. In particular, how much a player likes an outcome depends on some other player's payoff as well. One way to describe a decision-maker's preferences is to specify, for each possible pair of actions, the action the decision-maker prefers, or to note that the decision-maker is indifferent between the actions. Alternatively we can represent the preferences by a payoff function, which associates a number with each action in such a way that actions with higher numbers are preferred. More precisely, the payoff function u represents a decision-maker's preferences if, for any actions $a, b \in A$, $u(a) > u(b)$ if and only if the decision-maker prefers a to b. The payoff function is also called a preference indicator function. In economic theory, a payoff function that represents a consumer's preferences is often called a utility function.

The Theory of Rational Choice. The theory of rational choice states that, in any given situation the decision-maker chooses the element of the available subset of A that is best according to its preferences [52]. The response of the player should give the highest payoff compared to choosing any other element of the subset, given the choices of other players. The function that represents the response yielding the highest payoff is called the best response function of the player, and the corresponding strategy is called the best response strategy. If there are several equally good actions, the theory of rational choice demands that, the action chosen by a decision-maker should be at least as good, according to its preferences, as every other available action. The theory of rational choice, the best response, etc., are the fundamental concepts used in game theory. We use these concepts in many of our studies.

4.2 Typical Game Models and Equilibria

In this section we describe typical game models and the equilibrium concepts used in the papers included in this thesis.

4.2.1 Strategic game

A strategic game is a model of interacting decision-makers. The decision-makers are the players. Each player has a set of possible actions. The model captures interactions between the players by analyzing the effect of the actions of all other players on each player, in addition to its own action. Specifically, each player has preferences about the action profile, i.e., the list of all the players' actions. More precisely, a strategic game is defined as follows.

A strategic game consists of

- a set of players, usually denoted by $\mathscr{N} := \{1, 2, \ldots, N\}$, where $N := |\mathscr{N}|$,
- for each player $n \in \mathscr{N}$, a set of actions $\mathbf{A}_n := \{a_n\}$
- for each player $n \in \mathscr{N}$, preferences over the set of action profiles or payoff functions or utility functions, usually written as u_n.

4.2.2 Nash equilibrium

In the idealized setting in which the players in any given play of a strategic game are drawn randomly from a collection of populations, a Nash equilibrium (NE) corresponds to a steady state, and is the most widely used equilibrium concept for the non-cooperative games modeling a variety of problems from different fields. The definition of an NE follows.

For a finite N-player game in strategic form $\mathscr{G} := \{\mathscr{N}, \{\mathbf{A}_n, n \in \mathscr{N}\}, \{\mathbf{u}_n, n \in \mathscr{N}\}\}$, where $\mathbf{u}_n$ is the utility function of player n for taking different actions available in the action set of player n, $\mathbf{A}_n$. Let us represent the actions taken by player n and the rest of the players as a_n and $\mathbf{a}_{-n}$, respectively, and let $\mathbf{A}_{-n}$ denote the action set of all players other than n.. Then, for all $n \in \mathscr{N}$, given the strategies of all other players $\mathbf{a}^*_{-n}$, the strategy $a^*_n \in \mathbf{A}_n$ is an NE for player n if for every alternative strategy $a_n \in \mathbf{A}_n$, $(a^*_n \neq a_n)$, the following relation holds:

$$u_n(a^*_n, \mathbf{a}^*_{-n}) \geq u_n(a_n, \mathbf{a}^*_{-n}). \tag{2}$$

Equation (2) indicates that at the NE, a player cannot improve its payoff by deviating alone from the equilibrium, given the strategies of all other players.

4.2.3 Correlated equilibrium

When a game has multiple Nash equilibria, the choice of the equilibrium that yields maximum payoff for each player is an issue. A correlated equilibrium (CE) is a solution concept widely used to improve the efficiency of NE. It is more general than NE and is based on the assumption of the presence of a trusted authority, which instructs the players which strategy to choose based on an unbiased probabilistic rule. If no individual player has an incentive to deviate from the recommended strategy assuming that the others do not deviate either, such a distribution is called correlated equilibrium.

A probability distribution P_d over the strategy space $A_1 \times ..... \times A_N$ is a CE if for every strategy $a_n \in \mathbf{A}_n$ such that $p(a_n) > 0$, and for every alternative strategy $a_n^{'} \in A_n$,

$$\sum_{\mathbf{a}_{-n} \in \mathbf{A}_{-n}} p(a_n, \mathbf{a}_{-n}) u(a_n, \mathbf{a}_{-n}) \geq \sum_{\mathbf{a}_{-n} \in \mathbf{A}_{-n}} p(a_n, \mathbf{a}_{-n}) u(a_n^{'}, \mathbf{a}_{-n}), \tag{3}$$

where $\mathbf{a}_{-n}$ denotes the strategy chosen by player $m : m \in \mathscr{N}, m \neq n$, and $p()$ represents the probability of the event inside the parenthesis occurring. CE is a very general concept, and it can be applied quite flexibly to optimize the playoff of the system or a particular player(s), as required, through the objective function employed by the trusted authority. For instance, the solution of (3) with an objective function based on social welfare, provides the solution that maximizes the total payoff of the system. If instead, the objective function is based on max-min fairness, the solution will maximize the payoff for the user with lowest payoff.

4.2.4 Stackelberg game and Stackelberg equilibrium

The Stackelberg game, also called Stackelberg leadership model [54], is a strategic game in which there is at least one player defined as the leader who can make the decision and commit the strategy before other players who are called followers. The players usually engage in Stackelberg competition if one has some kind of incentive to move first. The strategy chosen by the leader can be observed by the followers, and the followers can adapt their decisions (also called optimal reaction or best response), accordingly. For instance, let us consider a two player game, where both of them aim to maximize their profits. In this scenario, the leader can choose a strategy such that its profit is maximized, given that the followers will choose their best responses. This solution is called the Stackelberg equilibrium (SE), and can be obtained by backward induction technique. With backward induction, the best response of the follower is obtained first given the price set by the leader, i.e., $y_f^* = \mathscr{B}_f(y_l^*)$, where y_f^*, y_l^* are respectively, the price set by the follower based on its best response, and the price set by the leader to maximize its profit, and $\mathscr{B}_f()$ is the best response function of the

follower. Then, this best response is used to compute the profit of the leader, and the leader chooses a strategy for which the profit is maximized i.e.

$$y_l^* = \max_{y_l} u(y_l, \mathscr{B}_f(y_l)). \tag{4}$$

4.3 Applications for Solving Economic Problems in NGNs

Game theory presents a rich framework that can be used to model various aspects of resource management in NGNs. One can utilize various solution concepts of game theory for studying the competition and cooperation among the entities in NGNs.

The application of game-theoretic approaches for solving resource management problems in CRNs has been well explored. Thus a good volume of literature is available on resource management and optimization in CRNs using game-theoretical framework. We provide a comprehensive survey on economic approaches for resource management and optimization in CRNs in [60]. However, in the spectrum sensing aspect, which is crucial for awareness of the constantly changing radio environment, the application of games, have not been explored sufficiently. In the case of the SG, DRM problems have been well investigated using centralized optimization techniques. Nonetheless, game-theoretical models can be applied to improve the payoffs and the reliability of both the demand and the supply sides. Moreover, the existing analyses focus mainly on either only the supply side, or only the demand side. In this regard, they lack realistic elements, since in a real scenario there are many end-users and also there will be multiple providers in the context of the SG. Furthermore, in most of the existing studies, the results and claims are based on the simulations. They lack analytical evaluation, which saves time for evaluating the proposed schemes, architectures or algorithms, and which strengthens the claims of a study.

Motivated by this state of the art, it is of interest to us to: (i) - propose advanced applications of game theory in NGNs, and (ii) - derive analytical solutions with justifiable assumptions for analytical tractability, discover new solutions to improve the performances for the well studied problems, and (iii) - provide more general solutions to the existing problems by integrating various elements into a single framework.

Various game models and solution concepts can be employed to capture interactions among players for a wide variety of problems related to cooperation enforcement in the NGNs, E.g., non-cooperative games, NE, CE, etc. Furthermore the resource optimization problems for entities at different levels of NGNs (e.g., PUs and SUs, electricity providers and consumers, etc.,) can be characterized under a single framework using models such as a Stackelberg game. Hence, using economic and game-theoretical approaches, one can study different problems in the NGNs such as (but not limited

to): (i)- QoS improvement and payoff enhancement for SUs through collaboration for spectrum sensing in CRNs, (ii)-User centric cost minimization oriented DRM in the SG, (iii)- Joint utility maximization for electricity consumers and profit maximization for the providers in the SG, and (iv)- Opportunistic spectrum access based smart grid communications.

Naturally, users in any network are selfish and rational and they are concerned more about maximizing their own payoffs rather than the system payoff. Interestingly, however, when there exists a cost-benefit tradeoff for cooperation, cooperation can be enforced among the players even if they are selfish. For instance, in [61], cooperation is enforced among the SUs for spectrum sensing due to the associated cost for not sensing. Similarly, in [62] we use a non-cooperative game framework to characterize the behavior of the electricity consumers but the scheduling is cooperative because of the pricing scheme close to the cost of power generation.

5 Research Methodology

The research methods widely adopted for modeling various systems and networks in computer science and communication engineering can be broadly classified into three categories [63], [64]:

- Theoretical/analytical methods based modeling
- Computer simulation based modeling
- Experimental evaluation based modeling

Theoretical/analytical methods based modeling.

Theoretical/analytical methods based modeling follows the classical approach of building mathematical theories supported by logical reasoning with stringent definitions of objects and operations for deriving and proving mathematical expressions for functions, values or bounds. The results obtained from theoretical methods based modeling in computer science are judged by the insights they reveal about the mathematical nature of various models of computing and/or by their utility to the practice of computing and their ease of application, e.g., the mathematical upper and lower resource bounds for the solutions of various problems.

Computer simulation based modeling. Computer simulation based modeling can efficiently handle large data sets, can access a variety of distributed resources and collaborate with other expert systems or resources over the internet. This method is very efficient to tackle problems of great complexities where exact mathematical forms for

prerequisite functions are not available, and where the analytical solutions can not be derived without making a great deal of simplifications in the mathematical model(s).

Experimental evaluation based modeling. Experiments can be conducted, and the evaluation can be used for both theory testing and exploration. In this approach, the researchers observe phenomena, formulate explanations and theories, and test them in real devices operating in real networks, to verify their theoretical solutions. Besides, experiments can also be used in areas where theory and deductive analyses are difficult to apply, and can help researchers derive theories from observation.

Theoretical/analytical methods are typically less resource demanding than their simulation and experiment based counterparts and thus it is possible to obtain results faster. However, given the approximations that are usually undertaken in these methods, the applicability of the obtained results under generic scenarios in real systems may be limited.

Computer simulation can be used as a tool to strengthen or support the validity of the findings from an analytical model (and vise versa), where such a model exists. The simulator can also be used to provide a set of results, making it possible to make both qualitative and quantitative claims on the performance of the proposed schemes or models or algorithms.

The third option, of implementing the protocol in a real device, is often done as the very last step, to verify the performance of the protocol in a real network. In the case of experimental evaluations, the associated equipment costs, manpower cost and costs due to time required are usually big. Consequently, it is common to use the analytical and simulation based research methods in computer science and communication engineering.

In this thesis also, we employ analytical and computer simulation based research methods for modeling the behavior of different entities, and for deriving the equilibrium of various game models, and the optimal solutions of different schemes. For instance, in [65], we use queuing theory to derive the average packet delays for the SUs and validate them with computer simulations using a discrete event simulation technique. All computer simulations included in this thesis were performed using Matlab [66]. In [61], [62], [67], [68], we use different kinds of games, introduce the equilibrium concepts such as the NE, CE, SE etc., for specific game models, and derive closed form or analytical solutions. In [69], we employ an optimization based framework and verify the results with computer simulations.

6 Contributions of the Included Papers

We proposed advanced applications of game theory for NGNs such as CRNs and SGs. In the specific research problems where a system or a network benefit is more important and realistic, and multi-player optimization is not the central theme, we deployed optimization techniques to achieve *the greater good* of the system. The problems we solved can be mainly categorized into three parts. In the first part, our work deals with the tradeoff between spectrum sensing and spectrum access in CRNs. We design a mechanism based on the mutual cooperation among SUs of a CRN to reduce the delay caused by spectrum sensing, and thus to increase the spectrum access and utilization time relative to spectrum sensing time. Next, we deploy a game-theoretical framework in order to enforce cooperation among the SUs for spectrum sensing in CRNs and use the concept of CE to improve the payoff of the SUs. In the second part, we shed light on DR programs enabled by smart grid communications, in the SG. Specifically, we build an analytical framework using different game models, as required by the scenario of interest, for the context of multiple electricity providers and multiple electricity consumers, considering payoff maximization for both the supply and the demand sides. Moreover, we provide large population approximations for DRM under the Stackelberg game framework. In the third part of our study, we propose a cognitive radio enabled smart grid and present a thorough analysis of the impact of opportunistic spectrum sensing and access in the control performance of the SG. We also study the dependability issues associated with DRM, caused by the vulnerability of smart grid communications towards external attacks.

This thesis consists of seven papers: Paper I to Paper VII. In the rest of this section, we elaborate on our contributions by presenting a brief summary of these papers.

6.1 Paper I

Title: Economic Approaches for Cognitive Radio Networks: A Survey
Authors: S. Maharjan, Y. Zhang and S. Gjessing
Published In: Wireless Personal Communications, Springer, Special Issue on Cognitive Radio Networks and Communications, Vol. 57, No. 1, pp. 33-51, 2011.
(Submitted on 3rd October 2009, Accepted on 9th February, 2010 published online: May 2010).

This paper presents a survey of the applications of economic theories such as game theory, price theory and market theory, for solving the resource management and optimization problems in CRNs. (An extension of the paper can be found in [70]). In

this paper, we focus on providing an application-oriented state of the art analysis of economic approaches for solving a wide variety of problems in CRNs from spectrum sensing, to spectrum access, to spectrum sharing.

In this paper, we use the term *spectrum sharing* and the term CR technology interchangeably. Based on the existing literature, spectrum sharing can be facilitated by two different techniques: (i)- spectrum sensing, and (ii)- provision of explicit information from PUs or primary service providers to the SUs about the spectrum usage. For spectrum sharing enabled by either of these techniques, we provide a novel classification of spectrum sharing based on the issues addressed in the literature. First, we point out two distinct streams of research problems addressed in the context of spectrum sharing: (i)- resource allocation, and (ii)- spectrum trading.

Resource allocation here, refers to the case of opportunistic spectrum access by the SUs. In this scenario, there is no motivation for the PUs to participate in the spectrum sharing process because they do not get any benefit by letting the SUs use their spectrum. In this approach, the primary users are inflexible and the overall responsibility of maintaining peaceful coexistence with PUs is on the SUs, thus making the implementation aspect more complex and guaranteeing the performance harder. Resource allocation can be in terms of frequency bands, access time for the same channel(s), transmission power etc. We further divide the research issues addressed under *resource allocation* scenario as: (i)- resource allocation between PUs and SUs, and (ii)- resource allocation among SUs. Both of these problems have been well investigated using both cooperative and non-cooperative games.

Spectrum trading is the economic aspect of spectrum sharing in an incentive driven framework of the coexistence of primary and secondary users. In the spectrum trading based approaches, the PUs can lease the spectrum to the SUs whenever and wherever they are not using the particular bands, which in turn gives the PUs monetary or other benefits from the SUs. In a spectrum trading scenario, while PUs compete to sell the spectrum in order to maximize their revenues, SUs compete to get the spectrum according to their needs at better prices to maximize their satisfaction. Spectrum trading can be between PUs and SUs i.e., where the PUs sell the spectrum to the SUs, or it can be among SUs only i.e., a central authority such as a BS or a spectrum broker, purchases a certain amount of spectrum from a primary system and the SUs compete to get the spectrum from this intermediary agent. Thus, we further classify the spectrum trading models as (i)- competition among PUs to sell the spectrum, and (ii)- competition among SUs to buy the spectrum.

We develop another classification for spectrum sharing based on approaches used for solving the above mentioned research problems. We classify the approaches and solutions into three categories: (i)- the research problems solved using game theory, (ii)- the research problems solved using market theory and price theory, and (iii) the

research problems solved using *joint strategy*. i.e., a combination of game theory with market structure and/or pricing schemes. While game theory provides us modeling tools for understanding the behavior of rational players when their payoffs are interdependent with the actions, strategies and payoffs of other players, price theory explains how relative prices are determined and how prices function to coordinate the economic activities, and market theory describes how market structure affects the setting up of the price of commodities and/or how pricing schemes affect the stability of the market. The classification *joint strategy* in this paper refers to games with pricing in order to enforce cooperation among non-cooperative users, or to avoid unnecessary competition, or to incentivize the players to act according to the benefit of the system, etc.

After developing this classification based on the research issues and solution approaches, we provide a comprehensive taxonomy of the existing literature in terms of the *main research issue addressed*, the *main approaches or specific game/pricing/market models used*, the *structure* such as the centralized or distributed design, the *solution concepts derived* such as the equilibrium or the optimal solutions, fairness indices, etc. Finally we provided some open research directions.

Author Contribution. For this paper, I was the main contributor. I was responsible for the literature survey and the taxonomy. I got help from Yan Zhang and Stein Gjessing in making the paper contents clear and precise.

6.2 Paper II

Title: Delay Reduction for Real Time Services in IEEE 802.22 Wireless Regional Area Network
Authors: S. Maharjan, J. Xiang, Y. Zhang and S. Gjessing
Published In: Proc. IEEE 21st International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC 2010), pp. 1836-1841, 2010.

In this paper, we address the problem of improving the strict QoS requirement of a certain type of data such as voice and video, arising from the SUs of a CRN. We focus on an IEEE 802.22 WRAN based secondary system, i.e., where the CRN accesses the TV spectrum opportunistically. As specified in IEEE 802.22 WRAN standard, the SUs should sense the spectrum in two stages. In the first stage, the fast sensing, based on energy detection, is performed. If an alarm is generated by fast sensing, then, the SUs perform fine sensing, i.e., using the feature detection. Fast sensing durations are short, but the fine sensing durations are usually long, and they can affect the transmission of

the SUs severely, especially if the SUs have traffic with strict QoS requirements such as voice and video. The durations of spectrum sensing (both fast and fine sensing) are called quiet periods (QPs) in IEEE 802.22 WRAN, since the SUs listen to the transmission of PUs, without transmitting their own data, during these periods. We classify the SUs carrying traffic with strict delay requirements as real time traffic users, and those with more relaxed delay requirements as non-real time traffic users. We propose collaboration among the SUs of a CRN in order to facilitate the latency requirements of real time traffic, without compromising the detection performance of spectrum sensing. The meaning of *collaboration* in this paper is slightly different than the conventional meaning of *CSS*. We exploit the non-real-time traffic users to perform fine sensing (if an alarm is issued by fast sensing) while the real-time traffic users continue their data transmission. Moreover, when there are different types of traffic with different requirements on tolerable latency, priority based packet scheduling schemes are normally used in order to reduce the queuing delay for real time services. Thus, we combine our collaborative fine sensing scheme with the priority queuing scheme, where a certain data rate is reserved for the real-time traffic users in order to facilitate their QoS requirements, thus, significantly reducing the overall packet delay, including the delay due to sensing for real time services in CRNs. We deploy an M/D/1 queue, and derive analytical solutions for average packet delay for the proposed scheme, and verify them with the simulation results. We assess the impact of the real time packet arrival rate on the average packet delay faced by the traffic in the IEEE 802.22 WRAN. We compare our proposed scheme (PROP-Conv) with two other schemes: FCFS-Conv, the conventional sensing approach defined in IEEE 802.22 WRAN with first-come-first-serve (FCFS) based queuing system, and PRIO-Conv, the conventional sensing approach defined in IEEE 802.22 WRAN with priority queuing. The numerical results show that the priority based scheduling scheme combined with our sensing scheme (PROP-Conv) substantially reduces the average packet delay for real time services. The results indicate that, for a given total data rate, the average packet delay for non-real time services increases considerably when the real-time packet arrival rate rises, in all three schemes. However, the delay for real-time traffic is almost not affected by the increase in the real-time packet arrival rate in PROP-Conv, while the delay increases proportionally in FCFS-Conv and PRIO-Conv schemes.

Author Contribution. For this paper, I was the main contributor for the idea, implementation and simulations. Jie Xiang, Yan Zhang and Stein Gjessing contributed through regular discussions on improving the presentability of the paper.

6.3 Paper III

Title: Distributed Spectrum Sensing in Cognitive Radio Networks with Fairness Consideration: Efficiency of Correlated Equilibrium
Authors: S. Maharjan, Y. Zhang, C. Yuen and S. Gjessing
Published In: Proc. IEEE 8th International Conference on Mobile Ad-hoc and Sensor Systems (MASS 2011), pp. 540 - 549, 2011.

In this paper, we study CSS in a CRN, where the cooperation is enforced among the SUs, for improving spectrum utilization and to improve the payoffs of the SUs. Given a bunch of selfish SUs, who want to access channels from the PUs (as long as they have data to transmit), and who wish to be the *free riders* without participating in spectrum sensing if possible, we exploit the fact that not cooperating for spectrum sensing can incur a heavy cost to all the SUs, and design a framework to enforce cooperation among them, taking into account the spectrum sensing costs such as time and energy.

We develop a game theoretical model among the SUs for CSS, and introduce the concept of the CE, which improves the payoff of the SUs, compared to the case of mixed strategy NE. Moreover, we characterize the SUs with a traffic density factor which captures the traffic dynamics of each SU in terms of the necessity to access the available channel. We, then develop the CE of the spectrum sensing game for the heterogeneous SUs. We discover that the social welfare can be significantly improved when the users have heterogeneous traffic dynamics. While maximizing system payoff is important, fairness is also equally important in systems with dissimilar users. We find that the SUs with relatively less traffic density may be exploited to a severe extent for benefitting the SUs with relatively higher traffic density. In order to address fairness issue, we propose a new fair social welfare correlated equilibrium (f-SW CE), which maximizes the system utility and ensures that the less well-off users do not starve. We employ a no-regret learning algorithm for distributed implementation of the CE so that each SU can reach the CE of the game locally. Finally, we propose a neighborhood based learning (NBL) algorithm that maximizes the total utility using the information from one-hop neighbors.

The numerical results verify that the total system payoff increases in the CE compared to the mixed NE. The payoff for relatively higher traffic density follows similar pattern. The relatively light traffic users may have to sacrifice their payoffs though. We compare the performance of our proposed f-SW CE scheme with the three standard fairness based schemes: social welfare CE (SW CE), proportional fairness CE (PF CE) and max-min fairness CE (MMF CE). The results suggest that the total utility at f-SW CE is higher than at MMF CE but lower than at SW CE, and can be comparable with

the payoffs at PF CE depending on the value of the traffic density parameter. By appropriately choosing the tradeoff between the system efficiency and fairness, our f-SW CE scheme can yield system utilities very close to the SW schemes for fairly reasonable spectrum sensing durations. The results on the NBL algorithm show that it achieves better performance than the no-regret algorithm in terms of both individual payoff and the total payoff of the SUs.

Author Contribution. For this paper, I was the main contributor for the idea, implementation and simulations. Yan Zhang, Chau Yuen and Stein Gjessing contributed through regular discussions on improving the contents and presentability of the paper.

6.4 Paper IV

Title: Dependable Demand Response Management in the Smart Grid: A Stackelberg Game Approach
Authors: S. Maharjan, Q. Zhu, Y. Zhang, S. Gjessing and T. Başar
Published In: IEEE Transactions on Smart Grid, Volume 4, Issue 1, pp. 120-132, March 2013.

In this paper, we study the market level activities for DRM in the SG, glueing together multiple electricity providers' and multiple consumers' behavior, into a single framework. Given a number of residential, commercial and industrial electricity consumers and multiple electricity providers in an oligopolistic market framework, where each entity is concerned about maximizing its own payoff, we provide an interaction model between the providers and the consumers, that captures the coupling among the providers as well as the (indirect) coupling among the consumers.

We associate a budget with each consumer, and the power available with each provider is given. When there are multiple providers in a market, users prefer to buy power from the cheapest provider. This fact introduces competition among the providers. As a consequence, the providers compete to set reasonable unit prices for the power available with them. Similarly, since, the budget available with each user is fixed, they may compete only for the cheapest provider. For fair allocation of the resources, we assign logarithmic functions as the utility of the consumers so that each consumer can purchase the maximum from the cheapest provider, but every user must purchase from all providers, proportional to their budgets. This makes the coupling among the users rather indirect, through the prices.

The problem is modeled as a Stackelberg game with two levels: the providers play

a non-cooperative game to determine their unit prices for given power with them, to maximize their revenues, and the users respond with their optimal reaction to the unit prices, according to the budget they have, to maximize their utilities. In the Stackelberg game, the providers are the leaders and the end-users act as the followers. We derive analytical solutions for the strategies of both the supply and the demand sides under the holistic Stackelberg game approach. In particular, we obtain analytical results for the SE of the game and prove that a unique SE exists. We derive closed form solutions for the unit prices offered by different providers, and also for the demands of each consumer from each of those providers. With the help of the closed form solutions, we derive lower bound on the budget of the consumers in order to be able to purchase power from the market. Through numerical results we illustrate that the change in the affordable budget of a single user affects the unit price of all providers, and hence the demands and utility of every other user, and the revenues of all providers. We discover that the providers with lower amount of power available, are likely to set higher unit prices, and vice versa. We develop a distributed algorithm which converges to the equilibrium with only local information available for both providers and end-users.

Though DRM helps facilitate the reliability of power supply, the smart grid communications can be susceptible to attacks because of the communication links between the utility companies/providers and the consumers. We investigate the impact of an attacker who can manipulate the price information from the providers, thus altering the demands from the consumers, and hence, creating over or under supply situation. We also propose a scheme based on the concept of shared reserve power to cover the power demands from the users even in the presence of an attacker, to improve the grid reliability and ensure its dependability. The results indicate that a single attacker who can manipulate the price information from only one provider, can also threaten the dependability of the grid seriously, both in terms of economic and physical impact to the grid. Nonetheless, the results also suggest that maintaining a common reserve power for the providers can save a good amount of power compared to the individual reserve scheme, yet maintaining the reliability of the grid.

Author Contribution. For this paper, I was the main contributor for the idea, theoretical analysis and simulations, but got help from my collaborators Quanyan Zhu and Tamer Başar for theoretical analysis through discussions. All authors contributed to improve the organization of the contents in the paper.

6.5 Paper V

Title: Hierarchical Demand Response Management in the Smart Grid with Large Population Approximations
Authors: S. Maharjan, Q. Zhu, Y. Zhang, S. Gjessing and T. Başar
Submitted to: Journals on Selected Areas in Communications, Special Issue on Smart Grid Communications, 2012.

In this paper we investigate the DRM problem in a network of multiple providers and consumers, incorporating multiple processes from power generation, to market activities, and to power consumption. This paper extends the two-level DRM framework of **Paper IV** to a three-level framework. In addition, instead of comparing the residential, commercial and industrial consumers using their available budget, we introduce different criteria for these users with a motivation of capturing the essential differences in the power requirement and affordability of the different types of consumers. We differentiate the consumers as those who have strict power requirement but no budget constraints (e.g., commercial and industrial consumers), and with flexible power requirement but strict budget constraints (e.g., residential consumers) to characterize their reliability and efficiency requirements on power demands.

In order to holistically model the DRM problem, we propose a hierarchical system model that captures the decision processes involved at three different levels in the SG: (i)- the top level where the providers determine the optimal amount of power to be generated, (ii)- the middle level where the providers determine their unit prices, and (iii)- the bottom level where the residential consumers decide the optimal amount of power consumption and the amount of power to purchase from each provider. In particular, we establish a Stackelberg game between providers and end-users, where providers behave as leaders maximizing their profit by setting optimal unit price and by generating optimal amount of power, and the end-users act as followers maximizing their utility, taking into consideration the user-differentiation. We derive analytical results for the SE of the game and prove that a unique equilibrium solution exists for the game. We also discuss the applicability of our proposed hierarchical Stackelberg game model to a more generic case where the industrial and the commercial consumers also have some flexible demands and the residential consumers also have some minimum power requirement. We develop a distributed algorithm which converges to the equilibrium with only price information available to both the providers and the consumers.

We extend our analysis to a large population regime to obtain deeper insights into the sensitivities of the price and demands on the equilibrium solution. We portray a simplified picture of the behavior of the supply side and the demand side when the

influence of one type of users is strictly dominant over the other(s) for two specific cases:

- when the influence of the residential users is high,
- when the influence of the industrial users is high.

The numerical results illustrate that, although the industrial users have fixed demands, their impact on the equilibrium of the hierarchical Stackelberg game, is huge. If the demand from the industrial users increases from one of the providers, it will increase its unit price, consequently lowering the demands from the residential consumers from it. This kind of behavior might benefit other providers, who can also, then, increases their unit prices (correspondingly the amount of power generation). However, there is a limit on the amount of power that the industrial users can get from a provider for given budget of the residential consumers. Conversely, there is a minimum budget requirement of the residential consumers for given demands from the industrial users. Thus, the impact is mutual. In the large population regime, we derive some conditions on the budget of the residential users for a minimum requirement of power, given the demands from the industrial users. The results indicate the minimum required budget may be drastically affected by the demand of the industrial consumers.

Author Contribution. For this paper, I was the main contributor for the idea, theoretical analysis and simulations. Quanyan Zhu contributed through discussions for the theoretical analysis. All authors contributed to improve the organization of the contents in the paper through discussions.

6.6 Paper VI

Title: User-Centric Demand Response Management in the Smart Grid with Multiple Providers
Authors: S. Maharjan, Y. Zhang and S. Gjessing
Submitted to: IEEE 33rd International Conference on Distributed Computing Systems (ICDCS 2013), 2012.

In this paper, we incorporate the renewable energy sources into the SG and study the DRM problems in a network of multiple fossil fuel based and renewable energy sources, using a consumer centric approach where each consumer aims to minimize its bill.

We model the uncertainty introduced by the renewable energy sources as a discrete time Markov chain (DTMC), and deploy quadratic cost functions as the cost of power generation for each provider. The consumers are charged according to the cost of power generation. Given the prices from each provider, we characterize the interactions among the consumers for selecting the providers, as a non-cooperative game. We further explore the improvements in the cost for the users by exploiting the spatial and temporal dimensions of DRM, i.e., we develop a joint scheme where the consumers can schedule their load in time, as well as they can choose providers in each time slot, thus, making the scheme very generic for all types of electricity consumers. We model the joint scheme also as a non-cooperative game among the consumers, and characterize the NE of both the games. Furthermore, we develop distributed algorithms so that the consumers can reach the NE solution of the game with only local information.

We compare the performance of our provider selection game and the joint game with two other schemes: (i)- without any scheduling, and (ii) with load scheduling in time. The numerical results demonstrate that all three schemes: the provider selection, the load scheduling in time, and the joint scheme, offer improvements in the costs for the users compared to the scheme without any scheduling. The results further show that the joint scheme offers more reduction in the costs incurred to the consumers, compared to the provider selection scheme or the load scheduling in time scheme.

Author Contribution. For this paper, I was the main contributor for the idea and implementation. Yan Zhang and Stein Gjessing contributed to improve the organization of the contents in the paper through discussions.

6.7 Paper VII

Title: Sensing-Performance Tradeoff in Cognitive Radio enabled Smart Grid
Authors: R. Deng, J. Chen, X. Cao, Y. Zhang, S. Maharjan and S. Gjessing
Publication in: IEEE Transactions on Smart Grid, Volume 4, Issue 1, pp. 302-310, March 2013.

Communications are critical to the accuracy and optimality of DRM, and hence at the core of the control performance of the SG. We can not fully exploit the advantages and improvements offered by the DRM models and solutions without the underlying smart grid communications infrastructure and technology.

In this paper, we propose the deployment of CR technology to improve the communication quality for smart grid communications. By means of spectrum sensing and

channel switching, smart meters can decide to transmit data on either an original unlicensed channel or an additional licensed channel, so as to reduce the communication outage. Considering the energy cost taxed by spectrum sensing together with the control performance degradation incurred by imperfect communications, we formulate the sensing-performance tradeoff problem between better control performance and lower communication cost, paving the way towards green smart grid. By employing the energy detector, we prove that there exists a unique optimal sensing time which yields the maximum revenue, under the constraint that the licensed channel is sufficiently protected.

The impact of the communication outage on the control performance of DRM is also addressed. The problem is modeled as an optimization problem with the objective of maximizing the total revenue of the provider and the total gains of all consumers. We propose a distributed algorithm to reach the optimal solution by using only local computations at both the supply and the demand sides.

The numerical results indicate that the increase in sensing time lowers the false alarm probability, thereby increasing the probability of channel switching and reducing the probability of channel outage. Next, the results imply that the increase in the outage probability reduces the profit of the power provider and the social welfare of the SG, although it may not always decrease the profit of the power consumers. Furthermore, the results verify that there exists a unique sensing time at which the revenues are maximized.

Author Contribution. For this paper, the idea is from the first author, Ruilong Deng. I contributed in developing the model for the opportunistic spectrum access by regular discussion on formulating the problem, brainstorming about the validity of the assumptions especially in the channel switching part and the assumption that channel 2 has a better quality than channel 1. All authors contributed to improve the organization of the contents in the paper through discussions.

During PhD, I also published [71] and [72], but these papers are not included in the thesis since the main ideas of [71] were used in [61], and [72] was extended to the journal [69].

7 Conclusion

In this thesis we have studied novel problems for resource management and optimization, in NGNs. We discovered some new problems and new solutions, and also proposed improvements over existing solutions for well studied problems. Our contributions include problem modeling, formulation and solutions for CSS in CRNs, and

for DRM and the integration of communication and control performances in the SG. Our work paves a path towards the future deployment of the CR technology, and the implementation of efficient DR and communication schemes in the smart grid.

In CRNs, we have developed collaboration based schemes for spectrum sensing to improve QoS of SUs' transmission by utilizing the heterogeneity in traffic dynamics or QoS requirement of the SUs. In the IEEE 802.22 WRAN framework that we considered in our analysis, if fast sensing generates an alarm, while the SUs with non-real time traffic perform fine sensing, the SUs with real-time traffic continue their transmission. Our results show that the average packet delay is reduced significantly for real time services with our proposed scheme, at the cost of increased packet delay for non-real time services. Since, the delay requirement for the non-real time traffic is not severe, the cost is not significant. In the generic CR framework, we have proposed fair and efficient schemes for CSS, by holistically exploiting the traffic dynamics of the SUs, based on the required criteria such as efficiency and fairness. The results indicate that our fair-social welfare scheme yields great improvement in the payoff of the SUs, also setting a benchmark as a generic scheme that can be deployed for a range of systems demanding reasonable efficiency and fairness.

In the SG, one of our major contributions lies in designing efficient and more practical DRM paradigms. We have integrated entities at different levels of the power generation, distribution, and consumption processes, considering multiple players (including renewable energy sources), and have developed DRM schemes motivated by the payoff maximization objective for all the players. We have employed various game models and solution concepts in our studies on DRM, and have derived explicit closed-form and analytical solutions characterizing the equilibria, for both the supply and the demand sides. We have extended our DRM analysis to including both spatial and temporal dimensions, and for all proposed schemes, we have provided distributed algorithms so that they can be practically implemented. With extensive theoretical and numerical results, we have illustrated the performance of our schemes and algorithms, compared them with existing solutions and approaches to display the improvements, and verified the convergence of our distributed algorithms.

Moreover, we have also established some benchmarks by opening the doors to some new research problems such as the study of DRM in a large population regime, the dependability analysis of DRM in the presence of an attacker, etc. Although, the scenarios investigated for these contributions are simple, these problems are interesting, and we hope that they will serve the research community to investigate the related issues for these new problems into further detail, and to develop new and useful solutions.

Finally, we have proposed CR enabled smart grid communications based on opportunistic spectrum access. This aspect serves not only as an integrating element to the communication and control units in the smart grid, but also as a bridge between my

work on CR and the smart grid during my Ph.D. studies.

8 Suggestions for Future Research and Extensions

In this section, we discuss potential future directions for the different approaches proposed in this dissertation.

For the delay reduction problem [65], while this dissertation focused mainly on static real-time and non-real time SUs for a single superframe, one possible extension to this work is to study the dynamic case when the traffic of the SUs changes with time. Thus, the distinction between real-time and non-real time SUs as introduced in [65] will become rather blurred, and the concept will be valid in the average sense (over multiple superframes). While the queuing analysis may be more complicated for this case, this scenario brings into existence some flavor of fairness as it is not the same SUs who are always exploited for fine sensing. Nonetheless, each SU may have to estimate the future traffic that will be generated locally. The scenario can be modeled by using a repeated non-cooperative game where SUs will contribute in sensing only if they do not have data with severe QoS requirement for the current superframe, and also, if their weighted average payoff will be higher than a certain threshold.

For CRNs, in this dissertation, the SUs were characterized as selfish but trusted, i.e., non of them are malicious. Had any of the SUs been malicious or had any of them been even an eavesdropper, the sensing strategies/behavior would be strongly influenced. One possible direction of extending our work on spectrum sensing is to incorporate malicious nodes into our analysis. Eg. for the delay reduction problem [65], if a SU fakes their data as real time data, i.e., the data with stringent delay requirement, it will be exempted from fine sensing. This might motivate the origin of malicious behavior from the SUs. Similarly, in the enforcement based CSS study [61], the fact that the SUs with relatively smaller traffic density end up contributing more in sensing and those with heavy traffic enjoy free rides more often, may encourage the SUs to broadcast a *false status* as heavy traffic users and not cooperate for sensing. In either of these cases, learning algorithms would be necessary in order to maintain a data base of the trust value of each SU (verified by the nearest neighbors), so that the malicious nodes can be identified. Hence, it would be of interest to study the problems incorporating the learning algorithms in our spectrum sensing analysis.

As there are malicious nodes who intend to eavesdrop, jam, or manipulate the data, there are also mechanisms to detect them as long as certain conditions are fulfilled. Consequently, one important challenge for a malicious node is to be able to harm a system without being visible or detected. Thus, another possible future direction of research on spectrum sensing in CRNs, is to investigate the behavior of a more powerful

or flexible malicious node who can change its role from manipulating the spectrum sensing results sent to the FC, to being an eavesdropper, to being a trusted SU, and to even acting as a PU at times. The problem can be tackled from two perspectives: (i)- from the malicious node's side - choosing optimal strategies of switching different possible roles in order to create maximum harm to the system. E.g., act as the malicious node to manipulate the data, or to broadcast signals as a PU whenever the earned trust value is fairly high, and switch to being an eavesdropper or a trusted SU when the trust value starts getting fairly less. (ii)- from the honest nodes' perspective - establishing cooperation to detect or avoid such malicious nodes by deploying jammers to confuse them etc. The problem can be modeled as a non-cooperative game between the trusted and the malicious/compromised nodes where each of them aim to maximize its own payoff.

For the DRM scheme in [67], a logarithmic utility function was assigned to the electricity consumers, and the budget of each user was given. The electric power consumption is valued differently by different types of consumers. Although the logarithmic function has nice mathematical properties, empowering analytical tractability, more complicated functions might better capture the actual utility for consuming electricity, especially for commercial and industrial consumers. In [68], we improved this aspect by using the logarithmic utility function only for the residential consumers and considering fixed demands from the industrial (and commercial consumers). We can use more complicated but more practical and very generic utility functions such as $1-\frac{1}{1+e^{\alpha_n d_n-\beta_n}}$, where n is the consumer index, d_n is the demand of user n, and α_n, β_n are the consumer-specific parameters. If $\alpha_n d_n \ll \beta_n$, the utility is very small since the power is much less than the minimum required power, if $\alpha_n d_n = \beta_n$, the utility is 0.5, and when $\alpha_n d_n \gg \beta_n$, the utility is close to 1. With consumer specific values for α_n, β_n, this function can capture the electricity needs and the associated utilities of a diverse type of electricity consumers, and thus, some insights about the DR models using this kind of utility function, would be a contribution of significant practical value.

Moreover, the incorporation of EVs into the DRM framework is an interesting direction to extend our work on DRM, since the EVs serve as both a challenge and an opportunity to the DRM in the SG. The EVs bring opportunities because they can act as the storing units (at times when electricity from the grid is cheap). However, incorporating EVs to the grid, adds unique challenges and thus, brings into existence many interesting research problems, mainly because the EVs are different from the usual home appliances that we consider in the DRM analysis. Notably, with the implementation of EVs and PHEVs, which are capable of selling the power back to the grid, the power flow can also be bi-directional, thus bringing into surface, numerous interesting research challenges. Moreover, the EVs are mobile, and the mobility aspect has not

been investigated much. An interesting research problem in this direction is to design the schemes and algorithms for optimal charging/discharging of the EVs from the nearest charging stations. When the EVs are moving, finding an optimal charging station in the presence of multiple other EVs and multiple charging stations, is essentially a non-cooperative game where each EV aims to minimize the total cost of charging.

Although, the SG is a complex system where the communication and control units are closely coupled, the work on DRM, smart grid communications, security etc. have been mostly done independently, and the connection among the communication and control units has not been well investigated. In [69], we proposed the deployment of CR technology for improving smart grid communications and also illustrated the coupling of the sensing performance with DR. We, however, assumed that the channel from the licensed system has a better communication quality than the unlicensed channel. A possible extension for this problem, is to study the integration of communication and control aspects relaxing this assumption, when both the licensed and unlicensed channels can be used to transmit the data.

Moreover, while this dissertation mainly focused on theoretical and algorithmic approaches for various problems in NGNs, corroborated by analytical and numerical simulation results, an important and interesting future extension is to consider practical aspects of deploying these algorithms in real-life networks. For doing so, the use of test beds or advanced simulators would be needed. Finally, beyond the economic approaches presented in this dissertation, the economic frameworks can also be explored in numerous potential future systems such as specific CPSs, IoTs, machine-to-machine (M2M) communications, device-to-device (D2D) communications etc., for scheduling, optimal resource allocation and resource sharing.

9 Book Chapter, Journal and Paper Contributions during Ph.D. Studies

During my Ph.D. studies, I contributed to the following journals and conference publications:

List of Book Chapter Publications:

1. S. Maharjan, Y. Zhang, and S. Gjessing. *Economic Approaches in Cognitive Radio Networks*, Cognitive Radio Mobile Ad Hoc Networks, ed. by F. Richard Yu, Springer, Chap. 16, pp. 403-432. (ISBN: 978-1-4419-6171-6), 2011.

List of Journal Publications:

1. S. Maharjan, Q. Zhu, Y. Zhang, S. Gjessing and T. Başar, "Dependable Demand Response Management in the Smart Grid: A Stackelberg Game Approach", *IEEE Transactions on Smart Grid*, Vol. 4, Issue 1, pp. 120-132, March 2013.
2. R. Deng, J. Chen, X. Cao, Y. Zhang, S. Maharjan and S. Gjessing, "Sensing-Performance Tradeoff in Cognitive Radio enabled Smart Grid", *IEEE Transactions on Smart Grid*, Vol. 4, Issue 1, pp. 302-310, March 2013.
3. S. Maharjan, Q. Zhu, Y. Zhang, S. Gjessing and T. Başar, "Hierarchical Demand Response Management in the Smart Grid with large Population Approximations", Submitted to *Journals on Selected Areas in Communications, Special Issue on Smart Grid Communications*, 2012.
4. S. Maharjan, Y. Zhang and S. Gjessing, "Economic Approaches for Cognitive Radio Networks: A Survey", *Wireless Personal Communications, Springer, Special Issue on Cognitive Radio Networks and Communications*, Vol. 57, No. 1, pp. 33-51, 2011, (Submitted on 3rd October 2009, Accepted on 9th February, 2010 published online: May 2010).

List of Conference Publications:

1. S. Maharjan, Y. Zhang and S. Gjessing, ”User-Centric Demand Response Management in the Smart Grid with Multiple Providers”, Submitted to *IEEE 33rd International Conference on Distributed Computing Systems (ICDCS 2013)*, 2012.

2. S. Maharjan, Y. Zhang, C. Yuen and S. Gjessing, ”Distributed Spectrum Sensing in Cognitive Radio Networks with Fairness Consideration: Efficiency of Correlated Equilibrium”, *IEEE 8th International Conference on Mobile Ad-hoc and Sensor Systems (MASS 2011)*, pp. 540 - 549, 2011.

3. R. Deng, S. Maharjan, X. Cao, J. Chen, Y. Zhang and S. Gjessing, ”Sensing-delay Tradeoff for Communication in Cognitive Radio Enabled Smart Grid”, *IEEE 2nd International Conference on Smart Grid Communications (SmartGridComm 2011)*, 2011.

4. S. Maharjan, Y. Zhang and S. Gjessing, ”Distributed Spectrum Sensing for Cognitive Radio Networks with Heterogeneous Traffic”, (Invited Paper), *3rd International Symposium on Applied Sciences in Biomedical and Communication Technologies, Workshop on Cognitive Radio and Advanced Spectrum Management (CogART 2010)* , 2010.

5. S. Maharjan, J. Xiang, Y. Zhang and S. Gjessing, ”Delay Reduction for Real Time Services in IEEE 802.22 Wireless Regional Area Network”, *IEEE 21st International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC 2010)*, pp. 1836-1841, 2010.

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Part II: Papers

Paper I

Economic Approaches for Cognitive Radio Networks: A Survey

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Wireless Pers Commun
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Economic Approaches for Cognitive Radio Networks: A Survey

Sabita Maharjan · Yan Zhang · Stein Gjessing

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Abstract Efficient resource allocation is one of the key concerns of implementing cognitive radio networks. Game theory has been extensively used to study the strategic interactions between primary and secondary users for effective resource allocation. The concept of spectrum trading has introduced a new direction for the coexistence of primary and secondary users through economic benefits to primary users. The use of price theory and market theory from economics has played a vital role to facilitate economic models for spectrum trading. So, it is important to understand the feasibility of using economic approaches as well as to realize the technical challenges associated with them for implementation of cognitive radio networks. With this motivation, we present an extensive summary of the related work that use economic approaches such as game theory and/or price theory/market theory to model the behavior of primary and secondary users for spectrum sharing and discuss the associated issues. We also propose some open directions for future research on economic aspects of spectrum sharing in cognitive radio networks.

Keywords Cognitive radio · Resource allocation · Spectrum trading · Spectrum sharing · Game theory · Price theory · Market theory

1 Introduction

Cognitive radio networks [1,9,20] have been proposed to overcome the ineffectiveness of the traditional static spectrum assignment policy [12] and to facilitate effective use of electromagnetic spectrum by coexisting with licensed users through spectrum sharing. The licensed users

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are called primary users and the users of the cognitive radio network are called secondary users. To peacefully coexist with primary users, secondary users should have timely and accurate information about the usage of primary user spectrum. There are two different approaches for secondary users to get this information:

- Through Spectrum Sensing
  In this case, secondary users perform sensing of primary user spectrum in order to detect the vacant spectra called spectrum holes. Spectrum sensing is a crucial function for such opportunistic spectrum access.
- Exclusive Information from Primary Users
  In this case, the primary users explicitly provide information about the available spectrum to secondary users. In this model, the primary users get monetary or some other kinds of benefit by allowing the secondary users to use the spectrum.

In an opportunistic spectrum access scenario, there is no motivation for primary users to participate in the spectrum sharing process because they do not get any benefit by letting secondary users use their spectrum. In this approach, the primary users are inflexible and the overall responsibility of maintaining peaceful coexistence with primary users is on the secondary users, thus making the implementation aspect more complex and guaranteeing the performance harder. On the other hand, a resource trading based approach of spectrum sharing is that primary users can lease the spectrum to secondary users whenever and wherever they are not using the particular bands which in turn gives the primary users monetary or other benefits from secondary users.

Figure 1 shows spectrum trading and resource allocation as two different issues in cognitive radio networks. Effective resource allocation is the key to efficient spectrum sharing. Resource allocation can be in terms of frequency band, channel access time, transmission power etc. and can be between primary users and secondary users and among secondary users. Spectrum trading is the economic aspect of spectrum sharing in an incentive driven framework of coexistence of primary and secondary users. In a spectrum trading scenario, while primary users compete to sell the spectrum in order to maximize their revenue, secondary users compete to get the spectrum according to their needs at better price to maximize their satisfaction. Spectrum trading can be between primary and secondary users or can be among secondary users only.

There is a crucial need to study the competitive and cooperative strategies of users for multiplayer optimization of the resource allocation problem. Meanwhile, understanding the pricing issues and market structures for spectrum trading is not less important either for practical implementation of cognitive radio networks. Thus, our motivation for this work stems from the need to establish a framework to understand the possibilities and challenges of using economic approaches for deploying cognitive radio networks.

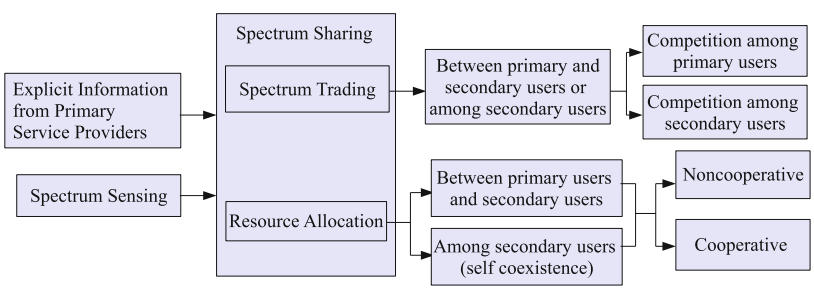

Fig. 1 Spectrum sharing issues in cognitive radio networks

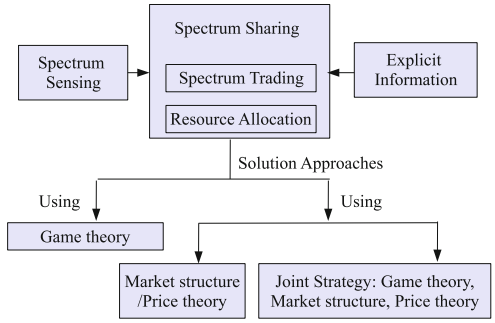

Fig. 2 Solutions for spectrum sharing in cognitive radio networks

Different approaches that have been used to model the strategic interactions for spectrum sharing are shown in Fig. 2. Game theory serves as a powerful tool in order to model the strategic behavior of primary and secondary users for their coexistence. The economic models include principles such as setting the price of the spectrum available aimed to maximize revenue of the primary users, choosing the best seller for the spectrum in order to maximize the satisfaction from the usage of the spectrum for secondary users, modeling market competition etc. Therefore, many of the existing literature on modeling the economic interactions in wireless networks use a combination of game theory, price theory and market structure.

The rest of the paper is organized as following. Different kinds of games that can be applicable to model economic interactions and decision making in cognitive radio networks and an overview of the related work are described in Sect. 2 along with the associated challenges. Price theory and market principles to model the trading activities in cognitive radio networks, related work and the corresponding research challenges are discussed in Sect. 3. An overview of the work using a combination of game theory, price theory and market theory is presented in Sect. 4. Section 5 shows the classification of the related work based on the issues and solutions. Open problems for future research are introduced in Sect. 6. Section 7 concludes the paper.

2 Game Theory

Game theory is the study of conflict and cooperation among individuals, groups or firms. It provides an analytical framework with a set of mathematical tools for the analysis of interactive decision-making processes. It is a multi-player optimization approach and the concept applies whenever the actions of several players are interdependent.

A game is formed by three fundamental components: a set of players, a set of strategies and a set of payoffs for given set of actions. A player is the one that makes decisions in the game. A strategy is a complete contingent plan, or a decision rule that defines an action that a player will select in every distinguishable state of the game. Payoff is the revenue or satisfaction of the player for a given strategy. Payoff is often expressed through utility functions. Game theory combined with market principles and price theory serves as a strong ground for modeling the economic activities of cognitive radio networks for spectrum sharing.

2.1 Cooperative and Non-cooperative Games

Games can be classified into different categories based on different criteria. A common approach is to classify games as cooperative and non-cooperative games.

2.1.1 Cooperative Game

In a cooperative game, there is no competition between players in a group and they act as a single entity to maximize the total group utility. An example is a bargaining game, which is often used to formulate the interaction among cooperative players provided that a player can influence the action of other players. In a bargaining game, the players can negotiate and bargain with each other. A general solution of the bargaining game is the Nash bargaining solution, which can ensure efficiency as well as fairness among the players.

2.1.2 Non-Cooperative Game

A non-cooperative game is the one in which players are selfish and each individual player makes decisions independently. In a non-cooperative environment, players have different (often conflicting) interests. Non-cooperative game theoretical framework is used to obtain an equilibrium solution that optimizes the payoff of all players. One of the most widely used solutions for non-cooperative games is the Nash equilibrium. Nash equilibrium is the solution at which any player in the game cannot achieve a better solution by deviating unilaterally, given the actions of the other players.

2.2 Different Game Models

Some game models that have been used extensively for analyzing the strategic interactions among users for spectrum sharing are as following.

2.2.1 Stackelberg Game

The Stackelberg leadership model [5] is a strategic game in which there is at least one player defined as the leader who can make the decision and commit the strategy on the price before other players who are defined as followers. The players engage in Stackelberg competition if one has some kind of incentive to move first. The strategy chosen by the leader can be observed by the followers, and the followers can adapt their decisions accordingly. The leader can choose a strategy such that its profit is maximized, given that the followers will choose their best responses. This solution is called the Stackelberg equilibrium.

2.2.2 Bertrand Game

In a Bertrand game [5], there are a finite number of firms that decide on the service prices simultaneously. Given the price offered by a service provider, based on a demand function, the amount of commodity requested from the users can be determined. Then, the profit is computed and used in a profit maximization problem for a service provider to obtain the best response in terms of setting the service price. For a spectrum trading scenario, the service providers are the primary users, the consumers are the secondary users and the size of the spectrum will change according to the price set by the primary users. When the service providers offer their prices simultaneously (i.e. imperfect information), Nash equilibrium is the solution. The interaction in Stackelberg game is more dynamic due to the timing in strategy adaptation compared to the Bertrand model. If the assumption of perfect information is released and all firms decide their service prices simultaneously, Stackelberg model reduces to Bertrand model.

2.2.3 Cournot Game

In Cournot game [5], the competition is in terms of the quantity of the commodity. The decision of each user is affected by the strategies of other users and the decisions are made simultaneously.

2.2.4 Coalition Game

Cooperative game theory provides analytical tools to study the behavior of rational players when they collaborate. The group of cooperating players that can strengthen the players' position in the game, is called a coalition, and all players forming a coalition act as a single entity. Based on application oriented approach, coalitional games can be classified into three categories [31]: canonical (coalitional) games, coalition formation games and coalitional graph games. In canonical games, no group of players can do worse by joining a coalition than by acting non-cooperatively. In coalition formation games, forming a coalition brings advantage to its members but the gains are limited by a cost for forming the coalition. In coalitional graph games, the coalitional game is in graph form and the interconnection between the players strongly affects the characteristics as well as the outcome of the game.

2.2.5 Game with Learning

In a competitive resource market, the information available may be incomplete and/or imperfect. In such cases, the players have to evolve and learn the behavior of other players from history. Such evolutionary games are often called games with learning and are closely related to the concept of "repetition" in games (described next). If the games are "repeated", players can learn and adapt their behaviors and strategies in subsequent rounds of the game.

2.2.6 Repeated Games

A repeated game [5] is an important tool in order to understand the concepts of reputation and punishment in game theory. A repeated game allows a strategy to be contingent on the past moves, thus allowing threats and promises about future behavior to influence current behavior, which create possibilities for cooperation among greedy users. If a greedy user behaves selfishly and chooses the strategy to optimize his/her individual payoff, it can enjoy the benefit in one round. However, if this user has to depend on others as well for future rounds of the game, it will be punished by them. Players must therefore consider the effects that their chosen strategy in any round of the game will have on opponents' strategies in subsequent rounds.

2.3 Applications of Game Theory in Spectrum Sharing

Game formulations can be used for multiplayer optimization to achieve individual optimal solution for resource allocation. The use of game theoretic models for resource allocation has mainly focused on issues such as admission control, throughput optimization, power control, channel allocation etc. Table 1 summarizes the related work on the use of game theory for resource allocation, in terms of the specific issue addressed, approach/model(s) used and the solution proposed. These works are explained next.

Table 1 Summary of related work on resource allocation using game theory

Paper	Issue(s) addressed	Approach/specific model(s) used	Solution
[6]	Power control, throughput control	Non-cooperative, distributed game	Nash equilibrium (Optimal for power, optimal or suboptimal for network throughput)
[7]	Power control, rate adaptation, subchannel assignment	Non-cooperative, distributed game	Nash equilibrium (Transmitted power)
[10]	Channel/power allocation	Non-cooperative/cooperative, distributed game	Nash equilibrium (Radio range)/ nash bargaining solution
[33]	Spectrum allocation (Channel switching)	Non-cooperative, distributed game (Modified minority game)	Nash equilibrium (Channel switching probability)
[36]	Channel access time	Cooperative Stackelberg game (between primary and secondary users), non-cooperative payment selection game (among secondary users), distributed game	Nash equilibrium (Payment vector)
[3]	Spectrum assignment	Cooperative, distributed, bargaining game	Bargaining based solution (Spectrum usage)
[19]	Channel allocation	Cooperative, distributed game	Correlated equilibrium (Spectrum access)
[8]	Packet forwarding	Cooperative, distributed game (Repeated coalitional game)	Min-max fairness, average fairness and market fairness investigated
[32]	Spectrum sensing	Cooperative, distributed game (Non-transferable utility coalitional game)	Performance compared with non-cooperative and centralized scheme

In [6], a game theoretical approach is proposed for distributed resource allocation in wireless networks. Power control at the user level and throughput control at the system level are linked through non-cooperative games.

In [7], a distributed non-cooperative game is proposed for joint subchannel assignment, adaptive modulation and power control for multi-cell multi-user OFDMA networks. In order to improve the performance of Nash equilibrium points, a virtual referee is introduced in the system that can modify the rule of the resource competition game for efficient resource sharing.

In [10], the authors modeled the channel/power allocation for cognitive radios considering IEEE 802.22 [13] framework. The strategic behavior of the system was studied considering the limit on the total interference from all opportunistic transmissions for each primary user as well as the minimum SINR requirement of the cognitive radios and a cooperative scheme based on Nash bargaining solution was proposed for optimal channel/power allocations.

In [33], the self-coexistence of multiple overlapping IEEE 802.22 networks operated by multiple wireless service providers that compete for resources and try to seek a spectrum band without any interference from other coexisting IEEE 802.22 networks, was investigated from a game theoretic perspective. The dynamic channel switching was modeled as a distributed modified minority game (MMG), in which each user has to decide whether to leave a particular band or to continue using it when another user also appears in the same band.

In [36], a cooperative cognitive radio framework is formulated as a Stackelberg game where primary user acting as the leader, selects some of the secondary users to be the cooperative relay, and in return, leases portion of the channel access time to them for their own data transmission. Selected secondary transmitters, acting as the followers, can use the wireless channel only if they cooperate with the primary link and meanwhile make a certain amount of payment to the primary system.

In [3], the authors propose a local bargaining approach to achieve distributed conflict free spectrum assignment adapted to network topology changes assuming that there is a collaboration between network nodes to improve system utility. In this paper, the authors propose Fairness Bargaining with Feed Poverty to improve fairness in spectrum assignment and have derived a lower bound on the spectrum assignment (poverty line) that each node can get from bargaining.

In [19], a decentralized dynamic spectrum access scheme is proposed for cognitive radios considering the application domain as a set of collision channels from game theoretical perspective. The authors proposed the use of an adaptive procedure called Regret Tracking, which converges even when multiple users are adapting their behavior simultaneously, for which correlated equilibrium (in terms of channel allocation) is investigated.

In [8], an approach based on coalition games is proposed for symbiotic cooperation between boundary nodes and backbone nodes in selfish packet forwarding wireless networks. Different fairness criteria are investigated including market fairness. In addition, a joint protocol is designed using both repeated games and coalition games for packet-forwarding and it has been shown that the network connectivity can be significantly improved using the proposed protocol compared to using pure repeated game approach.

In [32], a distributed collaborative spectrum sensing algorithm is developed based on a dynamic coalition formation game among secondary users to improve the overall probability of miss detection with increase in the probability of false alarm as the cost for coalition formation.

2.4 Research Challenges of Using Game Theory

Although, game theory is a powerful tool to model and analyze the interactions among primary and/or secondary users for spectrum sharing, how realistic are the assumptions in the models and how close the models are to the possible implementation of cognitive radio networks, are yet open issues. These issues are challenging especially because of the need of dynamic access of the spectrum and heterogeneous requirements on the quality/quantity of the offered spectrum for different users in the network. Some of such challenges for specific game models, are explained next.

The availability of perfect knowledge in Stackelberg model may be quite costly for distributed implementation of cognitive radio networks. How to efficiently manage the information flow among leaders and followers in a dynamic scenario is a big challenge. Even if the information can be made available to all players, the overhead due to this information exchange (which increases with the number of players in the game) can not be ignored. Although, the prices and profits of some service providers may be higher at Stackelberg equilibrium than at Nash equilibrium [25], Bertrand/Cournot game models (thus Nash equilibrium solution) may incur relatively less overhead for the distributed approach because of the simultaneity of moves.

Cooperative framework is often used to model the spectrum sharing scenario in cognitive radio networks. However, finding users with common interest to form coalitions and get them

to act cooperatively, itself is a big issue. Even if there exist users that can be symbiotic, the change in network topology, change in channel conditions, motivation towards cooperating with other users etc. may cause the coalition not to be stable. Introducing incentive or monetary gain based schemes can be quite expensive and inefficient due to divergence of user interests.

The concept of repeated games provides a good direction towards counteracting the possibility of collusive behavior in a network by certain selfish players and to adapt one's strategies accordingly. However, algorithms are necessary to effectively estimate long-term profits. In a practical network, the overhead to maintain the database of the strategies of each player and to update it is a huge challenge in a distributed scenario. In addition, when the game is being played the second last time or the last time, the future profits are not meaningful. So, in such a case, the concept of repeated games may not be effective enough. Besides, if an individual deviates from the optimal strategy for the system for its own benefit, to find out which user it was, is another issue. It may be possible to locate the group from which the deviating action occurred but locating the exact player from the group may still be difficult considering the time limitations, especially when users have the right not to disclose their strategies.

3 Price Theory and Market Theory

Price theory explains how relative prices are determined and how prices function to coordinate the economic activities. For incentive/monetary gain based spectrum sharing, appropriate pricing schemes are necessary for setting up the price of the spectrum, formulating economic models and maximizing the payoffs of both primary and secondary users. Pricing is an important issue not only to maximize the revenue of the service providers but also to prevent unnecessary competition (to reserve the resources) and to allocate the radio resource efficiently.

3.1 Price Theory

Auction and bargaining are the popular pricing schemes for resource trading. These are explained next.

3.1.1 Auction Theory

An auction [16] is a decentralized form of trading, widely known for providing efficient allocation of scarce resources. Sellers use auctions to improve revenue by dynamically pricing based on buyer demands. Buyers benefit since auctions assign resources to buyers who value them the most. In a game-theoretic auction model, the action set of each player is a set of bid functions or reservation prices. Each bid function maps the player's value (in case of a buyer) or cost (in case of a seller) to a bid price.

There are different kinds of auctions such as English auction, sealed first price auction, Vickrey auction, double auction etc. English auction is the ascending price auction in which bidders bid openly against one another, with each subsequent bid higher than the previous bid and the highest bidder gets the commodity at his/her bid. In sealed first-price auction, all bidders simultaneously submit sealed bids so that no bidder knows the bid of any other participant. The highest bidder pays the price he/she submitted. Vickrey auction is the sealed bid second-price auction, in which the bidders submit sealed bids and the highest bidder

wins, but pays only as much as the second-highest bid. Vickrey-Clarke-Groves auction [16] is a generalization of Vickrey auction for multiple items. When there are multiple items and multiple buyers, double auction is often used to model the double competition.

3.1.2 Bargain Theory

Bargaining is a type of negotiation in which the buyer and the seller of a commodity or service dispute the price that will be paid and the exact nature of the transaction that will take place, and eventually come to an agreement. Bargaining is an alternative pricing strategy to fixed prices. In a bargaining scenario, the buyer's willingness to pay is dominant over the actual price of the commodity.

3.2 Market Theory

A market is the most efficient known mechanism for the allocation of goods and services. A market consists of sellers and buyers of a commodity or service. Appropriate pricing schemes are necessary to maintain the stability of the market. Some of the useful concepts of market theory used for spectrum trading are as following.

3.2.1 Monopoly, Oligopoly and Competitive Equilibrium

Monopoly is the simplest market structure when there is only one seller in the system. Since there is a single seller in this market structure, the seller can optimize the trading to achieve the highest profit based on the demand from buyers. In an oligopoly market structure, a small number of firms dominate the market. The firms compete with each other independently to achieve the highest profit by controlling the quantity or the price of the supplied commodity. Oligopoly differs from monopoly in the sense that there are a multiple (few) firms providing the same service, thus making it necessary for each firm to take into account the strategies of all other firms. In a monopolistic context, the pricing is a single level of game between users (buyers). However, when there are multiple sellers in the market for the same commodity, the competition between them can highly affect the results of price determination. This competition introduces an additional level of game among the service providers. Buyers demand less as the price of a commodity increases. On the other hand, sellers tend to produce more as the price increases. The price at which the quantity supplied of a product/service and the quantity of it demanded are equal, is called the market equilibrium price and when the environment is competitive with flexible prices and many traders, the equilibrium is called competitive equilibrium.

3.3 Applications of Price Theory and Market Theory in Spectrum Sharing

In this section, an overview of the existing work using price theory and market principles to model economic interactions for spectrum sharing is presented (summarized in Table 2 in terms of the specific issue addressed, structure, approach/model(s) used and the solution proposed). Although price theory and market theory provide us models to address pricing issues and market stability, to model the interdependency of the sellers and buyers in the market and their strategic interactions, game theory is used. So, many literature use a combination of price theory, market theory and game theory for analyzing spectrum sharing. An overview of these works is presented in Sect. 4.

Table 2 Summary of related work on spectrum sharing using price theory/market theory

Paper	Issue(s) addressed	Structure	Approach/specific model(s) used	Solution
[30]	Spectrum allocation	Cooperative, centralized	Optimization (Multi-unit Vickrey auction)	Optimal price
[4]	Spectrum access	Cooperative, centralized	Optimization	Optimal price
[21]	Spectrum trading	Competitive (among secondary users), Distributed	Joint pricing and marketing theory	Pricing solution obtained through market equilibrium and disequilibrium

In [30], the problem of a CDMA operator participating in a dynamic spectrum allocation scheme is addressed in a cooperative framework based on multi-unit Vickrey auction. A spectrum manager implements DSA by periodically auctioning short-term spectrum licenses and a pricing driven solution based on the willingness to pay of each user is introduced.

In [4], a framework based on an auction mechanism was presented for dynamic spectrum access using classical optimization approach. In the system model considered in [4], multiple spectrum buyers submit spectrum demand function, which is based on piecewise linear price demand (PLPD), to the spectrum owner that formulates an optimization problem to maximize revenue under an interference constraint. The authors propose to restrict the interference constraints and reduce them into a number that grows linearly with the number of buyers.

In [21], a spectrum trading model based on multiple markets for different frequency bands is proposed between the primary and secondary services. The authors have investigated two different cases: the first one with equilibrium pricing where spectrum supply is equal to spectrum demand and the second one is the case where the sellers do not offer the equilibrium price, and have proposed models for both cases using linear feedback time-invariant control systems. Classical control system stability techniques are used to analyze the dynamics of market behavior under both cases.

3.4 Research Challenges of Using Price Theory and Market Theory

Though price theory and market theory can be applicable to model the spectrum trading for cognitive radio networks, the information exchange required for pricing and negotiation is a big challenge. Eg. Vickrey-Clarke-Groves auction is one of the most used auctions for resource trading. It can be used to achieve a socially optimal allocation. However, it requires gathering global information from the users, which is a huge challenge for distributed implementation of cognitive radio networks especially when the available information is incomplete. The communication overhead and computational complexity to gather and manage global information in a distributed scenario may be quite costly.

To perform bargaining, users form groups and bargain with other groups. However, the larger the groups are, the more is the complexity of bargaining due to high costs of synchronization and communication overhead. So, efficient formation of bargaining groups and effective communication between them in a distributed spectrum sharing scenario is also another issue. In addition, the stability of the bargaining groups formed in a network with rapidly changing topology and other underlying conditions is another issue. To dynamically

split and merge to firm optimal coalitions in a network with only local information available, is also a huge challenge.

The pricing theory with per unit price of the resource same for all users for any amount of resource demanded may not produce highest revenue for the sellers. So, a discriminatory pricing scheme as proposed in [4] may be better in terms of maximizing the revenue. However, the computational complexity of this kind of scheme is yet a big issue.

4 Joint Strategy: Game Theory, Market Theory and Price Theory

While market principles and price theory are needed to model economic activities, game theory is necessary to analyze the interdependency and strategies of the users for spectrum sharing. So, many literature use a combination of these to investigate spectrum trading and resource allocation in cognitive radio networks. Table 3 summarizes the related work using joint strategy in terms of the specific issue addressed, structure, approach/model(s) used and the solution proposed, each of which is briefly explained next.

Table 3 Summary of related work on spectrum sharing using joint strategy

Paper	Issue(s) addressed	Structure	Specific model(s) used	Solution
[2]	Power control	Non-cooperative, distributed	Non-cooperative game, pricing mechanism	Nash equilibrium (For uniformly strictly convex pricing function)
[11]	Channel access	Non-cooperative, distributed	Non-cooperative game, SINR auction, power auction	Nash equilibrium (Bidding profile, power profile)
[34]	Joint power/ channel allocation	Non-cooperative/cooperative (among CR pairs), distributed	Non-cooperative game, pricing based cooperation	Nash equilibrium/ Pareto optimum boundary (power vector)
[23]	Spectrum trading	Competitive (among primary services), centralized/distributed	Non-cooperative game, Bertrand game, repeated game, Oligopoly market	Nash equilibrium, optimal spectrum price
[22,24]	Spectrum trading	Competitive (among secondary users), centralized/distributed	Non-cooperative game, Cournot game, Oligopoly market	Nash equilibrium (Spectrum size)
[25]	Spectrum pricing	Competitive (among service providers), centralized	Non-cooperative game, Stackelberg game, Bertrand game, oligopoly market	Stackelberg equilibrium, Nash equilibrium (Spectrum price)
[26]	Spectrum trading	Market equilibrium/ competitive/cooperative (Among primary service providers), distributed	Non-cooperative game, optimization, oligopoly market	Market equilibrium, Nash equilibrium, optimal price
[28]	Spectrum trading	Competitive (among primary users, among secondary users), distributed	Non-cooperative game, (among primary users), evolutionary game (among secondary users), Oligopoly market	Nash equilibrium (Spectrum price), evolutionary equilibrium (Spectrum size)

Table 3 Continued

Paper	Issue(s) addressed	Structure	Specific model(s) used	Solution
[15]	Spectrum allocation	Non-cooperative, distributed	Non-cooperative game, double auction, pricing based allocation	Nash bargaining solution (lower bound on payoff), competitive equilibrium (without user collusion)
[17]	Slotted resource allocation	Competitive (among providers), distributed	Non-cooperative two stage pricing game	Nash equilibrium of the pricing game (under mild conditions)
[35]	Spectrum trading/access	Competitive (among primary users), centralized/distributed	Non-cooperative game	Nash equilibrium (spectrum price)
[14]	Spectrum leasing	Non-cooperative (between primary and secondary users)	Non-cooperative power control game	Nash equilibrium (Transmission power)
[29]	Joint spectrum bidding and pricing	Non-cooperative, centralized	Non-cooperative game, sealed-bid double auction	Nash equilibrium (Spectrum price)
[27]	Spectrum trading	Non-cooperative (among TV broadcasters), (among WRAN users), Distributed	Non-cooperative game, generalized fading memory scheme, microeconomic approach	Market equilibrium (Spectrum price)

In [2], the CDMA uplink power control in a multicell CDMA wireless network model is addressed as a non-cooperative game. The game incorporates a pricing mechanism that limits the overall interference and preserves battery energy of mobiles. The concept of outage probability was introduced as a performance metric for the quality of the channel. Distributed iterative power algorithms are analyzed using an outage probability based utility function for a generalized fading channel model.

In [11], a non-cooperative game was formulated to address the problem of spectrum sharing among users using spread spectrum signaling, in a distributed scenario to access the channel subject to a constraint on the interference temperature at a measurement point and two auction mechanisms: : SINR auction and power auction, are proposed for allocating received power.

In [34], a joint power/channel allocation scheme is proposed using a distributed pricing approach for cognitive radio networks and a frequency dependent power mask constraint is introduced for secondary users in addition to maximum transmission power constraint and minimum SINR constraint.

In [23], the issue of spectrum pricing in cognitive radio network is addressed for multiple primary services and 1 secondary service. The trading of spectrum between primary and secondary services was modeled as an oligopoly market. A Bertrand game model was applied for price competition among primary services to obtain the Nash equilibrium pricing. Distributed algorithms were presented to obtain the solution of the dynamic game, when primary services have to make decisions based only on the spectrum demand from the secondary

service. A repeated game was formulated to analyze the behavior of selfish primary operators that try to deviate from the equilibrium point to increase their profit at the cost of lower profit to other primary operators.

In [22,24] the competition among multiple secondary users for spectrum offered by 1 primary user was modeled using Cournot game. In [24], the problem is formulated as an oligopoly market competition and the spectrum allocation for secondary users is obtained through non-cooperative game. The competition is in terms of the size of the spectrum they request. A dynamic game is formulated in which the selection of strategy by the secondary users is solely based on the pricing information obtained from the primary user.

In [25], the authors modeled service competition and pricing in a WiMAX and WiFi based heterogeneous wireless access network using non-cooperative Stackelberg and Bertrand game models respectively and the performance was compared for the two models.

In [26], spectrum trading for cognitive radio networks was investigated considering multiple primary services that are willing to sell the available spectrum to the secondary service. Distributed algorithms were presented for three different pricing schemes: market equilibrium, competitive and cooperative pricing models and the performance of all three schemes were compared.

In [28], the problem of spectrum trading with multiple primary users selling spectrum opportunities to multiple secondary users is considered. The competition among primary users is formulated as a non-cooperative game where each primary user sets the size of spectrum to be shared and the price of the spectrum such that its own payoff is maximized. It is assumed that the secondary users can evolve over time to buy the spectrum opportunities that provide the best payoff in terms of performance and price.

In [15], spectrum allocation among primary and secondary users is modeled as a bilateral pricing process to maximize the utilities of both primary and secondary users and a distributed collusion-resistant dynamic pricing approach with optimal reserve prices was proposed to achieve efficient spectrum allocation while combating user collusion. Double auction scenario was considered for the pricing game. A belief function was introduced that builds up certain belief of other players' future possible strategies for each user to assist its decision making.

In [17], the competition among providers was studied on a non-cooperative game theoretic framework. The authors introduced a pricing model dealing with how for fixed prices, total demand is split among providers following Wardrop's principle and determined the existence and uniqueness of Nash equilibrium under mild conditions.

In [35], the economic interactions are modeled considering both price of the offered spectrum and its quality. The analysis scenario consists of multiple self interested spectrum providers operating with different technologies and offering the spectrum at different costs that compete with each other to get potential customers that are grouped into two categories: quality sensitive group and price sensitive group. In [35], the authors have proposed a practical price updating strategy using structured stochastic learning for which the price is shown to converge to the optimal equilibrium.

In [14], a game theoretic framework was developed to facilitate dynamic spectrum leasing (DSL) in cognitive radio networks in which primary users are also included as active decision makers in a non-cooperative game with secondary users by selecting an interference cap on the total interference they are willing to tolerate.

In [29], the joint spectrum bidding and pricing scheme was proposed for dynamic spectrum access in the exclusive usage model for IEEE 802.22 based cognitive radio network. Multiple TV broadcasters offer the available TV bands and WRAN service providers bid for these TV bands. A sealed bid double auction scenario was considered for the procurement

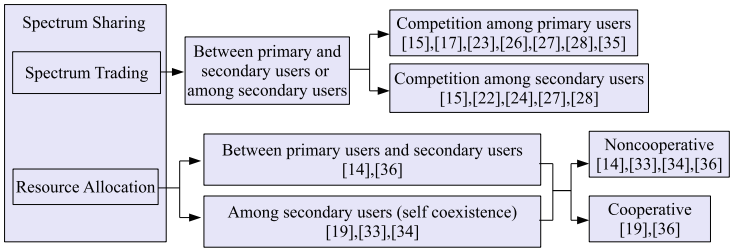

Fig. 3 Classification of related work on spectrum sharing based on resource allocation and spectrum trading

of TV bands from TV broadcasters in terms of the number of TV bands and the trading price. After buying TV bands, multiple WRAN service providers compete with each other to sell the spectrum to WRAN users. A non-cooperative game was formulated to model the competitive environment for bidding and pricing strategies.

In [27] the authors have proposed a market-equilibrium based model for spectrum trading between primary and secondary services using supply and demand functions. A non-cooperative game is formulated between primary and secondary users where a distributed generalized fading memory algorithm is used by the secondary service to estimate spectrum price and adjust spectrum demand accordingly so that the market equilibrium can be reached for the price and size of the spectrum allocated for the secondary service by the primary service.

5 Classification of Related Work Based on Issues and Solutions

In this section, we categorize the related work on modeling the economic interactions in cognitive radio networks on three different bases as following.

Figure 3 shows the classification based on different aspects of spectrum trading and resource allocation as described in Sect. 1. The related work on spectrum trading between primary and secondary users address competition among primary users and competition among secondary users. While [17,23,26,35] deal with price competition among primary users/service providers for profit maximization, [22,24] address the competition among secondary users in terms of the size or quality of spectrum demanded. On the other hand [15,27,28] address the competition among primary services as well as among secondary users for spectrum trading.

In terms of resource allocation, while [14,36] consider the coexistence between primary service providers and secondary users, [19,33,34] explore self coexistence among secondary users. Although both [14,36] consider the coexistence between primary and secondary systems, [36] is based on the cooperation between primary and secondary users, while in [14], the framework is non-cooperative. The payment selection game among secondary users in [36] however is a non-cooperative game. For self coexistence among secondary users, while [33] proposes a non-cooperative dynamic channel switching game, [19] considers a cooperative approach for channel allocation. On the other hand, [34] considers a non-cooperative framework and a pricing based cooperative approach for self coexistence of secondary users.

Figure 4a shows the classification based on the particular game models described in Sect. 2. Both [25,36] use Stackelberg game model. However, while the approach considered in [36] is a cooperative framework between primary and secondary users, [25] uses

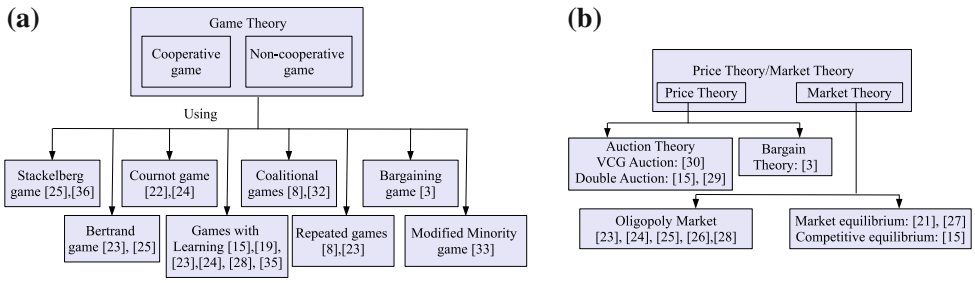

Fig. 4 Classification of related work on spectrum sharing based on solution approaches

Stackelberg game for price competition among WiMAX and WiFi service providers. However, when all players in the game should act simultaneously, the price competition game can be modeled using Bertrand game as in [23,25]. When the competition is not in terms of price, but in terms of the size of the spectrum, Cournot game was applied to model the competition among secondary users [22,24].

In a distributed scenario with incomplete information, the players have access to local information only and they have to learn about the strategy of other players adaptively. Such interactions can be modeled using games with learning [15,19,23,24,28,35]. In [23], when a primary service provider has the information about the demand from secondary service but no information about the current pricing strategy of other primary services, a non-cooperative game with learning was used to decide its strategy based on the past strategy of other primary services. Similar approach is applied but in case of secondary users in [24]. [28] addresses the issue for both primary and secondary users when there are multiple primary services and multiple secondary users. In [15], belief assisted approach (using belief function) was used for selfish users to reduce pricing overhead. In [35], a price updating strategy was proposed using structured stochastic learning when the sellers have no knowledge about each others' strategy and also about the consumer population. In [19], the authors have used a learning based game called Regret Tracking for channel allocation among cognitive radio users.

Coalitional game was used in [32] for collaborative spectrum sensing. In [8], coalition game combined with repeated game was used to make the boundary nodes and backbone nodes collaborate for packet forwarding. In [23], repeated games were used to prevent selfish primary services from deviating unilaterally for individual profit that may lower the profit for other primary services. The combination of repeated and coalitional games can be a very effective way to introduce cooperation among network users. A bargaining game was used in [3] in a cooperative and distributed framework for spectrum assignment. An MMG was used in [33] in a non-cooperative framework for channel switching.

Figure 4(b) depicts the classification based on the particular models/approaches from price theory and market structure described in section 3. In [30], VCG auction was used for dynamic spectrum allocation in a cooperative and centralized framework. Double auction scenario was considered in [15,29] in a non-cooperative frame- work. Bargaining game was used in [3] in a cooperative and distributed framework for spectrum assignment. Oligopoly market structure was considered for analyzing pricing schemes in [23–26,28]. While [21,27] considered market equilibrium as the pricing solution, [15] used competitive equilibrium for the case without user collusion in the network.

The applicability of particular game models or pricing schemes/market structure depends on many factors. The selfish behavior of network users and the limited amount of information available usually requires distributed schemes for spectrum sharing. On the other hand, the synchronization issues and the information exchange overhead in a distributed system is a

huge research challenge. The implementation of pricing schemes can also cause significant communication overhead to the system. In many cases, Nash equilibriums are not efficient and Pareto optimal solutions provide higher revenues/payoffs. However, the cost to make the users act cooperatively in a distributed environment may be quite high, which leaves users no other choice but Nash equilibrium. The trade off is in terms of information exchange and computation complexity versus distributedness.

6 Open Research Problems

Some possible directions for future research in investigating the economic interactions for spectrum sharing in cognitive radio networks are as following.

6.1 Coalition Formation and Communication Overhead

Many of the literature that propose cooperative strategies for primary/secondary users have not considered the cost for cooperation. The cost can be the power required for negotiation, delay because of the information exchange etc. In a practical scenario, it is not reasonable to neglect this cost for spectrum management especially in a resource constrained network like cognitive radio. Therefore, investigating the overhead caused by the communication to form a coalition for cooperation for spectrum sharing can be an interesting direction for future research.

6.2 Bidder Collusion

While the objective of primary services to participate in spectrum trading is to maximize their revenue, the purpose of secondary services is to get the spectrum as cheap as possible such that the required quality of service is also maintained. So, it is likely that the bidders may attempt to lower the price of the spectrum offered by the primary services by acting collusively. Bidder collusion [18] is probably the most serious practical threat to the revenue of the primary users but there has not been much work on this. So, another possible direction for future research may be to study the interactions in presence of bidders' collusion and to explore its effect on the payoff of primary and secondary users.

6.3 Incentive Driven Spectrum Sensing

Spectrum sensing, a crucial function for opportunistic spectrum access, requires significant share of the network resources (energy, time etc.). So, selfish secondary users tend to take a free ride as far as possible by overhearing the sensing results from other secondary users. To prevent such behavior and to maintain the motivation to perform sensing, incentive or reward driven sensing schemes can be another possible direction for further research.

6.4 Trust and Security in Collaborative Sensing

Trust and security is one of the key issues in cognitive radio networks. The presence of malicious sensors and their possible attack against honest sensors can pose a serious threat to the reliability of the results obtained from collaborative spectrum sensing. Investigating

different issues such as how can malicious sensors attack the honest sensors, how to detect them, how to counteract such attacks, the criteria that the reliability of the results obtained from collaborative sensing can be maintained despite the presence of malicious sensors etc. using game theory can be a very interesting research area.

6.5 Assumption of Rationality and Complete Information

Although game theory is a powerful tool for modeling resource allocation problems, it inherently assumes that the players are rational and often the payoff functions of all players are assumed to be common knowledge, which in many scenarios may not be valid. Game theory merely suggests what strategies should be taken to maximize individual/total utility. It does not provide much insight into the analysis of what the players are most likely to do and how much rational the players are likely to be when the information available is incomplete or faulty. Therefore, further study is needed to investigate the factors that may make the players irrational and to predict the natural behavior of players in addition to the strategies that are towards maximizing the individual or total utility.

7 Conclusion

We have described different types and models of games, price theory and market principles that have been used to model the economic activities of primary and secondary users for resource allocation and spectrum trading. An extensive summary of the related work on economic approaches has been presented with the classification based on the spectrum sharing issues and solutions. We also discussed the research challenges of using game theory and price theory/market theory for their application in cognitive radio research. We discussed the open research problems and proposed some interesting directions for future research about economic approaches in cognitive radio networks.

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Author Biographies

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Paper II

Delay Reduction for Real Time Services in IEEE 802.22 Wireless Regional Area Network

Sabita Maharjan, Jie Xiang, Yan Zhang and Stein Gjessing

Delay Reduction for Real Time Services in IEEE 802.22 Wireless Regional Area Network

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Abstract

Real time traffic such as voice and video have strict requirements on the acceptable end-to-end packet delay. When there are different types of traffic with different requirements on tolerable latency, priority based packet scheduling schemes are normally used in order to reduce the queuing delay for real time services. However, in cognitive radio networks, the time that the system spends on spectrum sensing adds further delay to the packet transmission. In this paper, we propose a new scheme to significantly reduce the overall packet delay, including the delay due to sensing for real time services in cognitive radio networks. We derive the expression for average packet delay for the proposed scheme and the simulation results match well with the analytical results. The numerical results show that the priority based scheduling scheme combined with our scheme substantially reduces the packet delay for real time applications.

I. Introduction

Spectrum sensing is a crucial function for cognitive radio networks [1] to detect available channels from primary user spectra and also to make sure that the primary users returning to a channel being used by secondary users are detected so that the secondary users can vacate the channel immediately. Spectrum sensing can be classified into two categories based on the duration it needs to detect the primary users: fast sensing and fine sensing. Fast sensing takes less time as it uses energy detection to detect the primary users. Fine sensing takes longer as it uses feature detection [7]. In the IEEE 802.22 Wireless Regional Area Network

(WRAN) standard [2][3], quiet periods are scheduled periodically to detect returning primary users in the channel being used by secondary users. During quiet periods, all secondary users stop their transmission and listen for primary users. In conventional scheme, fast sensing is performed at regular intervals of time and fine sensing is performed if fast sensing gives an alarm.

The typical duration of energy detection is less than 1 ms while feature detection takes much longer (e.g. 24.2 ms for field synchronous detector) [6]. The acceptable packet delay for real time applications is quite small. The typical values for voice communication is 20 ms and for video applications is 40 ms. In order to reduce the delay for real time services, priority based packet scheduling schemes are often used where real time packets have a higher priority over non-real time packets (e.g. [8]). However, such schemes reduce queuing delay only. In cognitive radio networks, apart from queuing delay, the delay induced by sensing also comes into the picture. The duration of fast sensing is relatively small. Hence, the quiet period due to fast sensing is not a big issue. However, if the transmission is stopped for the duration of fine sensing, it is a critical issue for real time applications in terms of the tolerable latency. Therefore, it is very important to reduce the fine sensing induced delay for real time services.

In [4], a packet scheduling algorithm based on the estimation of packet loss amount was proposed to reduce average packet loss rate of real time traffic for IEEE 802.22 WRAN. In [5], the authors proposed a sensing scheme that consists of a series of consecutive energy detections followed by feature detection to suppress the effect of false alarms generated by energy detection. However, the issue of the duration of fine sensing (when performed) for real time packets was not addressed in these works. The motivation for our work stems from the need to reduce the effect of sensing induced delay for real time traffic in IEEE 802.22 WRAN.

In this paper, we propose a new scheme to significantly reduce the packet delay due to sensing for real time traffic. In our proposed scheme, during fine sensing, only non-real time users listen for primary users while real time users continue their transmission. Thus, the packet delay due to fine sensing is eliminated for real time packets. We combine this scheme to reduce fine sensing induced delay for real time traffic with the priority based packet scheduling scheme. The numerical results show that the proposed scheme can drastically reduce average packet delay for real time services with insignificantly increased delay for non-real time services. We derive an analytical expression for the average packet delay including the delay due to sensing and verify the analytical results with the simulation results.

The rest of the paper is organized as following: Section II shows the system model and explains our assumptions. We describe the proposed model in section III. Results are presented and analyzed in section IV. Section V concludes the paper.

II. System Model

We consider one primary base station (BS) with a single TV channel of bandwidth 6 MHz, an IEEE 802.22 system with one secondary base station and N secondary users as shown in Figure 1. Among N users, N_{r} is the number of secondary users with real time traffic and N_{n} is the number of secondary users with non-real time traffic.

The dark gray area in the primary user spectrum in Figure 1 means that the channel is being used by the primary user. The white spaces indicate that the channel is vacant. We assume that the primary user activity is a two-state birth-death process such that the on-time and off-time are exponentially distributed with rate β and α respectively. Let p_a be the probability that the primary user is active. Then,

$$p_a = \frac{\beta}{(\beta + \alpha)}. \tag{1}$$

Fast sensing (energy detection) is performed by all secondary users once every superframe at the beginning of the superframe. We assume that all secondary users are synchronized for sensing. One common approach to combine the results of spectrum sensing from secondary users is to use OR rule of data fusion. If fast sensing gives an alarm, the base station schedules fine sensing for more reliable detection. The probability of miss detection is defined as the probability that the channel is claimed to be vacant although it is actually occupied by the primary user. The probability of false alarm is defined as the probability that the channel is claimed to be occupied although it is actually vacant. Let $P_{\mathrm{MD,c}}$ and $P_{\mathrm{FA,c}}$ denote the probability of miss detection and the probability of false alarm, respectively of fast sensing. Then, using cooperative energy detection based on OR rule, we have,

$$\begin{cases} P_{\mathrm{MD,c}} = \prod_{i=1}^{N} P_{\mathrm{MD,c,i}}, \\ P_{\mathrm{FA,c}} = 1 - \prod_{i=1}^{N} (1 - P_{\mathrm{FA,c,i}}). \end{cases} \tag{2}$$

where $P_{\mathrm{MD,c,i}}$ and $P_{\mathrm{FA,c,i}}$ are the probability of miss detection and the probability of false alarm, respectively of secondary user i for fast sensing. With more number of users performing sensing, $P_{\mathrm{MD,c}}$ decreases but $P_{\mathrm{FA,c}}$ increases. Thus the false alarm rate of fast sensing based on OR rule is often very high. IEEE 802.22 WRAN requires in-band sensing to achieve

the probability of miss detection and the probability of false alarm of at most 10%. The achievable values of $P_{\mathrm{MD,c}}$ and $P_{\mathrm{FA,c}}$ depend on the number of secondary users from which the sensing results are combined. The base station chooses certain number of sensing users to combine the sensing results from in order to maintain $P_{\mathrm{MD,c}} <= 0.1$ and $P_{\mathrm{FA,c}} <= 0.1$.

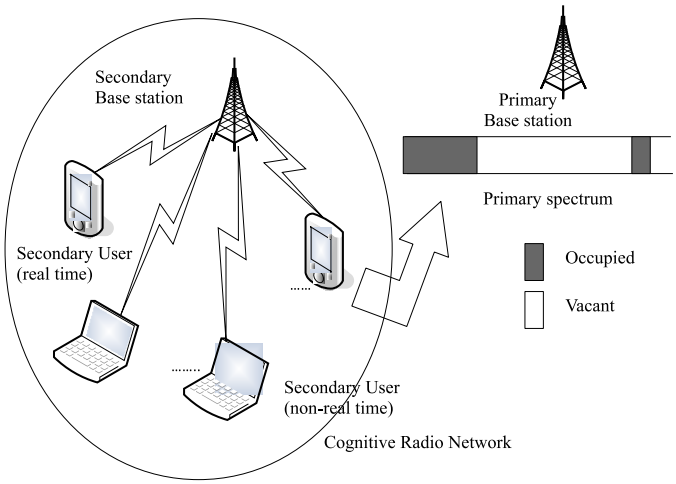

Fig. 1: System Model

III. Delay Reduction Scheme

A. Fine Sensing by Non-Real Time Users

Figure 2 shows the proposed delay reduction scheme. The white spaces are IEEE 802.22 WRAN transmission, the light colored areas indicate fast sensing and the dark areas indicate fine sensing. The duration of the sensing interval is fixed as 1 superframe. Each superframe consists of 1 frame for fast sensing, which is a quiet period and 10 data frames each of duration 10 ms. In our scheme, fast sensing is performed by all secondary users at the beginning of each superframe. When an alarm is issued by fast sensing, feature detection is performed for reliable detection. However, different from the conventional approach [2], we use non-real time users only to perform fine sensing while all real time users continue their transmissions during this period. During fine sensing, even though there is background traffic from real time secondary users, the non-real time secondary users can still detect primary

user activity because feature detection can differentiate signals based on their modulation type and cyclic frequencies. The result obtained from fine sensing is considered accurate enough and based on that, the base station decides whether to continue transmission or to stop/move to another available channel, which is followed by all secondary users (including real time users).

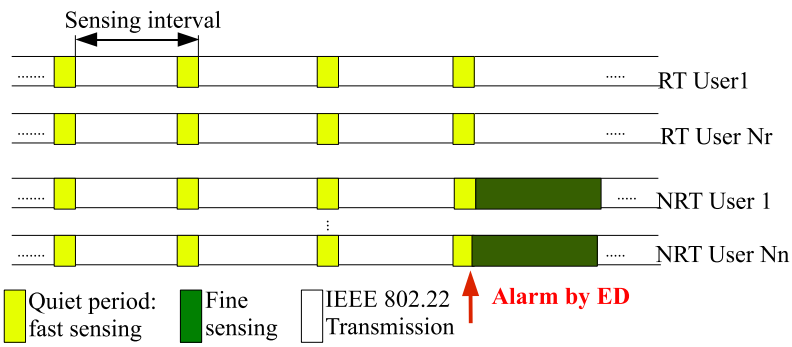

Fig. 2: Proposed Scheme to Reduce Fine Sensing Induced Packet Delay

B. Priority Based Scheduling

In order to maintain relatively low queuing delay for real time packets, we reserve certain data rate for them based on the real time traffic intensity. Let $r_{\min}$ be the minimum data rate reserved for real time users. Then, we use the following linear function for the data rate reserved for real time users (R_{r}):

$$R_{\mathrm{r}} = (aN_{\mathrm{r}} + b)r_{\min} \tag{3}$$

where a and b are constants and $r_{\min}$ is chosen such that the service rate is at least greater than the total arrival rate of real time packets. The total data rate left for non-real time users (R_{n}) is then given by

$$R_{\mathrm{n}} = R_{\mathrm{total}} - (aN_{\mathrm{r}} + b)r_{\min} \tag{4}$$

where R_{total} is the total data rate of the channel. As the number of real time users increases, the data rate left for non-real time users decreases which increases the delay for non-real

time packets. Such a strategy of reserving data rate for real time services might even lead to data rate starvation for non-real time users [8]. We, however, consider R_{total}, a, b and $r_{\min}$ such that the effective service rate for non-real time users with the leftover data rate is at least greater than the total non-real time packet arrival rate.

C. Proposed Delay Reduction Scheme

In our proposed delay reduction scheme, only non-real time secondary users perform fine sensing and real time secondary users continue their transmission even during fine sensing. To further reduce the delay, we combine the priority based packet scheduling scheme with the sensing delay reduction scheme. Let t_{QP}, t_{FD} and t_{SF} be the duration of fast sensing, fine sensing and that of a superframe, respectively as shown in Figure 3. Then the average time spent on sensing per superframe by real time users ($t_{\text{s,r}}$) and by non-real time users ($t_{\text{s,n}}$) are

$$\begin{cases} t_{\text{s,r}} = t_{\text{QP}}, \\ t_{\text{s,n}} = t_{\text{QP}} + p_{\text{Alarm}} t_{\text{FD}}. \end{cases} \tag{5}$$

where p_{Alarm} is the probability that fast sensing issues an alarm. This probability is the sum of the probabilities that the primary user is detected when it is active and that a false alarm is issued although the primary user is not active, i.e.

$$p_{\text{Alarm}} = P_{\text{FA,c}}(1 - p_a) + (1 - P_{\text{MD,c}})p_a. \tag{6}$$

We consider Poisson arrival of packets for both real time users and non-real time users with mean arrival rates of λ_{r} and λ_{n}, respectively. The size of a real time packet is S_r bytes and the size of a non-real time packet is S_n bytes. The secondary BS holds separate queues of infinite length for real time and non-real time packets. The packet arrival process is Poisson, the packet size is constant, thus the service time is deterministic for both real time and non-real time traffic and there is a single server (base station). Hence, we can employ M/D/1 model for the queues. The effective service rates for real time traffic ($\mu_{\text{r,eff}}$) and for non-real time traffic ($\mu_{\text{n,eff}}$) are

$$\begin{cases} \mu_{\text{r,eff}} = \frac{R_{\text{r,eff}}}{S_r}, \\ \mu_{\text{n,eff}} = \frac{R_{\text{n,eff}}}{S_n}. \end{cases} \tag{7}$$

where $R_{\text{r,eff}} = R_{\text{r}}(1 - \frac{t_{\text{s,r}}}{t_{\text{SF}}})$ and $R_{\text{n,eff}} = R_{\text{n}}(1 - \frac{t_{\text{s,n}}}{t_{\text{SF}}})$.

The M/D/1 queuing delay for real time packets ($d_{\text{r,q}}$) and non-real time packets ($d_{\text{n,q}}$) are

$$\begin{cases} d_{\mathrm{r,q}} = \frac{(2-\rho_r)}{2\mu_{\mathrm{r,eff}}(1-\rho_r)}, \\ d_{\mathrm{n,q}} = \frac{(2-\rho_n)}{2\mu_{\mathrm{n,eff}}(1-\rho_n)}. \end{cases} \tag{8}$$

where $\rho_r = \frac{\lambda_{\mathrm{r}}}{\mu_{\mathrm{r,eff}}}$ and $\rho_n = \frac{\lambda_{\mathrm{n}}}{\mu_{\mathrm{n,eff}}}$.

Within a superframe, the arrival time of a particular packet u is the sum of the inter-arrival times of the first packet until the uth packet. Because the inter-arrival time between any two packets is exponentially distributed, the probability density function (pdf) of the arrival time of the uth packet is Erlang distribution. Thus, the pdf of the uth real time packet $f_r(t; u, \lambda_r)$, can be written as

$$f_r(t; u, \lambda_r) = \frac{\lambda_r^u t^{(u-1)} e^{(-\lambda_r t)}}{(u-1)!}. \tag{9}$$

Let t_1, t_2, t_3 and t_4 be the beginning of a typical superframe, the end of fast sensing of the superframe, the end of fine sensing and the duration of the superframe, respectively as shown in Figure 3. For real time traffic, let $p_{r_{u,1}}$ and $p_{r_{u,2}}$ be the probabilities that the uth real time packet arrives in the interval (t_1, t_2) and (t_2, t_4), respectively. Then,

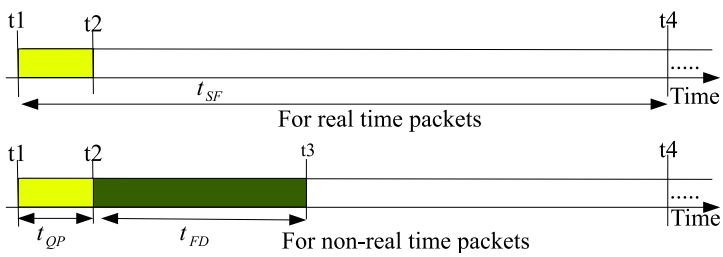

Fig. 3: A typical superframe for real time and non-real time users

$$\begin{cases} p_{r_{u,1}} = \int_{t_1}^{t_2} f_r(t; u, \lambda_r) dt, \\ p_{r_{u,2}} = \int_{t_2}^{t_4} f_r(t; u, \lambda_r) dt, \end{cases} \tag{10}$$

where $p_{r_{u,1}} + p_{r_{u,2}} = 1$. Let $E(ta_{r_{u,1}})$ be the expected arrival time of the uth real time packet in the interval (t_1, t_2). Then,

$$E(ta_{r_{u,1}}) = \frac{\int_{t_1}^{t_2} t f_r(t; u, \lambda_r) dt}{\int_{t_1}^{t_2} f_r(t; u, \lambda_r) dt}, \qquad t_1 < ta_{r_{u,1}} < t_2 \tag{11}$$

where $ta_{r_{u,1}}$ is the arrival time of the uth real time packet if it comes in the interval (t_1, t_2). Let $d_{r_{u,1}}$ be the expected delay that the uth real time packet experiences because of stopping transmission during fast sensing. Then, $d_{r_{u,1}}$ is the time from $E(ta_{r_{u,1}})$ until t_2 and can be expressed as

$$d_{r_{u,1}} = t_2 - E(ta_{r_{u,1}}). \tag{12}$$

If m packets arrive during a superframe duration, the total delay faced by all real time packets that arrive during sensing is $\sum_{u=1}^{m} p_{r_{u,1}} d_{r_{u,1}}$. The average packet delay is therefore $\{\frac{1}{m}\sum_{u=1}^{m} p_{r_{u,1}} d_{r_{u,1}}\}$. Let p_{r_m} be the probability that the total number of real time packets arriving in a typical superframe is m, given by

$$p_{r_m} = \frac{(\lambda_r t_{\text{SF}})^m e^{(-\lambda_r t_{\text{SF}})}}{m!}. \tag{13}$$

Now, considering all packets that arrive during a superframe duration, the average delay for a real time packet that arrives during sensing can be written as

$$d_{r_1} = \sum_{m=1}^{\infty} p_{r_m} \{\frac{1}{m}\sum_{u=1}^{m} p_{r_{u,1}} d_{r_{u,1}}\}. \tag{14}$$

The packets that arrive at the base station during sensing period of a superframe can be transmitted only after sensing is over. As soon as sensing ends, some packets will already have accumulated. These packets need to be transmitted before transmitting the packets that come after sensing. This is depicted in Figure 4. Let $M_{r,1}$ denote the average number of real time packets that arrive during $t_{\text{s,r}}$, where $M_{r,1} = \lambda_r t_{\text{s,r}}$. Let $T_{r,1}$ be the service time for $M_{r,1}$ real time packets, given by $T_{r,1} = \frac{M_{r,1}}{\mu_{\text{r,eff}}}$. Let $M_{r,2}$ be the average number of real time packets that arrive during $T_{r,1}$, where $M_{r,2} = \lambda_r T_{r,1}$. Then $M_{r,2}$ real time packets face a delay of $T_{r,1}$ before their transmission begins. Similarly, $M_{r,3}$ real time packets face a delay of $T_{r,2}$ before their transmission begins and so on until $T_{r,l}$ is so small that no packet arrives in that duration. For real time packets the total delay due to this effect is $d_{r_2 tot} = M_{r,2}T_{r,1} + M_{r,3}T_{r,2} + ... = \lambda_r((T_{r,1})^2 + (T_{r,2})^2 + ...) = \lambda_r \sum_{l=1}^{\infty} (T_{r,l})^2$. Therefore, if d_{r_2} be the average delay faced by all real time packets that arrive during t_{SF}, due to this effect, then

$$d_{r_2} = \frac{1}{\lambda_r t_{\text{SF}}} \lambda_r \sum_{l=1}^{\infty} (T_{r,l})^2, \tag{15}$$

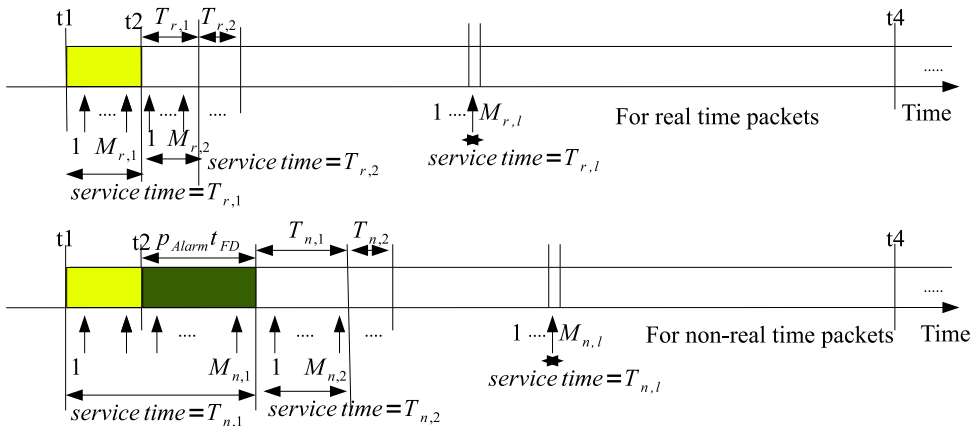

Fig. 4: Effect of Sensing in Packet Delay

which gives $d_{r_2} = \frac{1}{t_{\mathrm{SF}}} \sum_{l=1}^{\infty} (T_{r,l})^2$.

The average delay (due to M/D/1 queue and due to sensing) for real time packets (d_{r}) is

$$d_{\mathrm{r}} = d_{r_1} + d_{r_2} + d_{\mathrm{r,q}}. \tag{16}$$

For non-real time packets, the pdf of the wth non-real time packet $f_n(t; w, \lambda_n)$, is

$$f_n(t; w, \lambda_n) = \frac{\lambda_n^w t^{(w-1)} e^{(-\lambda_n t)}}{(w-1)!}. \tag{17}$$

Let $p_{n_{w,1}}$, $p_{n_{w,2}}$ and $p_{n_{w,3}}$ be the probabilities that the wth non-real time packet arrives in the interval (t_1, t_2), (t_2, t_3) and (t_3, t_4), respectively. Then,

$$\begin{cases} p_{n_{w,1}} = \int_{t_1}^{t_2} f_n(t; w, \lambda_n) dt, \\ p_{n_{w,2}} = \int_{t_2}^{t_3} f_n(t; w, \lambda_n) dt, \\ p_{n_{w,3}} = \int_{t_3}^{t_4} f_n(t; w, \lambda_n) dt. \end{cases} \tag{18}$$

where $p_{n_{w,1}} + p_{n_{w,2}} + p_{n_{w,3}} = 1$. Let $E(ta_{n_{w,1}})$ and $E(ta_{n_{w,2}})$ be the expected arrival time of the w_{th} non-real time packet in the interval (t_1, t_2) and (t_2, t_3), respectively. Then,

$$\begin{cases} E(ta_{n_{w,1}}) = \frac{\int_{t_1}^{t_2} t f_n(t; w, \lambda_n) dt}{\int_{t_1}^{t_2} f_n(t; w, \lambda_n) dt}, & t_1 < ta_{n_{w,1}} < t_2 \\ E(ta_{n_{w,2}}) = \frac{\int_{t_2}^{t_3} t f_n(t; w, \lambda_n) dt}{\int_{t_2}^{t_3} f_n(t; w, \lambda_n) dt}. & t_2 < ta_{n_{w,2}} < t_3 \end{cases} \tag{19}$$

where $ta_{n_{w,1}}$ and $ta_{n_{w,2}}$ are the arrival times of the wth non-real time packet in the intervals (t_1, t_2) and (t_2, t_3), respectively. Let $d_{n_{w,1}}$ and $d_{n_{w,2}}$ be the expected delays that the wth non-real time packet experiences because of stopping transmission during sensing if it comes in the range (t_1, t_2) and (t_2, t_3), respectively. In case of a non-real time packet, the delay due to fine sensing is also added to packet delay but only if fine sensing is performed. Hence, $d_{n_{w,1}}$ and $d_{n_{w,2}}$ can be expressed as

$$\begin{cases} d_{n_{w,1}} = t_2 - E(ta_{n_{w,1}}) + p_{\text{Alarm}} t_{\text{FD}}, \\ d_{n_{w,2}} = p_{\text{Alarm}}(t_3 - E(ta_{n_{w,2}})). \end{cases} \tag{20}$$

Similar to the case of real time packets, we can derive the average delay for a non-real time packet that arrives during sensing as

$$d_{n_1} = \sum_{m=1}^{\infty} p_{n_m} \{ \frac{1}{m} \sum_{w=1}^{m} (p_{n_{w,1}} d_{n_{w,1}} + p_{n_{w,2}} d_{n_{w,2}}) \}, \tag{21}$$

where $p_{n_m} = \frac{(\lambda_n t_{\text{SF}})^m e^{(-\lambda_n t_{\text{SF}})}}{m!}$, is the probability that the total number of non-real time packets arriving in a typical superframe is m.

Let $M_{n,1}$ be the average number of non-real time packets that arrive during $t_{\text{s,n}}$ as shown in Figure 4, where $M_{n,1} = \lambda_n t_{\text{s,n}}$. Let $T_{n,1}$ denote the service time for $M_{n,1}$ non-real time packets, given by $T_{n,1} = \frac{M_{n,1}}{\mu_{\text{n,eff}}}$. Let $M_{n,2}$ be the average number of non-real time packets that arrive during $T_{n,1}$, where $M_{n,2} = \lambda_n T_{n,1}$. Then $M_{n,2}$ non-real time packets face a delay of $T_{n,1}$ before their transmission begins. Similarly, $M_{n,3}$ non-real time packets face a delay of $T_{n,2}$ before their transmission begins and so on until $T_{n,l}$ is so small that no packet arrives in that duration. If d_{n_2} be the average delay faced by all non-real time packets that arrive during t_{SF}, due to the propagating effect of sensing, with similar analysis as in case of real time services, it can be expressed as

$$d_{n_2} = \frac{1}{t_{\text{SF}}} \sum_{l=1}^{\infty} (T_{n,l})^2. \tag{22}$$

The average delay (due to M/D/1 queue and due to sensing) for non-real time packets (d_{n}) is

$$d_{\text{n}} = d_{n_1} + d_{n_2} + d_{\text{n,q}}. \tag{23}$$

TABLE I: Parameters Used for Analysis

Parameters	Values
Bandwidth	6 MHz
Considered transmission	Downlink
Data rate (R_{tot})	10 Mbps
Real time packet size	60 bytes
Non-real time packet size	500 bytes
Mean on time for PU	0.1s
Mean off time for PU	0.9s
$P_{\mathrm{MD,c}}$	0.1
$P_{\mathrm{FA,c}}$	0.1
t_{QP}	1 ms
t_{FD}	25 ms
a	0.4
b	10
$r_{\min}$	0.2 Mbps

IV. Numerical Results

The parameters used for numerical analysis are shown in Table I.

Figure 5 shows the average packet delay for real time and non-real time packets for the proposed scheme with respect to λ_r. The theoretical curves were plotted using (16) and (23) and the simulations were performed in matlab using 100 trials. The analytical results match well with the simulation results. This validates the correctness of the analysis model and simulation as well.

We compare the performance of the proposed scheme with two different schemes: First Come First Serve (FCFS) scheme that uses all secondary users to perform fine sensing, (hereafter referred to as FCFS-Conv) and priority based scheme with data rate reservation for real time packets that uses all secondary users to perform fine sensing (hereafter referred to as PRIO-Conv).

Figure 6 and Figure 7 show the average packet delay with respect to λ_r for real time services and non-real time services, respectively. The packet delay increases with higher packet arrival rate, which is expected. PRIO-Conv has lower delay for real time packets compared to FCFS-Conv. However, the improvement is not significant. The proposed scheme shows drastic improvement in delay for real time packets because it allows real time users to escape the significant part of sensing delay due to feature detection. The delay in the

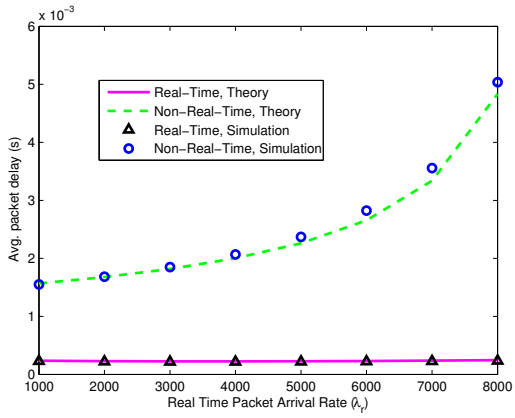

Fig. 5: Average packet delay for real time and non-real time services for the proposed scheme ($\lambda_n = 1000$ packets/s)

proposed scheme is almost constant for real time users because a packet suffers almost no sensing delay. Hence, the dominant delay in this case is only queuing delay.

The delay for non-real packets becomes higher in PRIO-Conv compared to FCFS-Conv (Figure 7). The delay in PRIO-Conv starts increasing at a higher rate for non-real time packets after $\lambda_r = 6000$. This is because of the increase in the reserved data rate for real time users thus leaving relatively less data rate for non-real time users. The delay is same for PRIO-Conv and the proposed scheme for non-real time packets.

Figure 8 and Figure 9 show the packet delay with respect to λ_n for real time packets and non-real time packets, respectively. The delay is almost constant with respect to λ_n for both PRIO-Conv and the proposed scheme because the data rate reserved for real time traffic is independent of λ_n. The proposed scheme yields lower delay compared to PRIO-Conv, which is maintained at about 0.2 ms. An interesting observation here is that the delay in FCFS-Conv is smaller than that in PRIO-Conv when $\lambda_n < 900$. This can be explained as follows. When λ_n is much smaller than λ_r, reserving a fixed data rate does not give real time users as much data rate as they can get out of total data rate in FCFS-Conv.

For non-real time users, the delay increases with higher value of λ_n and is higher in PRIO-Conv and the proposed scheme compared to FCFS-Conv. The delay is the same in PRIO-Conv and the proposed scheme, which indicates that our scheme to reduce sensing delay does not add extra cost for non-real time packets, compared to PRIO-Conv. The delay

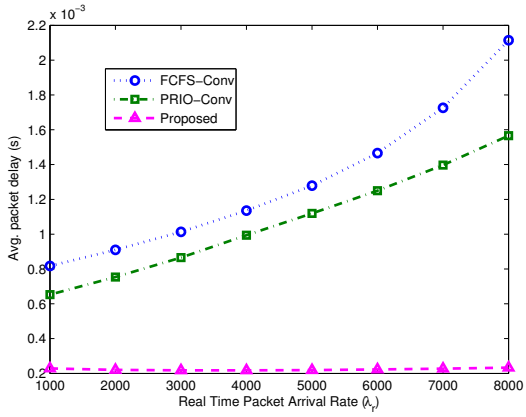

Fig. 6: Average packet delay for real time services ($\lambda_n = 1000$ packets/s)

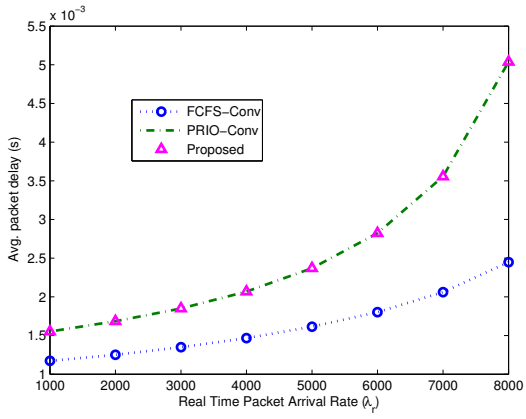

Fig. 7: Average packet delay for non-real time services ($\lambda_n = 1000$ packets/s)

starts increasing rapidly after about $\lambda_n > 1300$. This sharp increase is because the data rate leftover for non-real time users is fixed while the packet arrival rate is increasing.

Thus, compared to FCFS-Conv, our proposed scheme drastically reduces the delay for real time packets with higher delay for non-real time packets. However, the non-real time packets are not as sensitive to delay as are real time packets. Therefore, non-real time services can tolerate this delay. Compared to PRIO-Conv, the proposed scheme significantly reduces the delay for real time services without increasing the delay for non-real time services.

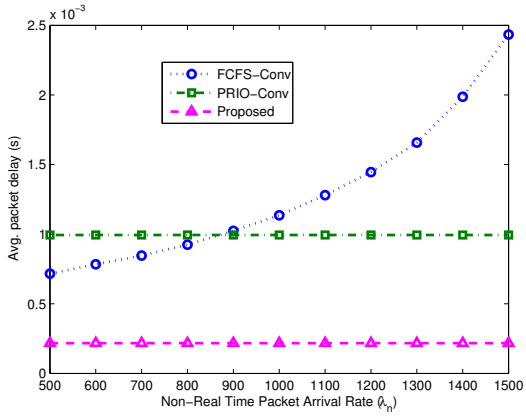

Fig. 8: Average packet delay for real time services ($\lambda_r = 4000$ packets/s)

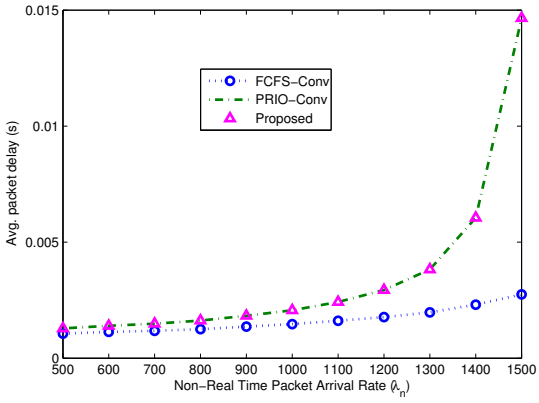

Fig. 9: Average packet delay for non-real time services ($\lambda_r = 4000$ packets/s)

V. Conclusion

We proposed a scheme to reduce delay induced by fine sensing for real time services in IEEE 802.22 WRAN. We derived a close form expression for average packet delay for both real time and non-real time services for the proposed scheme. We compared the performance of the proposed scheme with two existing schemes. The results show that our proposed scheme yields drastic reduction in delay for real time services without worsening the performance for non-real time services or with insignificant cost in terms of packet delay

for non-real time services.

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Paper III

Distributed Spectrum Sensing in Cognitive Radio Networks with Fairness Consideration: Efficiency of Correlated Equilibrium

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Distributed Spectrum Sensing in Cognitive Radio Networks with Fairness Consideration: Efficiency of Correlated Equilibrium

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Abstract

Cooperative spectrum sensing improves the reliability of detection. However, if the secondary users are selfish, they may not collaborate for sensing. In order to address this problem, Medium Access Control (MAC) protocols can be designed to enforce cooperation among secondary users for spectrum sensing. In this paper, we investigate this problem using game theoretical framework. We introduce the concept of correlated equilibrium for the cooperative spectrum sensing game among non-cooperative secondary users and formulate the optimization problem for the case where secondary users have heterogeneous traffic dynamics. We show that the correlated equilibrium improves the system utility, as compared to the mixed strategy Nash equilibrium. While maximizing system payoff is important, fairness is also equally important in systems with dissimilar users. In order to address fairness issue, we propose a new fair social welfare correlated equilibrium, which maximizes the system utility and ensures that the less well-off users do not starve. We employ a no-regret learning algorithm for distributed implementation of the correlated equilibrium. Finally, we propose a neighbourhood based learning algorithm and show that it achieves better performance than the no-regret algorithm.

I. Introduction

Cognitive radio networks (CRNs) [1][2] have been proposed to address the problem of spectrum scarcity and to make effective use of the electromagnetic spectrum by opportunis-

tically using the spectrum of the licensed users. The licensed users are called primary users and the users of the cognitive radio network are called secondary users. Spectrum sensing is a crucial function for cognitive radio networks to detect available channels from primary user spectra and also to prevent excessive interference to primary users from secondary users. Cooperative spectrum sensing [3]-[8] has been recognized as a powerful approach to improve detection reliability. In most of the existing literature on cooperative spectrum sensing, there is an inherent assumption that all secondary users cooperate with each other to maximize the payoff for the system [9]-[11]. However, if the secondary users are selfish, they will not collaborate with each other for cooperative sensing without any incentive [12]. This kind of selfish behavior comes into the picture in many situations e.g., if the secondary users belong to different systems, if their spectrum requirements are heterogeneous, if there is no central authority to assign the sensing responsibility etc. Under such situations, the decision of a secondary user to participate in cooperative sensing depends not only on the user's strategy but also on the strategies taken by other users. This framework can be modeled using non-cooperative game theory.

There is only a limited literature investigating the selfish behavior of secondary users for spectrum sensing using game theoretical framework. One of the pioneering works on modeling cooperation enforcement among selfish secondary users for cooperative spectrum sensing is [13]. In [13], the authors modeled cooperative sensing as an N-player horizontal infinite game and they proposed to use Carrot-and-Stick strategy, which results mutual cooperation as the Nash equilibrium (NE) of the game. In [14], the authors proposed mixed strategy NE as the solution of the non-cooperative game among secondary users for cooperative spectrum sensing. They deployed an evolutionary game [15] among secondary users for their strategies to converge to the equilibrium in distributed manner. However, the analytical framework was limited to the case of users with homogeneous traffic, and the mixed strategy NE does not always exist for the case when users are not homogeneous. In [19], we differentiated the heterogeneous users as heavy traffic and light traffic users, and designed the sensing game so as to encourage light traffic users take the sensing responsibility and allow heavy traffic users get a free ride. In this paper, we derive the analytical framework for a more general case with heterogeneous traffic dynamics and improve the system performance compared to the mixed strategy equilibrium, using correlated equilibrium (CE) as the solution of the game.

Medium Access Control (MAC) protocols can be designed to enforce cooperation among secondary users for spectrum sensing. In this paper, we investigate this problem using game

theoretical framework. CE can improve the payoff of users in both cooperative and non-cooperative games. The concept of CE has been used extensively for spectrum access and transmission control [21]-[24]. These works focus on the issues of opportunistic spectrum access such as rate control/adaption, power allocation etc. but none of them dealt with the issues of spectrum sensing. In this paper, we apply the concept of CE to design the cooperative spectrum sensing game among secondary users in order to improve the payoff of the secondary users and the network. This is a novel approach in the context of opportunistic spectrum access based on spectrum sensing performed by the non-cooperative secondary users. In most of the studies on spectrum sharing and spectrum sensing, it is assumed that the secondary users always have data to transmit and they want to use the channel as long as possible. In a real network, the users may not necessarily have data to transmit all the time. Taking this into account, we analyze the sensing game for users with heterogeneous traffic: some of which have data to transmit all the time while others do not. We exploit the heterogeneity among the secondary users and use CE to improve the system payoff. While maximizing the system payoff is important, fairness is also equally important in systems with dissimilar users. To incorporate fairness into the utility maximization problem, we propose fair social welfare correlated equilibrium (f-SW CE), which maximizes system utility while ensuring that the users with less traffic do not suffer severely. In addition, we employ a no-regret (NR) learning algorithm for the game so that each secondary user's strategy converges to the CE using only local information. Furthermore, we propose a neighbourhood based learning (NBL) algorithm which yields considerable improvement in the performance compared to the NR algorithm. Thus, our three major contributions in this paper are as follows:

1) We formulate the optimization problem for obtaining CE of the spectrum sensing game among heterogeneous users and show that CE improves the system performance considerably.
2) We propose f-SW CE scheme, which maximizes the system utility and ensures fairness to all secondary users.
3) We propose a distributed NBL algorithm that needs exchange of some information among users but yields higher performance compared to the NR algorithm.

The rest of this paper is organized as follows. In section II, we discuss the system model and the energy detection mechanism for spectrum sensing. We derive the mixed strategy

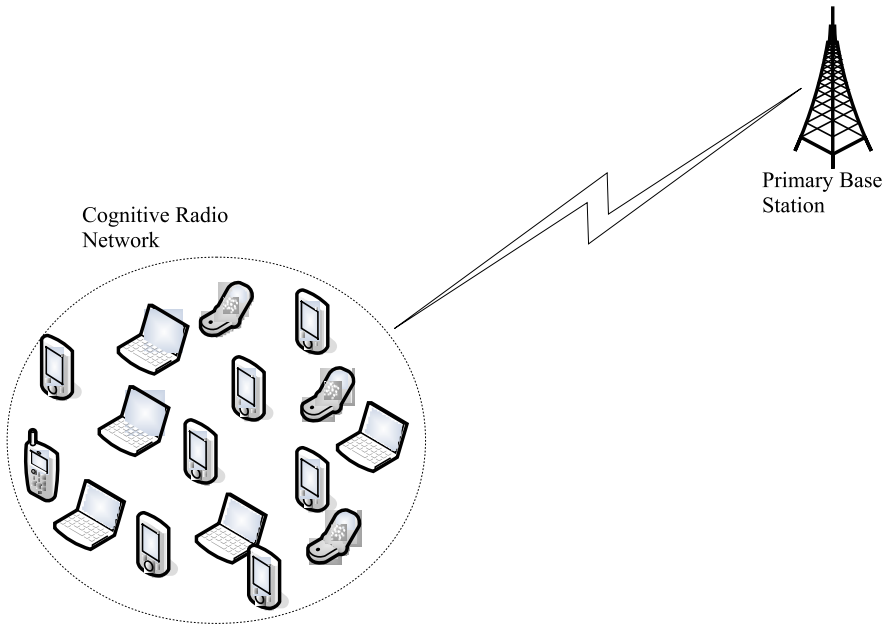

Fig. 1. Scenario and System Model

equilibrium for the game in section III. Our motivation to employ CE for the spectrum sensing game is presented in sections IV-A and IV-B. The optimization problem is formulated to maximize the system utility in section IV-C. We discuss schemes that incorporate fairness at different levels and propose the f-SW CE in section IV-D. We describe NR algorithm and propose an NBL algorithm in section V. The results are presented and analyzed in section VI. Section VII concludes the paper.

II. System Model

A. Network Model

We consider the sensing of M channels of the primary system. There are K secondary users, located far away from primary base station as shown in Figure 1. In this scenario, the received signal to noise ratio for secondary users from primary base station is very small.

We consider the frame based sensing and transmission scheme as shown in Figure 2. Spectrum sensing is performed at the beginning of each frame. The duration of each frame is T and the sensing duration is T_s. The users in the network have heterogeneous traffic. Among

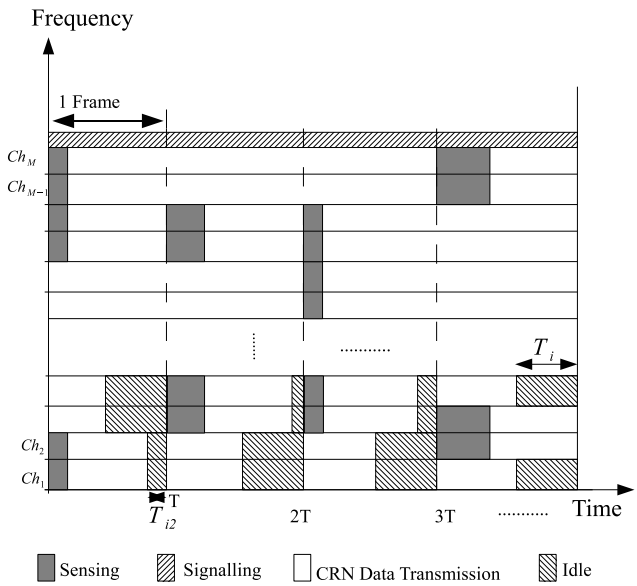

Fig. 2. Cooperative Spectrum Sensing Scheme

them, some users always have data to transmit, while others have lower input data rate, thus, they need less time to transmit their data and have idle time slots in a frame duration. $T_i \geq 0$ is the idle duration for user i if it does not perform sensing, and $0 \leq T_{i2} < T_i$ is the idle duration after it performs sensing, if it does. The signaling channel is used to share the sensing results.

The secondary users that choose to perform sensing cannot transmit data while sensing. In order to achieve a higher throughput, secondary users will tend to decide not to sense if they are selfish, and take the advantage by overhearing the sensing results from other users. However, if no user contributes in sensing, then, no user will achieve any payoff. Thus, the users are fundamentally non-cooperative, but cooperation can emerge among them as none of them desires zero or very low payoff. For this kind of scenario, it is important to study their behavior dynamics such that cooperation can be enforced among the secondary users to maintain the required level of detection performance.

B. Spectrum Sensing Technique: Energy Detector

We employ energy detection mechanism to perform sensing. For energy detector, the received signal $x(n)$ is subject to a hypothesis test as following:

$$\begin{cases} H_0 : x(n) = w(n) \quad \text{primary user is absent,} \\ H_1 : x(n) = gs(n) + w(n) \quad \text{primary user is present,} \end{cases} \tag{1}$$

where g is the gain of the channel between primary user transmitter and secondary user receiver, $s(n)$ is the signal from the primary user, which is assumed to be an i.i.d. random process with zero mean and variance σ_s^2 and $w(n)$ is an additive white Gaussian noise with zero mean and variance σ_w^2, $n = 1, 2, ..., N$ is the discrete time index and N is the total number of samples collected during sensing period.

Neyman-Pearson detection approach for this hypothesis test has two errors. The first is the error that occurs when the primary signal is absent but the detector declares that the signal is present, namely probability of false alarm (P_{FA}). The second error occurs when the channel is occupied, but the detector decides that the channel is vacant. This error is called probability of miss detection (P_{MD}). Each of these errors is associated with the particular threshold γ for comparing the power of the received signal with, and can be calculated as follows [20]:

$$P_{\text{FA}} = Q(\frac{\gamma - N\sigma_w^2}{\sqrt{N\sigma_w^4}}), \tag{2}$$

$$P_{\text{D}} = Q(\frac{\gamma - N(\sigma_w^2 + |g|^2\sigma_s^2)}{\sqrt{N(\sigma_w^2 + |g|^2\sigma_s^2)^2}}), \tag{3}$$

where $P_{\text{D}} = 1 - P_{\text{MD}}$ is the probability of detection and $Q(.)$ is the standard complementary Gaussian function. Let $\overline{P}_{\text{D}}$ denote the target probability of detection. Then, substituting γ from (3) to (2), P_{FA} of sensing user i can be written as,

$$P_{\text{FA,i}} = Q(Q^{-1}(\overline{P}_{\text{D}})(1 + \lambda) + \sqrt{N}\lambda), \tag{4}$$

where $\lambda = \frac{|g|^2\sigma_s^2}{\sigma_w^2}$ is the received signal to noise ratio of the primary user under H_1.

We use *OR* rule to combine the result of sensing from each secondary user in the sensing group. If $\mathcal{J} = \{s_1, s_2, ..., s_J\}$ be the set of secondary users that participate in cooperative sensing, then, the probability of false alarm for cooperative sensing is,

$$\widehat{P}_{\text{FA}} = (1 - \prod_{i=1}^{J} P_{\text{FA,i}}). \tag{5}$$

III. Game Theoretical Framework for Cooperative Spectrum Sensing: Mixed Strategy Equilibrium

We model the spectrum sensing as a non-cooperative game. The players of the game are the K secondary users $\mathcal{K} = \{1, 2, ..., K\}$. As in [14], each user gets one channel if the primary channel is found vacant. The strategy of player i is to choose from the binary strategy set $A_i = \{0, 1\}$ at the beginning of each sensing interval, where 0 means *not-sense* and 1 means *sense*.

Let p_{H_0} be the probability that the primary user is absent. Let us define $0 \leq \tau_i \leq 1$ as the idle duration that user i will have in each frame after transmitting its data. The achievable throughput for a sensing user can be written as

$$R_i(J) = \begin{cases} p_{H_0}(1 - \widehat{P}_{\text{FA}})C_{H_0}(1 - \tau_i) & \text{if } J \in [1, K] \text{ and } \frac{\tau}{J} <= \tau_i, \\ p_{H_0}(1 - \widehat{P}_{\text{FA}})C_{H_0}(1 - \frac{\tau}{J}) & \text{if } J \in [1, K] \text{ and } \tau_i < \frac{\tau}{J} < 1 \end{cases} \tag{6}$$

where $\tau = \frac{T_s}{T}$ is the normalized sensing duration and C_{H_0} is the data rate that one channel offers the secondary user under H_0.

We define the payoff as the difference between the obtained throughput and the associated energy cost. Therefore, the average payoff for a sensing user is

$$U_{i,s}(J) = \begin{cases} U_0(1 - \tau_i) - \frac{\sigma\tau}{J} & \text{if } J \in [1, K] \text{ and } \frac{\tau}{J} <= \tau_i, \\ U_0(1 - \frac{\tau}{J}) - \frac{\sigma\tau}{J} & \text{if } J \in [1, K] \text{ and } \tau_i < \frac{\tau}{J} < 1 \end{cases} \tag{7}$$

where $U_0 = p_{H_0}(1 - \widehat{P}_{\text{FA}})C_{\text{H}_0}$, σ is the energy consumed per unit of time for sensing.

If user i decides not to sense, its payoff can be derived in the similar manner and is given by

$$U_{i,ns}(J) = \begin{cases} U_0(1 - \tau_i) & \text{if } J \in [1, K - 1], \\ 0 & \text{if } J = 0. \end{cases} \tag{8}$$

For a finite K user game in strategic form $\mathcal{G} = \{\mathcal{K}, (A_i)_{i\in\mathcal{K}}, (u_i)_{i\in\mathcal{K}}\}$, where u_i is the utility function of user i, the strategy space for all users other than i is denoted as A_{-i}. Let us denote the actions taken by user i and the rest of the users as a_i and a_{-i}, respectively. Then, for all $i \in \mathcal{K}$, the strategy $a_i^* \in A_i$ is a *Nash Equilibrium* (NE) for user i if for every alternative strategy $a_i \in A_i$, $(a_i^* \neq a_i)$, the following relation holds:

$$u(a_i^*, a_{-i}^*) \geq u(a_i, a_{-i}^*). \tag{9}$$

Equation (9) indicates that at NE, a player cannot improve its payoff by deviating alone from the equilibrium, given the strategies of all other players.

If the achievable payoff for being a free rider is greater than the achievable payoff for contributing in cooperative sensing, each secondary user will tend to be a free rider [14]. Conversely, if the payoff of being a sensing user is greater, all users will take strategy *sense*. One of the equilibria for the game is the point where the obtainable payoff for being a sensing user is same as the obtainable payoff of being a not-sensing user. This point is called the *mixed strategy NE*. Let, $K_1, K_2, ..., K_L$ ($\sum_{l=1}^{L} K_l = K$) be the number of secondary users which choose to contribute in sensing with probabilities p_1, p_2,....,p_L respectively. Then, for user i which belongs to the group of K_l users, if it decides to contribute in sensing, the achievable average utility $\overline{U}_{i,s}$ will be,

$$\overline{U}_{i,s} = \begin{array}{l} \sum_{j_L=0}^{K_L} .. \sum_{j_l=0}^{K_l-1} .. \sum_{j_1=0}^{K_1} \begin{pmatrix} K_L \\ j_L \end{pmatrix} {p_L}^{j_L}(1-p_L)^{(K_L-j_L)} .. \begin{pmatrix} K_l - 1 \\ j_l \end{pmatrix} {p_l}^{j_l}(1-p_l)^{(K_l-1-j_l)} .. \\ \begin{pmatrix} K_1 \\ j_1 \end{pmatrix} {p_1}^{j_1}(1-p_1)^{(K_1-j_1)} U_{i,s}((\sum_{l=1}^{L} j_l)+1), \end{array} \tag{10}$$

The average utility for user i for taking pure strategy *not-sense* is given by $\overline{U}_{i,ns}$,

$$\overline{U}_{i,ns} = \begin{array}{l} \sum_{j_L=0}^{K_L} .. \sum_{j_l=0}^{K_l-1} .. \sum_{j_1=0}^{K_1} \begin{pmatrix} K_L \\ j_L \end{pmatrix} {p_L}^{j_L}(1-p_L)^{(K_L-j_L)} .. \begin{pmatrix} K_l - 1 \\ j_l \end{pmatrix} {p_l}^{j_l}(1-p_l)^{(K_l-1-j_l)} .. \\ \begin{pmatrix} K_1 \\ j_1 \end{pmatrix} {p_1}^{j_1}(1-p_1)^{(K_1-j_1)} U_{i,ns}(\sum_{l=1}^{L} j_l). \end{array} \tag{11}$$

Now, using equality of payoff theorem, the mixed strategy NE p_i for $i \in \mathcal{K}$ can be calculated by solving

$$\overline{U}_{i,s} = \overline{U}_{i,ns}. \tag{12}$$

We can get L such equations, solving which we can obtain p_1, p_2,....,p_L.

IV. Game Theoretical Framework for Cooperative Spectrum Sensing: Correlated Equilibrium

A. Preliminary

In this paper, we introduce the concept of *correlated equilibrium* (CE) for the cooperative spectrum sensing game among secondary users. The CE [16] is defined in a context where the

players are able to access certain common signals. These signals allow players to coordinate their actions and to perform joint randomization over their strategies according to a certain distribution. This probability distribution is called CE.

A probability distribution P_d over $A_1 \times ..... \times A_K$ is a CE of game $\mathcal{G}$ if and only if for all $i \in \mathcal{K}$, for every strategy $a_i \in A_i$ and for every alternative strategy $a_i^{'} \in A_i$,

$$\sum_{a_{-i} \in A_{-i}} p(a_i, a_{-i})[u(a_i^{'}, a_{-i}) - u(a_i, a_{-i})] \leq 0, \tag{13}$$

where $p(a_i, a_{-i}) \geq 0$, represents the probability that user i takes action a_i and the rest of the users take actions a_{-i}. The inequality (13) indicates that when the recommendation to user i is to choose action a_i, then choosing action $a_i^{'}$ instead of a_i cannot give a higher expected payoff to user i. The set of correlated equilibria is nonempty, closed and convex in every finite game. Moreover, it may include the distribution that is not in the convex hull of the NE distributions. In fact, every NE is a CE and Nash equilibria correspond to the special case where $p(a_i, a_{-i})$ is a product of each individual users probability for different actions. If we replace the right-hand side of inequality (13) by $\epsilon \geq 0$, the CE is called correlated ϵ-equilibrium.

The constraints (13) can be solved by using linear programming. This gives a set of equilibria. Thus, there can be multiple set of the solutions of correlated equilibria. An appropriate objective function should be defined in order to obtain a refined CE. In this paper, we focus on social welfare and fairness related objective functions, considering the unique characteristics of the spectrum sensing game.

B. Motivation

In this section, we explain the motivation behind introducing the concept of CE for the cooperative spectrum sensing game and present examples to demonstrate that the system performance can be improved by using CE. There are two reasons why CE is an important solution for such games. Firstly, some games do not have a pure NE while some games have multiple NE. Even the mixed strategy NE does not always exist but the set of correlated equilibria is nonempty, closed and convex in every finite game. Secondly, CE can achieve better utilities than NE. The following examples illustrate the efficiency of CE.

1) Spectrum Sensing Game Among Homogeneous Users: Let us consider the sensing game between two homogeneous users, which always have data to transmit. The payoff table for the two players is shown in Table I. The first element of each cell is the payoff of the first(/row)

TABLE I

PAYOFF TABLE FOR A HOMOGENEOUS 2 USER SENSING GAME ($\overline{P}_{\mathrm{D}} = 0.95$, $SNR = -14$ dB, $\tau = 0.5$, $f_s = 1$ MHz $C_{H_0} = 10$ Mbps)

	sense	not-sense
sense	$(6.75, 6.75)$	$(4.5, 9.0)$
not-sense	$(9.0, 4.5)$	$(0, 0)$

user and the second element is the payoff for the second(/column) user. The payoffs were calculated for $\sigma = 0$. The game has two pure Nash equilibria: $(4.5, 9.0)$ and $(9.0, 4.5)$. A mixed strategy equilibrium is that both user take the action *sense* with a probability of $\frac{2}{3}$, that yields average utilities of 6.0 for both of them and the total utility is 12.

In terms of CE concept, let, (q_1, q_2, q_3, q_4) be the probabilities of taking action pairs $\{0, 0\}, \{0, 1\}, \{1, 0\},$ and $\{1, 1\}$, respectively. The CE for this case to maximize the system utility is $(q_1 = 0.0, q_2 = 0.4702, q_3 = 0.4702, q_4 = 0.0596)$. Thus, the achievable utilities for both users will be 6.75, giving the system utility of 13.5, higher than in the case of mixed strategy equilibrium. Figure 3 shows the average utilities obtained at mixed strategy NE and CE. It is clear that CE improves the average utility of each user and hence, it improves the system utility.

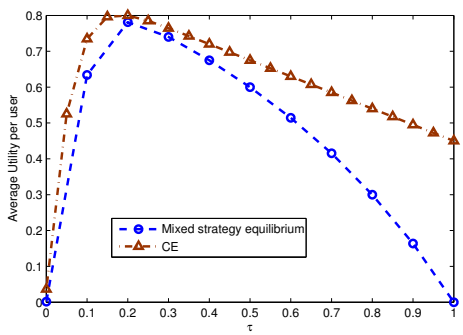

Fig. 3. Average utilities ($K = 2$)

2) Spectrum Sensing Game Among Heterogeneous Users: When the users have heterogeneity in traffic dynamics, the heterogeneity can be utilized to design a suitable approach for the case of cooperative spectrum sensing. The secondary users with lower traffic intensity do not need the full frame duration for their data transmission. The idle duration of such users

TABLE II

PAYOFF TABLE FOR A HETEROGENEOUS 2 USER SENSING GAME

	sense	not-sense
sense	$(6.75, 6.75)$	$(4.5, 7.2)$
not-sense	$(9.0, 4.5)$	$(0, 0)$

can be exploited for sensing to increase the network utility, as illustrated in the following example.

Let us consider the spectrum sensing game between two users with $\tau_1 = 0.0, \tau_2 = 0.2$. The payoff table for the two players is shown in Table II with other parameters same as for Table I. The game has two pure Nash equilibria: $(4.5, 7.2)$ and $(9.0, 4.5)$. A mixed strategy equilibrium is that user 1 takes the action 1 with a probability of 0.91 and user 2 takes the same action with a probability of 0.67. The average utilities obtainable at mixed strategy are 6.0 for user 1 and 6.5475 for user 2, respectively with the total utility of 12.5475. The CE for maximizing the system utility is $(q_1 = 0.0, q_2 = 1.0, q_3 = 0.0, q_4 = 0.0)$ and the achievable utilities for user 1 and user 2 will be 9.0 and 4.5 respectively, giving the system utility of 13.5. Unlike the case of homogeneous users, the CE probability distribution may be asymmetric in case of heterogeneous users as we saw in this example.

Always cooperating i.e. $\{1, 1\}$ is not an equilibrium here because of the lack of cooperation among selfish users. However, since the users adopt CE strategy, they jointly optimize their actions by considering the low utility of mutual non-cooperation and the high utility when both cooperate, thus the degree of cooperation is increased and the system utility improves.

C. Problem Formulation

There are 2^K possible combinations of strategies for the sensing game with K secondary users. Let us represent all possible combination of actions as matrix $B = [B_{i,j}]$, where the rows $i \in [1 \quad 2^K]$ represent the combination of actions of each user and the columns $j \in [1 \quad K]$ indicate the action chosen by each user, the matrix rows starts from $\{0, 0, .., 0\}$ and ends at $\{1, 1, .., 1\}$, the leftmost element being for user 1. For instance, $\mathrm{B}_{4,1}$ indicates the action of user 1 for strategy combination 0011 for a 4 user game. When a user, say k^{th} user, chooses an action: 0 or 1, it will be removed from matrix B and the number of possible combination of actions for the rest of the users will be 2^{K-1}. Let us represent this

matrix as $C = [C_{i,j}] : \forall i \in [1 \;\; 2^{(K-1)}], \forall j \in [1 \;\; K], j \neq k$, which is of size $2^{K-1} \times (K-1)$. The optimization problem ($\mathbf{P_1}$) for the cooperative spectrum sensing game to obtain CE that maximizes the *social welfare* (SW), can be defined as $\mathbf{P_1}$ (CE that maximizes social welfare)

$$\max_{p} \sum_{i=1}^{K} \sum_{l=1}^{2^{K-1}} \big\{ p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,ns}(K_s) \tag{14}$$
$$+ p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,s}(K_s + 1) \big\}$$

s.t.

$$\sum_{l=1}^{2^{K-1}} p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1})(U_{i,ns}(K_s) - U_{i,s}(K_s + 1)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{15}$$

$$\sum_{l=1}^{2^{K-1}} p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1})(U_{i,s}(K_s + 1) - U_{i,ns}(K_s)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{16}$$

$$p(a_i = 0, ..) \geq 0, \quad p(a_i = 1, ..) \geq 0.$$

where $K_s = \sum_{k=1,k\neq i}^{K} a_k$, is the total number of users performing cooperative sensing, for each combination of strategies, $U_{i,s}(.)$ and $U_{i,ns}(.)$ are the utilities given by (7) and (8) respectively. $\mathbf{P_1}$ is a linear programming problem and it can be solved in a centralized fashion by using either the simplex method (SM) or the interior point method (IPM).

It is noteworthy that the optimization problem $\mathbf{P_1}$ looks similar for the case of homogeneous as well as heterogeneous users. However, the difference in the two cases lies in the utility functions: $U_{i,s}(K_s + 1)$ and $U_{i,ns}(K_s)$. For the case of homogeneous users, the utility achievable for each user is the same for one value of K_s, irrespective of which users are contributing in sensing. This implies that the probabilities of choosing the combinations of strategies for which K_s is the same, are equal. On the other hand, when we consider users with heterogeneous traffic dynamics, the value of τ_i is different for each user. This means, different users can get different utilities even for the same value of K_s. Consequently, the probability distribution may also be asymmetric.

D. Fairness Consideration and Utility Maximization

When the users are heterogeneous, social welfare, although yields the highest system utility, it may lead users that have worst conditions or those that demand less resources to starve. The loss that the less well-off users suffer when the system utility is maximized, may be undesirable in some situations. Let's consider the example in section IV-B2. User 2 has 20%

less traffic compared to user 1. But the utility of user 1 is twice the utility of user 2. Even for smaller values of τ_2 e.g., $\tau_2 = 0.05$, the payoffs that the users obtain will be 9.0 and 4.5 respectively. The difference in the utilities they get, increases further for higher values of τ. Eg., when $\tau = 0.9$, user 1 still gets a utility of 9.0 but the utility of user 2 will be 0.9. The fairness issue is critical in this case and it is one of the worst consequences of maximizing the system utility. In such systems, the objective function should be fairness centric rather than social welfare. Next, we discuss two schemes that consider two standard definition of fairness, and then we propose a new scheme to maximize the social welfare incorporating fairness.

1) Max-min Fairness (MMF): One of the common approaches to focus on fairness, is to use max-min fairness as the objective function. The optimization problem ($\mathbf{P_2}$) for the cooperative spectrum sensing game to obtain CE that maximizes the utility of the worst user, can be defined as $\mathbf{P_2}$ (CE that maximizes the utility of the worst user)

$$\max_{p} \min_{i} \sum_{l=1}^{2^{K-1}} \big\{ p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,ns}(K_s) \tag{17}$$
$$+ p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,s}(K_s + 1) \big\}$$

s.t.

$$\sum_{l=1}^{2^{K-1}} p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1})(U_{i,ns}(K_s) - U_{i,s}(K_s + 1)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{18}$$

$$\sum_{l=1}^{2^{K-1}} p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1})(U_{i,s}(K_s + 1) - U_{i,ns}(K_s)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{19}$$

$$p(a_i = 0, ..) \geq 0, \quad p(a_i = 1, ..) \geq 0.$$

$\mathbf{P_2}$ is also a linear programming problem and it can be solved in a centralized fashion by using either the SM or the IPM.

When the difference in user requirements is not significant, using max-min fairness as the objective function in $\mathbf{P_2}$ yields almost the same total utility as in $\mathbf{P_1}$ but it improves the utility of the less-well off users significantly. This means, the max-min fairness greatly improves the fairness without considerably degrading the system utility. On the other hand, if the achievable utilities of the users have a vast difference, and we try to optimize the utility of the worst user(s), the total utility will be affected significantly.

2) Proportional Fairness (PF): There are several studies on rate control algorithms and dynamic spectrum allocation with fairness [25]-[27]. PF ensures more fairness than the SW scheme, while achieving better performance than the MMF scheme. The proportionally fair distribution is a feasible vector x such that if one element of x is increased by $y\%$, the total percentage of reduction that has to be applied to the other elements of x in order to get another feasible distribution vector must be more than $y\%$. The optimization problem (**P₃**) for the cooperative spectrum sensing game to obtain CE that maximizes proportional fairness, can be defined as

P₃ (CE that maximizes proportional fairness)

$$\max_{p} \sum_{i=1}^{K} \prod_{l=1}^{2^{K-1}} \big\{ p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,ns}(K_s) \tag{20}$$
$$+ p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,s}(K_s + 1) \big\}$$

s.t.

$$\sum_{l=1}^{2^{K-1}} p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1})(U_{i,ns}(K_s) - U_{i,s}(K_s + 1)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{21}$$

$$\sum_{l=1}^{2^{K-1}} p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1})(U_{i,s}(K_s + 1) - U_{i,ns}(K_s)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{22}$$

$$p(a_i = 0, ..) \geq 0, \quad p(a_i = 1, ..) \geq 0.$$

P₃ is also a linear programming problem and it can be solved in a centralized fashion by using either the SM or the IPM.

3) Fair Social Welfare (f-SW): System efficiency and fairness are two contradictory objectives. The key motivation behind deploying CE for the cooperative spectrum sensing game is to improve the overall system utility by utilizing the heterogeneity among the secondary users. However, addressing the fairness issue among the secondary users is of great importance too. We combine these two aspects and propose a new fair and efficient CE for the game, and we name it f-SW CE.

The average utility of user i at CE is

$$\begin{cases} \overline{U}_i = \sum_{l=1}^{2^{K-1}} p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,ns}(K_s) \\ + \sum_{l=1}^{2^{K-1}} p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,s}(K_s + 1). \end{cases} \tag{23}$$

Among K secondary users, let $\overline{U}_j$ be the utility of the highest traffic user(s) i.e. $\overline{U}_j = \max(\overline{\mathrm{U}}_\mathrm{i})$. In the proposed scheme, the highest traffic user gets the highest utility and the difference in the utility obtained by user i is related to their traffic dynamics by an additional constraint as following:

$$\frac{\overline{U}_j - \overline{U}_i}{\overline{U}_j} \leq d\tau_i \qquad \forall i, j \in \mathcal{K},\ i \neq j, \tag{24}$$

where $d \geq 1$ is a scaling parameter such that $d\tau_i \leq 1\ \forall i$. Equation (24) can be written as

$$\overline{U}_j(1 - d\tau_i) - \overline{U}_i \leq 0 \qquad \forall i, j \in \mathcal{K},\ i \neq j. \tag{25}$$

The proposed CE aims at maximizing the system utility and incorporates fairness criteria in it. The optimization problem (**P₄**) for the f-SW CE scheme can be defined as

P₄ (CE that maximizes system utility ensuring fairness)

$$\max_{p} \sum_{i=1}^{K} \Big(\sum_{l=1}^{2^{K-1}} p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,ns}(K_s) \tag{26}$$
$$+ \sum_{l=1}^{2^{K-1}} p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1}) U_{i,s}(K_s + 1) \Big)$$

s.t.

$$\sum_{l=1}^{2^{K-1}} p(a_i = 0, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1})(U_{i,ns}(K_s) - U_{i,s}(K_s + 1)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{27}$$

$$\sum_{l=1}^{2^{K-1}} p(a_i = 1, [a_k]_{k=1,k\neq i}^{K} = [C_{l,k}]_{k=1}^{K-1})(U_{i,s}(K_s + 1) - U_{i,ns}(K_s)) \geq 0 \quad \forall i \in \mathcal{K}, \tag{28}$$

$$\overline{U}_j(1 - d\tau_i) - \overline{U}_i \leq 0 \quad \forall i, j \in \mathcal{K}, i \neq j \tag{29}$$

$$p(a_i = 0, ..) \geq 0, \quad p(a_i = 1, ..) \geq 0.$$

As (25) is a linear constraint, **P₄** is a linear program, and thus it can also be solved using either the SM or the IPM.

Note that the proposed scheme is different and more flexible than PF CE scheme. In f-SW CE scheme, fairness can be controlled through parameter d for given τ and τ_i, which is not the case in PF CE.

V. Distributed Implementation of CE

A. No-Regret Algorithm

In order to solve the optimization problem for computing the CE, each player needs to know the payoff matrix of other players as well. This makes the concept of CE difficult to be implemented distributedly. We propose to use a *No-Regret* (NR) algorithm [17] so that the secondary users' strategy converges to the CE, by using only local information of each secondary user.

The sensing game is played repeatedly through time $t = 1, 2, 3, ...$ Given the history of play of each user until time t: $h(t) = (a^m)_{m=1}^t$, where a^m means the strategy taken at time index m, each player $i \in \mathcal{K}$ chooses $(a_i^{t+1}) \in A_i$ according to a probability distribution $p_i^{t+1} \in \Delta(A_i)$, where $\Delta(A_i) = A_1 \times A_2 \times .... \times A_K$, which can be calculated based only on the local information of each player.

Let $r, r^{'} \in A_i$ be two different strategies of player i. Then, for every action r that i took until time t, if it had taken strategy $r^{'}$ instead, the resulting difference in utility would have been

$$D_i^t(r, r^{'}) = \frac{1}{t} \sum_{m \leq t : a_i^m = r} [u_i(r^{'}, a_{-i}^m) - u_i(a^m)]. \tag{30}$$

The average regret until time t for not playing strategy $r^{'}$ every time it played strategy r in the past is

$$R_i^t(r, r^{'}) = max(D_i^t(r, r^{'}), 0). \tag{31}$$

The algorithm is shown in detail in Table III. The average regret in (31) can be calculated based only on the local information, which enables the NR algorithm to be implemented in a distributed manner.

At each stage of the game t, let $z^t \in \Delta(A)$ be the empirical distribution of the Q-tuples of strategies played until time t. Then, for every $a \in A$,

$$z^t(a) = \frac{1}{t} |m \leq t : a^m = a|, \tag{32}$$

where $|.|$ is the number of times the event inside $||$ occurs.

If every player plays according to NR learning algorithm, then the empirical distributions of play z_t converge almost surely (with probability one) to the set of correlated equilibria of the game as $t \to \infty$ [17]. A necessary and sufficient condition for this is that, all regrets

TABLE III

THE NO-REGRET LEARNING ALGORITHM

Choose the probability distribution $p_i^1 \in \Delta(A_i)$ randomly for each user i at the beginning of the game i.e. at $t = 1$.
for $t = 1, 2, , ...$ do Compute $D_i^t(r, r^{'})$ using (30). Compute $R_i^t(r, r^{'})$ using (31). If $r \in A_i$ be the strategy chosen by user i at time t i.e. $a_i^m = r$, then, the probability distribution $p_i^{t+1} \in \Delta(A_i)$ used by user i at time $t+1$ can be calculated using $p_i^{t+1}(r^{'}) = \frac{1}{\mu} R_i^t(r, r^{'})$ for all $r^{'} \neq r$, $p_i^{t+1}(r) = 1 - \sum_{r^{'} \in A_i, r^{'} \neq r} p_i^{t+1}(r^{'})$, where $\mu > 2M_i(n_i - 1)$ $\forall i \in \mathcal{K}$, M_i is the upper bound for $\lvert u_i(.) \rvert$ and n_i is the number of strategies of user i. **end for** when the probability distribution converges

converge to zero. In a more general form, this condition can be represented as the following proposition:

Proposition:Let $(a^t)_{t=1,2,..}$ be a sequence of plays $(a^t \in A \ \forall t)$ and let $\epsilon \geq 0$. Then, $\forall i \in \mathcal{K}$ and every $r, r^{'} \in A_i$ with $r \neq r^{'}$,

$$\lim_{t \to \infty} \sup R_i^t(r, r^{'}) \leq \epsilon \tag{33}$$

if and only if the sequence of empirical distributions z_t converge to the set of correlated ϵ-equilibria.

Proof: For each player i and every $r, r^{'} \in A_i$, using (30) and (32), we get,

$$D_i^t(r, r^{'}) = \sum_{a \in A, a_i = r} z^t(a)[u_i(r^{'}, a_{-i}^m) - u_i(a^m)]. \tag{34}$$

On any subsequence where z^t converges, say $z^{t'} \to \psi \in \Delta(A)$, (30) can be written as,

$$D_i^t(r, r^{'}) = \sum_{a \in A, a_i = r} \psi(a)[u_i(r^{'}, a_{-i}^m) - u_i(a^m)]. \tag{35}$$

The result is immediate from the definition of correlated ϵ-equilibrium and (31). This completes the proof.

This algorithm requires player i to know its own payoff matrix but not those of other players, and at time $t+1$, the history of play $h(t)$. In terms of computation, player i needs to keep a record of time t along with $2(2-1) = 2$ values of $D_i^t(r, r^{'})$ for $r \neq r^{'}$ in A_i, and has to update these numbers after each frame.

B. Neighbourhood Based Learning Algorithm

We propose a new algorithm based on learning from the neighbours to improve the performance of the NR algorithm. We call it *Neighbourhood Based Learning* (NBL) algorithm. In this algorithm, the strategy of each user is calculated based on the total regret rather than the user's individual regret.

Let $r, r^{'} \in A_i$ be two different strategies of player i. Then, for every action r that user i took until time t, if it had taken strategy $r^{'}$ instead its total utility would have been different. The resulting difference in total utility would have been

$$D^t_{i,\text{net}}(r, r^{'}) = \frac{1}{t} \sum_{m \leq t : a_i^m = r} \sum_{l \in \mathcal{K}} \{u_l(a_i^m = r^{'}, a_{-i}^m) - u_l(a^m)\}. \tag{36}$$

To compute $D^t_{i,\text{net}}(r, r^{'})$, at time $t+1$, each user should have the information about the history $h(t)$, as in NR algorithm. In addition, each user needs to compute the resulting difference in the utilities of all other users for not having played $r^{'}$ everytime it played r in the past . Each user gets the information required to compute this difference through its neighbours. This information exchange accounts for an additional cost for the proposed algorithm. However, in the spectrum sensing game, the only extra parameter that each user needs to exchange in the NBL algorithm is $\tau_i \forall i \in \mathcal{K}$, only once at the beginning of the game. Thus, the overhead due to this information exchange is negligible.The average net regret until time t for not having played $r^{'}$ every time it played r in the past is

$$R^t_{i,\text{net}}(r, r^{'}) = max(D^t_{i,\text{net}}(r, r^{'}), 0). \tag{37}$$

The detailed algorithm is described in Table IV. In terms of computation in the NBL algorithm, player i needs to keep a record of time t along with $2(2-1) = 2$ values of $D^t_{i,\text{net}}(r, r^{'})$ for $r \neq r^{'}$ in A_i and has to update these numbers after each frame. But calculating $D^t_{i,\text{net}}(r, r^{'})$, needs $K-1$ times more computation in this case.

Computing correlated equilibria for multi-player games becomes rapidly intractable in the general case when the number of players is large because the input length is exponential [18]. However, using the NBL algorithm, the users can start with an arbitrary strategy and then, converge to the CE, with some extra computations which is only linearly proportional to the number of users.

TABLE IV

THE NEIGHBOURHOOD BASED LEARNING ALGORITHM

Choose the probability distribution $p_i^1 \in \Delta(A^i)$ randomly for each user i at the beginning of the game i.e. at $t = 1$.
for $t = 1, 2, , ...$ do Compute $D_{i,\text{net}}^t(r, r^{'})$ using (36). Compute $R_{i,\text{net}}^t(r, r^{'})$ using (37). If $r \in A_i$ be the strategy chosen by user i at time t i.e. $a_i^m = r$, then, the probability distribution $p_i^{t+1} \in \Delta(A_i)$ used by user i at time $t+1$ can be calculated using $p_i^{t+1}(r^{'}) = \frac{1}{\mu} R_{i,\text{net}}^t(r, r^{'})$ for all $r^{'} \neq r$, $p_i^{t+1}(r) = 1 - \sum_{r' \in A_i, r' \neq r} p_i^{t+1}(r^{'})$, where μ is a sufficiently large number. **end for** when the probability distribution converges

TABLE V

PARAMETERS USED FOR ANALYSIS

Parameters	Values
$\overline{P}_{\text{D}}$	0.95
SNR	-14 dB
τ	0.5
f_s	1 MHz
C_{H_0}	10 Mbps

VI. NUMERICAL RESULTS AND ANALYSIS

The parameters used for evaluation are shown in Table V. Different values of the parameters will give different CE probability distributions. However, for the purpose of illustration, we use a set of realistic parameters in all analyses in this section. We consider $K = 3$ with $\tau_1 = \tau_2 = 0.0, \tau_3 = 0.2$, unless otherwise stated. We refer to users 1 and 2 as heavy traffic users and user 3 a light traffic user. In addition, for all plots in this section we consider $\sigma = 0$, but please note that, similar analysis can be done for $\sigma \neq 0$ as well using our analytical framework.

A. Utility Maximization for Heterogeneous Users: SW CE

The optimal CE probability distribution for this game can be obtained by solving $\mathbf{P_1}$. The utilities obtained are depicted in Figure 4. This figure indicates that the system utility is maximized if light traffic user(s) is(are) selected to contribute more in sensing. When SW

is considered, the light traffic user may get even less utility than in mixed strategy NE but the heavy traffic users get higher utilities compared to the mixed strategy NE. Thus the total utility is higher in SW CE than in mixed strategy NE.

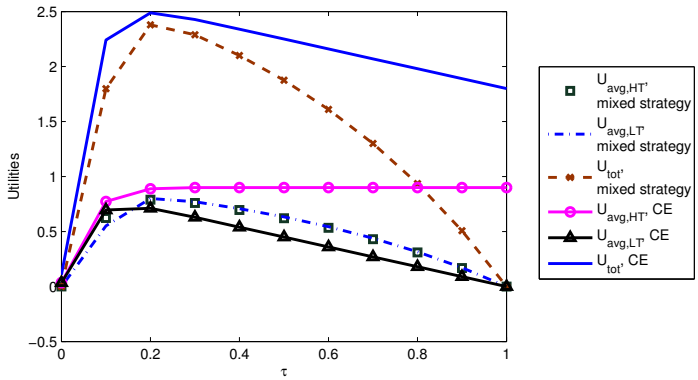

Fig. 4. Comparison of average utilities (CE and mixed strategy)

B. Comparison of SW CE, MMF CE and PF CE Schemes

We plot the utilities obtained by each user and the total utility in Figure 5, for the three schemes. The figure depicts that the utility of the heavy traffic users (users 1 and 2) and the total utility are high for the case of SW. In contrast, for the light traffic user (user 3), the utility is better when MMF is considered. The performance of PF is close to that of SW for smaller values of τ and it approaches MMF for higher values of τ.

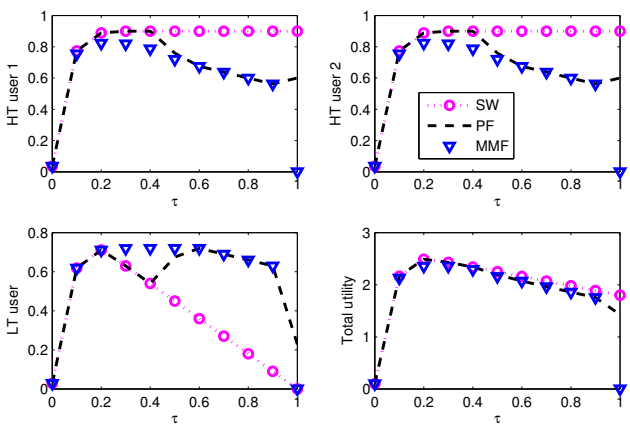

Fig. 5. Comparison of utilities (SW, PF, MMF)

C. Performance of the Proposed f-SW CE Scheme

We compare the total utilities obtained at f-SW CE with SW CE, PF CE and MMF CE schemes in Figure 6 for the scaling parameter $d = 2$. From the figure we see that the total utility at f-SW CE is less than in SW CE scheme, but it is higher than in MMF CE scheme for all values of τ. The total utility is better than in PF CE scheme as well for $\tau > 0.4$. Figure 7 depicts the utilities per user obtained using SW CE, MMF CE and f-SW CE schemes. One of the issues with MMF scheme is that the utility of users 1 and 2 is less than that of user 3 after $\tau > 0.5$. This is not fair to users 1 and 2. In the proposed scheme, this problem is eliminated as the utility of users 1 and 2 is always higher than that of user 3. The utilities of users 1 and 2 is lower than in SW CE but higher than in MMF CE. Moreover, the utility of user 3 is slightly less in the proposed scheme than in MMF CE but improves considerably compared to SW CE scheme. Thus, fairness is achieved in the proposed scheme keeping the total utility higher than in MMF CE scheme.

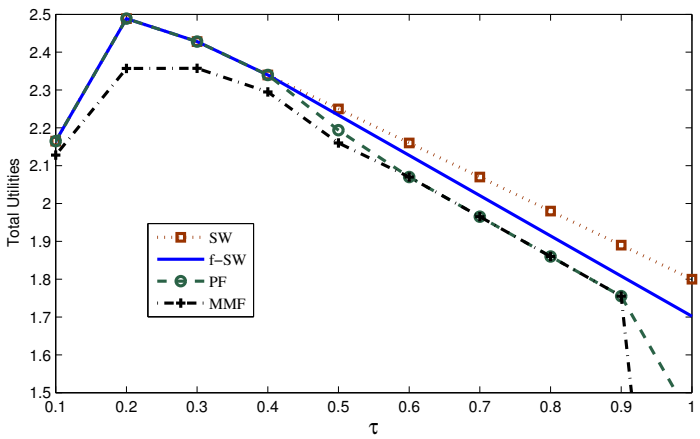

Fig. 6. Comparison of total utilities (SW, f-SW, PF and MMF)

D. Proposed NBL algorithm

The probability distribution for NBL algorithm is computed in similar manner as in case of the NR algorithm. Here, we consider a two user game with $\tau_1 = 0.0, \tau_2 = 0.2$. q_1, q_2, q_3, q_4 are the probabilities of the strategy combinations: $\{0, 0\}, \{0, 1\}, \{1, 0\}$ and $\{1, 1\}$, respectively. Figure 8 shows the performance of the NBL algorithm. The game converges to the probability distribution $(q_1, q_2, q_3, q_4) = (0.0, 0.37, 0.08, 0.55)$, indicating that the second user is involved

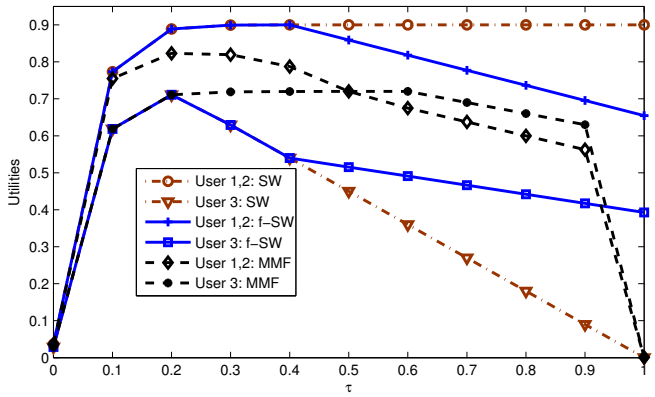

Fig. 7. Comparison of utilities per user (SW, f-SW and MMF)

more in sensing. Although not shown here, the sensing contribution from second user i is more if τ_i is higher.

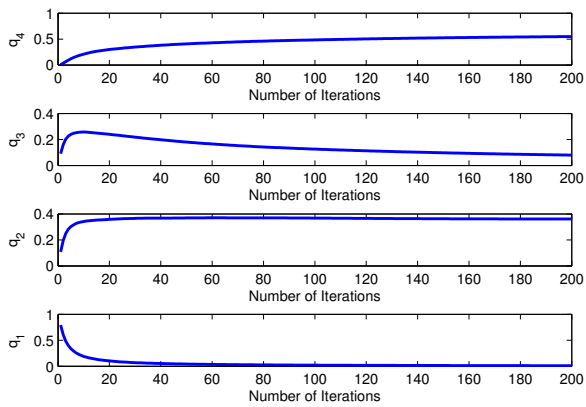

Fig. 8. Probability distribution obtained using NBL algorithm

Figure 9 compares the performance of the NR algorithm and the NBL algorithm for the case of two users with $\tau_1 = 0.0, \tau_2 = 0.2$ for different values of τ. The figure shows that the total utility obtained using the NBL algorithm is much closer to the total utility at SW CE compared to the total utility obtained using the NR algorithm. The improvement in the performance is more visible for larger values of τ.

One of the common questions for such schemes is, when each user is selfish, why would it exchange information and make more computations for social welfare if this does not benefit

itself. However, if the users act based on net the regret instead of the individual regret, the utility of each individual user also increases. This is what motivates the users to exchange information and to make extra computations. To consider more general case, we show the performance (average utility of each user) of the proposed algorithm for the case of 10 users in Figure 10 with $\tau_1, \tau_2, \tau_3 = 0.0, \tau_4, \tau_5 = 0.1, \tau_6, \tau_7, \tau_8 = 0.2, \tau_9, \tau_{10} = 0.3$. The figure shows that the performance of the NBL algorithm is close to that of the NR algorithm until about $\tau = 0.3$. The improvement brought by the proposed algorithm is significant for higher values of τ. As the average utility of each user improves in NBL algorithm, it is obvious that the total utility in NBL algorithm is higher compared to the NR algorithm.

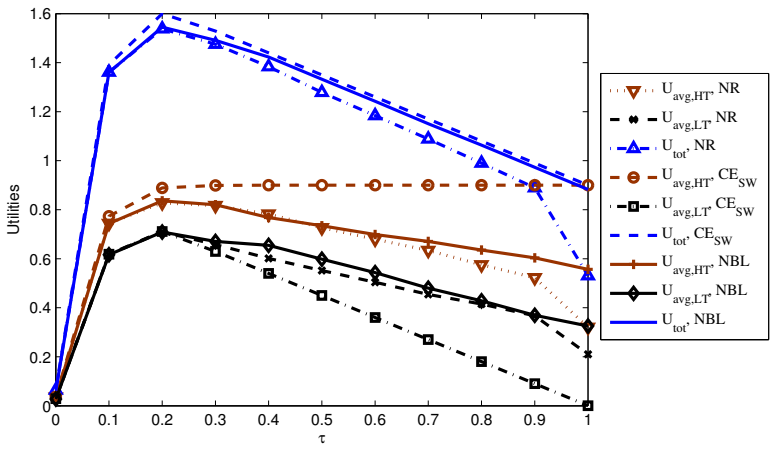

Fig. 9. Comparison of average utilities for 2-heterogeneous users game

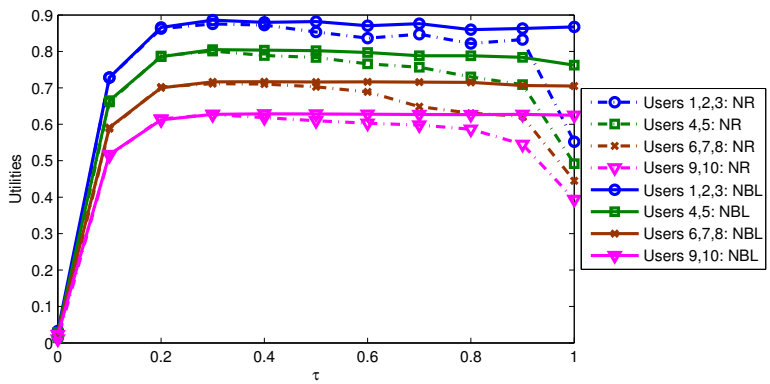

Fig. 10. Comparison of average utilities for 10-heterogeneous users game

VII. Conclusion and Future Work

We introduced the concept of correlated equilibrium for the cooperative spectrum sensing game among non-cooperative secondary users of heterogeneous traffic dynamics and illustrated that the correlated equilibrium improves the payoff of the users compared to the mixed strategy equilibrium. We proposed fair social welfare correlated equilibrium to maximize the social welfare without letting some users starve. We showed that the proposed scheme always performs better than the max-min fairness scheme in terms of total utility and ensures more fairness than the social welfare scheme. In order to implement correlated equilibrium in distributed manner, we proposed the neighbourhood based learning algorithm and the results show that this algorithm improves the performance compared to the no-regret algorithm significantly. As a future work, we would like to extend our study by focussing on the aspect of dynamic formation/splitting of the users for cooperative spectrum sensing and the infrastructure requirement for the same.

Acknowledgement

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Paper IV

Dependable Demand Response Management in the Smart Grid: A Stackelberg Game Approach

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Dependable Demand Response Management in the Smart Grid: A Stackelberg Game Approach

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Abstract

Demand Response Management (DRM) is a key component in the smart grid to effectively reduce power generation costs and user bills. However, it has been an open issue to address the DRM problem in a network of multiple utility companies and consumers where every entity is concerned about maximizing its own benefit. In this paper, we propose a Stackelberg game between utility companies and end-users to maximize the revenue of each utility company and the payoff of each user. We derive analytical results for the Stackelberg equilibrium of the game and prove that a unique solution exists. We develop a distributed algorithm which converges to the equilibrium with only local information available for both utility companies and end-users. Though DRM helps to facilitate the reliability of power supply, the smart grid can be succeptible to privacy and security issues because of communication links between the utility companies and the consumers. We study the impact of an attacker who can manipulate the price information from the utility companies. We

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also propose a scheme based on the concept of shared reserve power to improve the grid reliability and ensure its dependability.

Index Terms

Demand response management, dependability, reliability, reserve power, security, smart grid, Stackelberg game.

I. Introduction

The demand of electricity consumers has been growing due to increased use of machines and the new types of appliances such as plug-in hybrid electric vehicles. The concern towards the impact on environment and on the reliability of power supply, has also been rising. However, traditional power grids are not able to meet these demands and requirements because of their inflexible designs and lack of prompt communications between the supply and the demand sides. Recent blackouts [1] have indicated the inefficiency and serious reliability issues of the traditional grid. Therefore, it is essential to transform the traditional power grid into a more responsive, efficient and reliable system. Smart grid [2] is a future power grid system that incorporates a smart metering infrastructure capable of sensing and measuring power consumption from consumers with the integration of advanced information and communication technologies (ICT). Thus the power generation, distribution and consumption is efficient, more economical and more reliable in the smart grid network.

Demand Response Management (DRM), a key feature of the smart grid, is defined as changes in electric usage by end-users in response to changes in the price of electricity over time or across different energy sources. The importance of DRM can go far beyond reducing the electricity bills of consumers or the cost of generating power. It helps to balance the demand and supply in the power market through real-time pricing. It can also provide short-term reliability benefits as it can offer load relief to resolve system and/or local capacity constraints.

The recent studies on DRM can be categorized mainly into two areas: utility company (UC) oriented and end-user oriented. There has been considerable amount of work in power systems on supply-demand balance and market clearance [3], [4]. Such studies on power systems have focused on the economic aspects at the planning and generation level and have not considered user-utility as a significant component. On the other hand, the literature on user-utility has introduced schemes to maximize user utilities, without considering the power

generation costs or the revenue of the UCs. This has motivated us to consider the issue of benefit maximization for users alongside with the revenue maximization for the UCs. Our work aims to bridge the gap between the existing two research directions. In addition, with increasing concerns towards environment, incorporating renewable energy resources becomes important in the smart grid. This has motivated us to include in our work renewable energy sources in addition to traditional fossil fuel based sources.

We study the interactions among multiple UCs and multiple consumers, who aim to maximize their own payoffs. The UCs maximize their revenues by setting appropriate unit prices. The consumers choose power to buy from UCs based on the unit prices. The payoff of each consumer depends on the prices set by all the sources. In turn, the price set by each UC also depends on the prices of other UCs. These complicated interactions motivate us to use a game theoretical framework in our analysis. We develop a Stackelberg game between the UCs and the users where the UCs play a non-cooperative game and the consumers find their optimal response to the UCs' strategies. The interactions between the UCs and the users are enabled by the bidirectional communications between them.

An advanced metering infrastructure (AMI) is a communication infrastructure that enables meters and utilities to exchange information such as power consumption, price update, or outage awareness. Smart meters play the key role of gateway between the customers′ premises and the utility network. Their functionality make them an interesting target for attackers [5]. Therefore it is important to assess possible consequences of attacks and develop mechanisms to maintain the reliability and resilience of the grid in the face of unanticipated events. We assess the impact of an attacker that can manipulate the price of the UCs, and propose a scheme to ensure the reliability of power supply in the presence of an attacker, thus making the smart grid a dependable system.

We have three major contributions in this work.

1) We establish an analytical model for the multiple-UCs multiple-consumers Stackelberg game and characterize its unique Stackelberg equilibrium (SE).
2) We propose a distributed algorithm which converges to the SE with only local information of the users and the UCs.
3) We propose a scheme based on a common reserve to improve the dependability of the smart grid. We also discuss reliability of the grid when one of the sources gets disconnected from the grid due to occurance of some physical incidents.

The rest of the paper is organized as follows. Related work is described in Section II. We

introduce the system model and the communication model in Section III. In Section IV, we formulate the problem as a Stackelberg game and prove the existence and uniqueness of the SE. We propose a distributed algorithm for the game which converges to the SE, in Section V. In Section VI, we study the impact of an attacker as a possible threat to grid stability and propose a scheme based on maintaining a shared reserve power. We provide numerical results and discussion in Section VII. Section VIII concludes the paper.

II. Related Work

There are several studies on DRM in the smart grid [6]-[10]. In [6], the authors have formulated the energy consumption scheduling problem as a non-cooperative game among the consumers for increasing strictly convex cost functions. In [7], the authors have considered a distributed system where price is modeled by its dependence on the overall system load. Based on the price information, the users adapt their demands to maximize their own utility. In [8], a robust optimization problem has been formulated to maximize the utility of a consumer, taking into account price uncertainties at each hour. In [9], the authors have exploited the awareness of the end-users and proposed a method to aggregate and manage end-users' preferences to maximize energy efficiency and user satisfaction. In [10], a dynamic pricing scheme has been proposed to incentivize costumers to achieve an aggregate load profile suitable for utilities, and the demand response problem has been investigated for different levels of information sharing among the consumers in the smart grid. In [11], the unit commitment scheduling problem in smart grid communications has been studied using a partially observable Markov decision process framework for stochastic power demand loads and renewable energy resources. However, the analyses in [6]-[11], are limited in the sense that there is either only one source or a number of sources/utilities treated as one entity. Differently in our study, we include multiple UCs and consumers whose goal is to maximize their own payoffs, using the concept of Stackelberg game.

We note that there is rich literature using Stackelberg games in the context of congestion control, revenue maximization and cooperative transmission [14]-[15]. Our approach is similar to those in congestion control to model the behavior of end-users, but our study involves multiple UCs, and we adopt the non-cooperative game framework among UCs using the Stackelberg solution concept.

DRM enhances the reliability of the grid [16] when the data communications is perfect. However, the data communications in the smart grid may suffer attacks such as data manipula-

tion or false data injection [17] from malicious nodes. In such cases, the UC or the users may incur economic loss or physical impact e.g., grid instability. In [18], the authors have studied the utility-privacy tradeoffs of smart meter data and shed light on the impact of leakage of the data on the utilities of both the users and the suppliers. In [19], the authors have proposed a secure routing protocol incorporating delay due to queue building. They have investigated the tradeoffs between efficiency, reliability and resilience in centralized and decentralized approaches for secure routing. [20] proposes a six-layer hierarchical security architecture for the smart grid, identifying the security challenges present at each layer and addressing security issues at three different layers. In [21] the authors have developed a formal model for the C12.22 standard protocol to guarantee that no attack can violate the security policy without being detected based on the concept of specification-based intrusion detection. It is observed that there is no work that addresses the impact of attacks from an outsider on DRM through the information exchange between the users and the UCs. Because of the communications between the consumers and the UCs, there are inherent vulnerabilities that attackers can exploit to harm the utilities of either side or to even cause physical damage on the system.

III. System Model

We consider N end-users, which we also call customers, and K electricity UCs. Fig. 1 depicts an overview of the scenario. The utility side consists of the renewable and non-renewable energy sources. The fossil-fuel based energy generators have certain amount of power available all the time. The power generated by the fossil fuel generators creates pollution to the environment. On the other hand, the renewable energy sources can be seen as pollution free but they do not always have power available. When renewable energy sources are incorporated into the system, we add uncertainties to the utility side. There are many studies where discrete time Markov chain models have been used to model the availability of energy from the renewable sources (such as wind and solar energy) [11], [23], [24]. We incorporate the renewable energy sources too, and consider a stationary distribution for the states of the renewable energy generators. The end-user side consists of several consumers, which may be residential users, commercial users or industries. These different types of users have different needs for electricity. We differentiate them in terms of available budget which is an upper bound on their affordability to buy power. We employ a utility function for each user that increases with the amount of electricity the user can consume. At the same time,

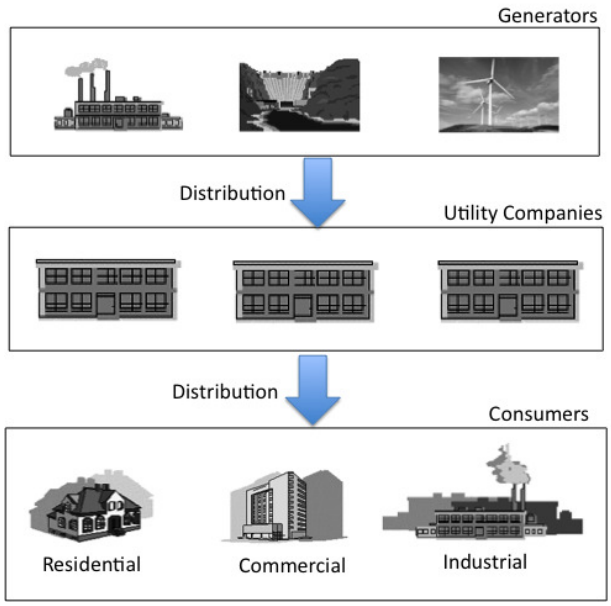


Fig. 1. Smart grid system model with multiple energy sources and end-users

we incorporate a cost constraint for each user. The UCs and the consumers have bidirectional communications for exchanging price and demand information as shown in Fig. 2. The UCs can also communicate with each other. The users receive the price information from the utility companies and transmit their demand to them. The data communication is carried out through the communication channel using wireless technologies, e.g., WiFi, WiMAX, or LTE.

In practice, the electricity generation, distribution and consumption can be decomposed into three layers as described in [22]: generators, aggregators or utility companies, and the end-users. The acquisition of power by the utility companies from the generators is a separate process. In this paper, we focus on the interactions between the UCs and the end-users. In practice, the unit price of a UC is determined through the market by the system operator. In this paper, the UCs play a non-cooperative game at the market level. Different from the traditional *perfect competitive market*, the UCs participate in an imperfect competition. In a perfectly competitive market, no market participant has the ability to influence the market price through its individual actions, i.e., the market price is a parameter over which firms have no control. Consequently, each firm should increase its production up to the point where its

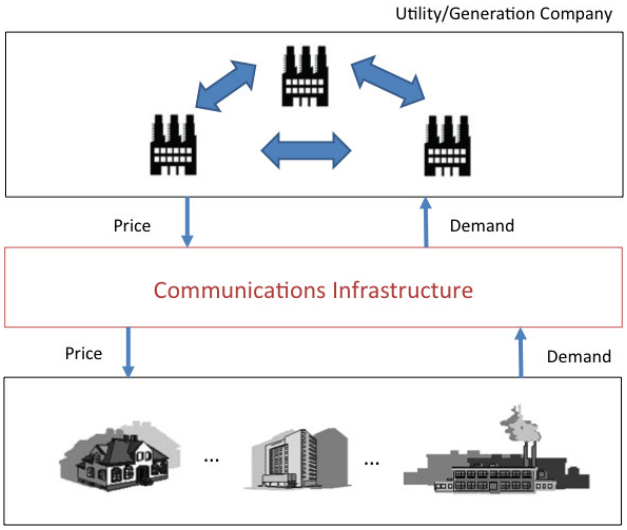


Fig. 2. Illustration of communications between utility companies and consumers

marginal cost equals the market price. This is valid when the number of market participants is large and none of the participants controls a large proportion of the production. However, in this paper, we consider a finite number of market participants (UCs) and each individual UC has non-infintesimal influence in the market. This leads to imperfect competition, where each firm determines its unit price based on its available power.

IV. Utility-User Interaction: Stackelberg Game

When there are multiple UCs with different energy prices, the cost to each user varies according to the prices set by each UC. In addition, the price set by a UC also depends on the prices of other UCs. Game theory provides a natural paradigm to model the behavior of the end-users and of the UCs in this scenario. The UCs set the price per unit power and announce it to the users. The users respond to the price by demanding an optimal amount of power from the UCs. Since the UCs act first and then the users make their decision based on the prices, the two events are sequential. Thus, we model the interactions between the UCs and the end-users as a Stackelberg game [25]. In our proposed game, the UCs are the leaders and the consumers are the followers. It is a multi-leaders and multi-followers game. The demand of the users depends on the unit price set by the UCs as well as their own

cost constraints. In turn, the UCs optimize their unit prices according to the response of the consumers.

A. User Side Analysis

Let $x_{n,k}$ be the demand of user n from UC k. We define the utility of user n, $U_{\text{user},n}$ as

$$U_{\text{user},n} = \alpha_n \sum_{k \in \mathscr{K}} \ln\left(\beta_n + x_{n,k}\right), \ \forall k \in \mathscr{K}, \tag{1}$$

where α_n and β_n are constants. The ln function has been widely used in economics for modeling the preference ordering of users and for decision making [12], [13].

The motivation behind choosing the utility function for user n as in (1) is that it is closely related to the utility function $\alpha_n \sum \ln(x_{n,k})$ that leads to proportionally fair demand response [12] [14]. If we use the utility function $\alpha_n \sum \ln(x_{n,k})$, then a user gets a payoff of $-\infty$ with respect to (w.r.t.) UC k when $x_{n,k} = 0$. With β_n, when $x_{n,k} = 0$, its benefit with w.r.t. that UC becomes finite. A typical value of β_n is 1.

Let y_k be the unit price set by UC k and let $C_n > 0$ denote the budget of user n. For a given set of prices from the UCs $\{y_1, y_2, \ldots, y_K\}$, user $n \in \mathscr{N}$ calculates its optimal demand response by solving the user optimization problem (OP_{user})

$$\max_{\mathbf{x}_n := \{x_{n,k}, \forall k \in \mathscr{K}\}} U_{\text{user},n} \tag{2}$$

$$\text{s.t.} \sum_{k \in \mathscr{K}} y_k x_{n,k} \leq C_n, \tag{3}$$

$$x_{n,k} \geq 0; \ \forall k \in \mathscr{K}. \tag{4}$$

OP_{user} is a convex optimization problem. Hence, the stationary solution is unique and optimal.

Let us start the analysis with N users and 2 UCs. We will later generalize the results to K UCs. The optimization problem for user n in this case is

$$\max_{\mathbf{x}_n := \{x_{n,1}, x_{n,2}\}} \alpha_n \sum_{k=1}^{2} ln(\beta_n + x_{n,k}) \tag{5}$$

$$\text{s.t. } y_1 x_{n,1} + y_2 x_{n,2} \leq C_n, \tag{6}$$

$$x_{n,1}, x_{n,2} \geq 0. \tag{7}$$

Using Lagrange's multipliers $\lambda_{n,1}, \lambda_{n,2}$ and $\lambda_{n,3}$ for constraints (6) and (7), we convert the constrained optimization problem (5) - (7) to the form

$$\begin{aligned} L_{\text{user},n} &= \alpha_n \sum_{k=1}^{2} ln\left(\beta_1 + x_{n,k}\right) \\ &- \lambda_{n,1}\left(\sum_{k=1}^{2} y_k x_{n,k} - C_1\right) + \lambda_{n,2} x_{n,1} + \lambda_{n,3} x_{n,2} \end{aligned} \tag{8}$$

and the complementarity slackness conditions

$$\lambda_{n,1}\left(\sum_{k=1}^{2} y_k x_{n,k} - C_n\right) = 0, \tag{9}$$

$$\lambda_{n,2} x_{n,1} = 0, \tag{10}$$

$$\lambda_{n,3} x_{n,2} = 0, \tag{11}$$

$$\lambda_{n,1} > 0, \lambda_{n,2}, \lambda_{n,3}, x_{n,1}, x_{n,2} \geq 0. \tag{12}$$

The first-order optimality condition for the maximization problem is $\nabla L_{\text{user}} = 0$, where $L_{\text{user}} = \{L_{\text{user},n}, \forall n \in \mathscr{N}\}$. Since the only coupling between the users is through y_k, $\nabla L_{user} = 0$ leads to

$$\frac{\partial L_{\text{user},n}}{\partial x_{n,k}} = 0, \forall n \in \mathscr{N}, k \in \mathscr{K}, \text{i.e., },$$

$$\begin{cases} \frac{\alpha_n}{\beta_n + x_{n,1}} - \lambda_{n,1} y_1 + \lambda_{n,2} = 0, \\ \frac{\alpha_n}{\beta_n + x_{n,2}} - \lambda_{n,1} y_2 + \lambda_{n,3} = 0. \end{cases} \tag{13}$$

The optimal demands of users can take one of the following forms.

1) Case $1 : x_{n,1}, x_{n,2} > 0$*:* In this case, $\lambda_{n,2} = \lambda_{n,3} = 0$. Substituting $\lambda_{n,2}$ and $\lambda_{n,3}$ into (13) yields

$$x_{n,k} = \frac{\alpha_n}{\lambda_{n,1} y_k} - \beta_n, \ \forall n \in \mathscr{N}, k = 1, 2. \tag{14}$$

Using (14) in (9) yields

$$\frac{1}{\lambda_{n,1}} = \frac{C_n + \beta_n \sum_{k=1}^{2} y_k}{2\alpha_n}. \tag{15}$$

Now substituting (15) into (14) yields

$$x_{n,k} = \frac{C_n + \beta_n \sum_{k=1}^{2} y_k}{2 y_k} - \beta_n, \ k = 1, 2. \tag{16}$$

2) Case $2 : x_{n,1} > 0, x_{n,2} = 0$*:* This is the case when $\frac{C_n + \beta_n \sum_{k=1}^{2} y_k}{2 y_2} = \beta_n$. Equation (10) implies $\lambda_{n,2} = 0$ and

$$x_{n,1} = \frac{\alpha_n}{\lambda_{n,1} y_1} - \beta_n. \tag{17}$$

Substituting $x_{n,1}$ into (9), we get,

$$\lambda_{n,1}\left(\frac{\alpha_n}{\lambda_{n,1}} - \beta_n y_1 - C_n\right) = 0.$$

Since $\lambda_{n,1} > 0$, $\frac{\alpha_n}{\lambda_{n,1}} - \beta_n y_1 - C_n = 0$ which gives $\lambda_{n,1} = \frac{\alpha_n}{C_n + \beta_n y_1}$. Using $\lambda_{n,1}$ in (17), we obtain

$$x_{n,1} = \frac{C_n + \beta_n y_1}{y_1} - \beta_n = \frac{C_n}{y_1}. \tag{18}$$

Equation (18) can be written as $x_{n,1} = \frac{C_n + \beta_n(y_1+y_2)}{2y_1} + \frac{C_n - \beta_n(y_1+y_2)}{2y_1}$. From $\frac{C_n + \beta_n \sum_{k=1}^{2} y_k}{2y_2} = \beta_n$ we get, $\beta_n(y_2 - y_1) = C_n$. Using this, we can write $x_{n,1} = \frac{C_n + \beta_n(y_1+y_2)}{2y_1} + \frac{\beta_n(y_2-y_1) - \beta_n(y_1+y_2)}{2y_1}$. After simplifying we get,

$$x_{n,1} = \frac{C_n + \beta_n(y_1 + y_2)}{2y_1} - \beta_n. \tag{19}$$

3) Case $3 : x_{n,1} = 0, x_{n,2} > 0$*:* Similar analysis can be performed as in case 2 to obtain

$$x_{n,2} = \frac{C_n}{y_2} = \frac{C_n + \beta_n(y_1 + y_2)}{2y_2} - \beta_n. \tag{20}$$

4) Case $4 : x_{n,1} = 0, x_{n,2} = 0$*:* In this case, $\lambda_{n,1} = 0$ and $\lambda_{n,2}, \lambda_{n,3}$ can be any non-negative real value. This is an extreme case, which does not happen unless $C_n = 0$ or $y_k = \infty \ \forall k \in \mathscr{K}$. Note that in cases 1-3 discussed above, both power constraint and the cost constraint are satisfied as equalities.

Thus using (16), (19) and (20), the demands for the general case of N users and K UCs that covers cases $1-3$ for a given set of $\{y_k\}$, can be formulated as

$$x_{n,k} = \frac{C_n + \beta_n \sum_{k \in \mathscr{K}} y_k}{K y_k} - \beta_n, \tag{21}$$

where $x_{n,k} \geq 0, \forall k \in \mathscr{K}, n \in \mathscr{N}$. Since $x_{n,k} \geq 0 \ \forall n \in \mathscr{N}, \forall k \in \mathscr{K}$, (21) implies that

$$C_n + \beta_n \left(\sum_{g \in \mathscr{K}, g \neq k} y_g \right) \geq \beta_n (K-1) y_k, \ \forall n \in \mathscr{N}, \forall k \in \mathscr{K}. \tag{22}$$

Conversely, user n will demand $x_{n,k} \geq 0$ from UC k if

$$y_k \leq \left[\frac{C_n + \beta_n \left(\sum_{g \in \mathscr{K}, g \neq k} y_g \right)}{\beta_n (K-1)} \right]. \tag{23}$$

We will derive a closed form for the necessary condition for $x_{n,k} \geq 0, \forall k \in \mathscr{K}, \forall n \in \mathscr{N}$ to be satisfied, in Section IV-B2.

B. Utility Side Analysis

Let $P_k > 0$ denote the available power of UC k. Each UC aims to sell all the available power. If it had been a single UC case, it could have a set a very high unit price to maximize its revenue. In this case however, there are two factors that limit the unit price of the UCs. The first one is the budget of the users and the second one is the competition among the UCs. The UCs play a non-cooperative *price selection game* with each other to decide the

optimal unit price. We assume that P_k is given for all $k \in \mathscr{K}$. For given P_k, since the cost of power generation is given, we define the revenue of UC k, $U_{\text{gen},k}$ as

$$U_{\text{gen},k}(y_k, \mathbf{y_{-k}}) = y_k \sum_{n \in \mathscr{N}} x_{n,k}, \tag{24}$$

where $\mathbf{y_{-k}}$ is the price of UCs other than k. Then, the optimization problem for a UC (OP_{gen}) is formulated as

$$\max_{\mathbf{y}:=\{y_k, \forall k \in \mathscr{K}\}} U_{\text{gen},k}(y_k, \mathbf{y_{-k}}) \tag{25}$$

$$\text{s.t.} \sum_{n \in \mathscr{N}} x_{n,k} \leq P_k, \tag{26}$$

$$y_k > 0, k \in \mathscr{K}. \tag{27}$$

Since the revenue of a UC is an increasing function in terms of the amount of power for a fixed y_k, (26) can be taken as an equality constraint. Since we do not assume the availability of storage with the UCs, when the available power is given, each UC prefers to sell all its power. In order to solve OP_{gen}, we start by relaxing the positivity constraint (27) but will show that the solution of (25)-(26) will lead to positive vector $\{y_k, \forall k \in \mathscr{K}\}$. Let us define $L_{\text{gen},k}$ as

$$L_{\text{gen},k} = y_k \sum_{n \in \mathscr{N}} x_{n,k} - \mu_k \left(\sum_{n \in \mathscr{N}} x_{n,k} - P_k \right) \tag{28}$$

The first order optimality condition for the UCs leads to $\frac{\partial L_{\text{gen},k}}{\partial y_k} = 0, \forall k \in \mathscr{K}$. Using (21) in $\frac{\partial L_{\text{gen},k}}{\partial y_k} = 0$ for UC k, we obtain

$$(K-1)By_k^2 - \mu_k \left[B \left(\sum_{g \in \mathscr{K}, g \neq k} y_g \right) + C \right] = 0, \tag{29}$$

where $B = \sum_{n \in \mathscr{N}} \beta_n$ and $C = \sum_{n \in \mathscr{N}} C_n$. Eqn. (29) gives K equations. Further, $\frac{\partial L_{\text{gen,k}}}{\partial \mu_k} = 0$ gives K equations, which are actually the original constraints: (26). Solving these $2K$ equations we can obtain $\mathbf{y}^* := \{y_1^*, y_2^*, \ldots, y_K^*\}$ and $\mu^* := \{\mu_1^*, \mu_2^*, \ldots, \mu_K^*\}$. Using $\mathbf{y}^*$, we can compute $\mathbf{x}^* := \{x_{n,k}^*\}$. Now using (21) in the equality form of (26), we get

$$y_k = \frac{C + B \left(\sum_{g \in \mathscr{K}, g \neq k} y_g \right)}{KP_k + B(K-1)} \tag{30}$$

Substituting y_k into (29), we arrive at

$$\mu_k = (K-1)B \left[\frac{C + B\left(\sum_{g \in \mathscr{K}, g \neq k} y_g\right)}{K(P_k + B)} \right] = (K-1)By_k \tag{31}$$

Here, $B, C > 0$. when $K = 1$, $\mu_k = 0$ and $y_1 = \frac{C}{P_k + B}$. This means that there is no game when there is only one UC. Our interest is in the case where $K \geq 2$. Then, $\mu_k > 0$ if $y_k > 0$. Equation (30) can be represented in the matrix form as

$$\mathbf{A}\mathbf{y} = \mathbf{F}, \tag{32}$$

where

$$\mathbf{A} := \begin{bmatrix} P_1 + D & -E & .... & -E \\ -E & P_2 + D & .... & -E \\ .. & .. & .... & .. \\ -E & -E & .... & P_K + D \end{bmatrix}, \tag{33}$$

$\mathbf{y} := \{y_1, y_2, ..., y_K\}'$, $D := \frac{B(K-1)}{K}, E := \frac{B}{K}$ and $F := \frac{C}{K}$. Thus, provided that $\mathbf{A}$ is invertible, the solution of (32) is

$$\mathbf{y} = \mathbf{A}^{-1}\mathbf{F}. \tag{34}$$

In order to obtain the closed-form of $\mathbf{y}$, let us consider the following cases:

1) Homogeneous case: When $P_1 = P_2 = ... = P_K = P$: If all UCs have the same amount of power available, then, (32) can be solved to obtain

$$y_k = y := \frac{F}{-(K-1)E + D + P} = \frac{C}{KP} > 0 \ \forall k \in \mathscr{K}. \tag{35}$$

As P increases, y decreases and vice versa. Now using (35) in (22), the condition for the demand of user n from all UCs to be positive is

$$C_n > \beta_n (K-1) y - \beta_n (K-1) y, \tag{36}$$

i.e., $C_n > 0$, which is always true. This indicates that when all UCs are homogeneous in terms of the available power, they set the same unit price and all users will buy at least some power from them.

2) Heterogeneous case: When different UCs have different available power: In this case, it is difficult to judge the existence of the solution without knowing the nature of $\mathbf{A}^{-1}$ (if it exists). Interestingly, matrix $\mathbf{A}$ possesses some special properties. Let us state the following definitions and properties.

Definition 1. *A real matrix $\mathbf{A} := \{a_{i,j},\ i, j = 1, 2, ..., K\} \in \mathscr{R}^{K \times K}$, is said to be strictly diagonally dominant if it satisfies the following condition:*

$$|a_{i,i}| - \sum_{j \neq i} |a_{i,j}| > 0 \quad i = 1, 2, \ldots, K. \tag{37}$$

Property 1. *A strictly diagonally dominant matrix is non-singular.*

Property 2. *If a matrix is strictly diagonally dominant by rows and has positive diagonal entries, then, its determinant will be positive [26].*

For matrix $\mathbf{A}$, since, $P_k + D - (K-1)E = P_k + \frac{(K-1)B}{K} - \frac{(K-1)B}{K} = P_k > 0$, $\mathbf{A}$ is a strictly diagonally dominant matrix. From Property 1, $\mathbf{A}$ is invertible and (32) yields a unique solution to $\mathbf{y}$.

Theorem 1. *The unique solution obtained from (33) is positive.*

Proof: The solution of (33) is given by

$$y_k = \frac{K^{K-1}C}{|\mathbf{A}|} \sum_{g \in \mathscr{K}, g \neq k} (B + P_g) \quad \forall k \in \mathscr{K}. \tag{38}$$

where $|\mathbf{A}|$ is the determinant of $\mathbf{A}$. The numerator of (38) is always positive. Property 2 implies that the denominator is positive, and hence the solution from (38) always yields $y_k > 0, \ \forall k \in \mathscr{K}$. ■

Theorem 2. *For the case of heterogeneous generators the necessary condition for the demands of all users from all UCs to be non-negative, is:*

$$C_n \geq \frac{K^{K-1}\beta_n C}{|\mathbf{A}|} \left[\left(\sum_{g \in \mathscr{K}, g \neq k} P_g \right) - (K-1)P_k \right] \forall k, \forall n. \tag{39}$$

Proof: The demand of user n from UC k is non-negative ($x_{n,k} \geq 0$) if $C_n + \beta_n \sum_{k \in \mathscr{K}} y_k \geq K\beta_n y_k, \forall n \in \mathscr{N}, \forall k \in \mathscr{K}$, i.e., if $C_n \geq \beta_n (K y_k - \sum_{k \in \mathscr{K}} y_k)$. Using (38), the condition can be written as $C_n \geq \beta_n \frac{K^{K-1}C}{|\mathbf{A}|} \left(K \sum_{g \in \mathscr{K}, g \neq k} (B + P_g) - \sum_{k \in \mathscr{K}} \sum_{g \in \mathscr{K}, g \neq k} (B + P_g) \right)$ After simplification, the required condition takes the form given by (39).

■

Theorem 3. *The values of price obtained from (34) maximize the revenue and are best responses of the UCs to other UCs' strategies.*

Proof: Suppose y_k is the solution obtained from (34) for UC k and it increases its price from y_k to $y_k^{'} = y_k + \delta_{y_k}$ while the prices of other UCs remains the same. Let us assume that $y_k, y_k^{'}$ satisfy (23). The demand of users from this UC will change from $x_{n,k}$ to $x_{n,k}^{'}$ given by

$$x_{n,k}^{'} = \left[\frac{C_n + \beta_n \left(\sum_{g \in \mathscr{K}, g \neq k} y_g + y_k^{'} \right)}{K y_k^{'}} - \beta_n \right].$$

The difference in the total demands from the users from UC k will be

$$x_{n,k} - x_{n,k}^{'} = \left[\frac{C_n + \beta_n \left(\sum_{g \in \mathscr{K}, g \neq k} y_g \right)}{K} \right] \frac{(y_k^{'} - y_k)}{y_k y_k^{'}}.$$

Clearly, $(x_{n,k} - x_{n,k}^{'} > 0)$. The users will demand less power than the power available with the UC. The difference in the revenue of the UC because of the increase in the price will be

$$U_{\text{gen},k}^{'} - U_{\text{gen},k} = y_k^{'} \sum_{n \in \mathscr{N}} x_{n,k}^{'} - y_k \sum_{n \in \mathscr{N}} x_{n,k} = -\tfrac{(K-1)}{K} B \delta_{y_k}. \tag{40}$$

From (40), it is clear that $U_{\text{gen},k}(y_k^{'}, \mathbf{y_{-k}}) < U_{\text{gen},k}(y_k, \mathbf{y_{-k}})$ if $\delta_{y_k} > 0$. If $\delta_{y_k} < 0$, $y_k^{'} < y_k$, but P_k is given. Consequently, the revenue will decrease. Therefore, the prices calculated using (34) are the best responses to each other and are the prices that maximize the revenue of each UC. ■

In addition to positivity, there are tighter limits on y_k such that $y_k \in [y_{k,\min}, y_{k,\max}]$. The lower limit $y_{k,\min}$ is due to the associated generation costs. The UCs will not reduce their price below $y_{k,\min}$. The upper bound $y_{k,\max}$ can be fixed according to government standards. The power available with the UCs implicitly take into account these price limits. Thus, the solution obtained by solving (34) should be within the specified limits. In case if the optimal prices obtained are outside the range, we propose the following algorithm to calculate the unit prices.

Algorithm 1:

1) If $\{y_k\}_{k \in \mathscr{K}}$ be the solution obtained from (38) but $y_i < y_{i,\min}$ or $y_i > y_{i,\max}$ for UC i, then its price will be set as $y_i = y_{i,\min}$ or $y_i = y_{i,\max}$.
2) The remaining UCs will use y_i as a known value in (30) to obtain a square matrix of size $K - K_i$, where K_i is the number of UCs for which the prices obtained are modified. Then, the matrix of the reduced dimension is solved to get the prices of the remaining $K - K_i$ UCs.
3) The process continues until all the prices come within the specified range.

C. Stackelberg Equilibirum

The UCs play the non-cooperative game with each other to set the unit price, which is at Nash equilibrium (NE) point, and announce the prices to the users. The equilibrium strategy for the followers in a Stackelberg game is any strategy that constitutes an optimal response to the one adopted (and announced) by the leader(s) [25].

Let $\Gamma_{\text{gen},k}$ and $\Gamma_{\text{user},n}$ be the strategy sets for UC k and user n respectively. Then, the strategy sets of all UCs and all users are $\Gamma_{\text{gen}} = \Gamma_{\text{gen},1} \times \Gamma_{\text{gen},2} \times ..... \times \Gamma_{\text{gen},K}$ and $\Gamma_{\text{user}} =$

$\Gamma_{\text{user},1} \times \Gamma_{\text{user},2} \times ..... \times \Gamma_{\text{user},N}$, respectively. Then, $y_k^* \in \Gamma_{\text{gen},k}$ is a Stackelberg equilibrium strategy for UC k if

$$U_{\text{gen},k}(\mathbf{y}^*, \mathbf{x}(\mathbf{y}^*)) \geq U_{\text{gen},k}(\mathbf{y}_k, \mathbf{y}_{-k}^*, \mathbf{x}(\mathbf{y}_k; \mathbf{y}_{-k}^*)), \forall k \in \mathscr{K}, \tag{41}$$

where $\mathbf{y}^* = \{y_k^*\}$, $\mathbf{x} := \{\mathbf{x}_1; \mathbf{x}_2; \ldots \mathbf{x}_N\}$ is the strategy of all users $1, 2, \ldots, N$ such that $\mathbf{x} \in \Gamma_{\text{user}}$, $\mathbf{x}(\mathbf{y}^*)$ is the optimal response of all users. The optimal response of user n for given $(y_1, y_2, ..., y_K) \in \Gamma_{\text{gen},1} \times \Gamma_{\text{gen},2} \times ..... \times \Gamma_{\text{gen},K}$ is

$$\begin{aligned} \mathbf{x}_n(\mathbf{y}) = \{ & \zeta_{\text{user},n} \in \Gamma_{\text{user},n}; U_{\text{user},n}(\mathbf{y}, \zeta_{\text{user},n}) \geq \\ & U_{\text{user},n}(\mathbf{y}, \mathbf{x}_n)\}, \forall n \in \mathscr{N}. \end{aligned} \tag{42}$$

where $\zeta_{\text{user},n} := \mathbf{x}_n^* \in \mathbf{x}_n(\mathbf{y}^*)$. The strategy $\mathbf{x}_n^*$ is a corresponding optimal strategy for user n, which is computed by using (21). The set $(\mathbf{y}^*, \mathbf{x}^*)$ is a Stackelberg equilibrium of the game between the UCs and the users.

D. Existence and Uniqueness of Stackelberg Equilibrium

OP_{user} has a unique maximum for a given $\mathbf{y}$. Therefore, the Stackelberg game possesses a unique SE if the price setting game among the UCs admits a unique NE.

Theorem 4. *A unique Nash equilibrium exists in the price selection game among the UCs, and thereby a unique Stackelberg equilibrium.*

Proof: A Nash equilibrium exists for the UCs in the price selection game if

1) $\mathbf{y}$ is a non-empty, convex, and compact subset of some Euclidean space $\mathscr{R}^K$.
2) $U_{\text{gen},k}(\mathbf{y})$ is continuous in $\mathbf{y}$ and concave in $y_k, \forall k \in \mathscr{K}$.

In the price selection game for the UCs, the strategy space $\Gamma_{\text{gen}} = \Gamma_{\text{gen},1} \times \Gamma_{\text{gen},2} \times ..... \times \Gamma_{\text{gen},K}$ where $y_k \in \Gamma_{\text{gen},k} := [y_{k,\min}, y_{k,\max}]$, $\forall k \in \mathscr{K}$. Thus the strategy set is a nonempty, convex and compact subset of the Euclidean space $\mathscr{R}^K$. From (24), we see that $U_{\text{gen},k}$ is continuous in y_k. Next, the second order derivative of $U_{\text{gen},k}$ w.r.t. y_k is

$$\frac{\partial^2 U_{\text{gen},k}}{\partial y_k^2} = 0, \ \forall k \in \mathscr{K}. \tag{43}$$

Hence, $U_k(\mathbf{y})$ is concave in y_k. Therefore, NE exists in this game.

As proven in Section IV-B, there exists only one positive solution for the price selection game given by (38). Therefore the NE of the UCs' game is unique and hence the Stackelberg game also admits a unique equilibrium. ■

V. Distributed Algorithm

In the previous section, the consumers calculate their optimal demands based on the prices provided by the UCs but the UCs play the best response to other UCs' strategies. In order to calculate the unit prices, UCs need to know the power available with other UCs too. We design a distributed algorithm that leads to the SE of the game without each UC knowing the parameters of the other UCs.

Each UC starts with an arbitrary price $y_{k,1} > 0$ and all of them send their price information to the consumers. This communication is enabled by the smart grid communication infrastructure between the UCs and the consumers. Each user decides how much to buy from each UC $\{x_{n,k,1}, \forall k \in \mathscr{K}\}$ using (21). The UCs get this demand matrix from all users. Then, one of the UCs will calculate the difference between the available power and the total power demanded from it by all users. Then, it will update its unit price using

$$y_{k,t+1} = y_{k,t} + \frac{\sum_{n \in \mathscr{N}} x_{n,k,t} - P_k}{\sigma_k}, \tag{44}$$

where t is the iteration number and σ_k is the speed adjustment parameter of UC k, which is a sufficiently large number. Whenever a UC updates its price, it sends this information to the users. The users again, update their demands vectors and inform the UCs. Then, other UCs will update their prices sequentially at alternative turns between users and UCs. This process continues until the price values converge.

We name this algorithm as **Algorithm** 2, which is shown in Table I. In the table, $n = 1$ indicates user number 1.

Theorem 5. *Provided that* $\forall k \in \mathscr{K}, \forall n \in \mathscr{N}, t = 1, 2, 3, \ldots,$

$$\sigma_k > \frac{(KP_k - \beta_n) y_{k,t} - \beta_n \sum_{g \in \mathscr{K}, g \neq k} y_{g,t} - C_n}{K y_{k,t}^2}, \tag{45}$$

Algorithm 2 *converges to the optimal solutions for both the users and the UCs as long as the individual strategies are updated sequentially.*

Proof: The users' response (21) is the optimal response to given $\{y_k\}$. The demand array of each user will converge to a fixed set once the price set converges to a fixed point. Consequently, it is sufficient to show the convergence of the price vector to prove the convergence of Algorithm 2.

The algorithm will diverge only if $y_{k,t}$ acquires a negative value in any iteration. If $\sum_{n \in \mathscr{N}} x_{n,k,t} - P_k < 0$, for any $k \in \mathscr{K}, \forall n \in \mathscr{N}, t = 1, 2, 3, \ldots,$ then the sufficient condition for

TABLE I

DISTRIBUTED ALGORITHM FOR PRICE AND DEMAND UPDATING

1: For t=1, arbitrarily choose $y_{k,1}$ $\forall k \in \mathscr{K}$
2: Repeat for t=2,3,.....
3: User **n=1,2,....N**
4: **Do**
5: Solve (2) - (4) for given $\mathbf{y}_t$ using (21).
6: Transmit $x_{n,k,t}$ to each k.
7: **end**
8: UC $k \in \mathscr{K}$, which has not updated the price for iteration $t+1$
9: Calculate $y_{k,t+1}$ using (44).
10: **If** $y_{k,t+1} - y_{k,i} = 0$
11: Send a no-change signal to all users.
12: Go to 8.
13: **else**
14: Send the new value of price to all users.
15: Go to 3.
16: **end**
17: **If** $y_{k,t+1} == y_{k,t} \forall k \in \mathscr{K}$,
18: stop.
19: **else**
20: Go to 2.
21: **end**.

$y_{k,t+1}$ not to acquire a negative value is $|\frac{\sum_{n\in\mathscr{N}} x_{n,k,t} - P_k}{\sigma_k}| < y_{k,t}$. Since the condition should be satisfied only when $\sum_{n\in\mathscr{N}} x_{n,k,t} - P_k < 0$, we can rewrite the condition as $\sigma_k > \frac{P_k - \sum_{n\in\mathscr{N}} x_{n,k,t}}{y_{k,t}}$ $\forall k \in \mathscr{K}, \forall n \in \mathscr{N}, t = 1,2,3,\ldots$. Using (21) and upon simplification, the sufficient condition takes the form (45).

Equation (44) implies that the price y_k of UC k increases if $\sum_{n\in\mathscr{N}} x_{n,k,t} - P_k$ is positive and the price decreases if $\sum_{n\in\mathscr{N}} x_{n,k,t} - P_k$ is negative. Equation (44) shows that when $\sum_{n\in\mathscr{N}} x_{n,k,t} - P_k = 0$, the price will remain unchanged. This is the fixed point to which Algorithm 2 converges. As discussed in section IV-B, the prices corresponding to stock clearance are the prices that maximize the UC revenue. Therefore, if $\{y_{k,T}\}$ are the prices at iteration T such that $\sum_{n\in\mathscr{N}} x_{n,k,T} - P_k = 0$, then, $U_{\text{gen},k}(y_{k,T}, \mathbf{y}_{-k,T}) \geq U_{\text{gen},k}(y'_{k,T}, \mathbf{y}_{-k,T})$ where $U_{\text{gen},k}(y_{k,T}, \mathbf{y}_{-k,T})$ and $U_{\text{gen},k}(y'_{k,T}, \mathbf{y}_{-k,T})$ are the utilities of UC k at price $y_{k,T}$ and $y'_{k,T}$ respectively. Thus the fixed point to which Algorithm 2 converges is the NE of the game among the UCs, i.e., SE of the game between the UCs and the consumers. Thereafter, the

UCs will not deviate from this point. ■

VI. Dependability of DRM

One important component of dependability of DRM is the vulnerability due to attacks from malicious agents. While the bidirectional communications between the UCs and the users facilitates demand response management in a more timely and effective manner, it also creates room for different kinds of threats to the system from attackers.

Next, we proceed to analyze this kind of attack. Let us assume that there is an attacker who is an outsider. The goal of the attacker is to harm the UCs to the largest extent possible. We constrain the attacker such that it can attack only one of the UCs at a time by injecting the data or by manipulating the price. This assumption is reasonable because the attacker has physical constraints to access the UCs.

A. Attack and Its Impact

An attacker can cause two kinds of harm to the UCs and the users: economic impact and physical impact. We consider the case when all UCs have the same power to supply (P). We assume that there is a range in which the price can vary. The users have the knowledge of this range. Users get this range during the service agreement. If the attacker manipulates the prices in such a way that it is out of the range, the manipulation will become obvious. So, the attacker will launch the attack such that the resulting prices are within the specified range. When the attacker does not manipulate any of the prices, the price set by each UC is

$$y_k = y = \frac{C}{KP}, \ \forall k \in \mathscr{K}. \tag{46}$$

Using (46) in (21), the demand of user n from UC k is

$$x_{n,k} = \frac{C_n}{C} P.$$

The total demand of user n from all UCs is

$$\sum_{k \in \mathscr{K}} x_{n,k} = \frac{C_n}{C} KP.$$

Let the price of UC k, y_k, be changed by the attacker through data injection or indirect manipulation to $y_k^{'} = y_k + \delta_y$ and all other prices are in-tact, where δ_y is a real number such that $|\delta_y| \in [0, \delta_{y,\max}]$. Positive value of δ_y means that the price is increased and negative value means the price is decreased, $\delta_y = 0$ corresponds to the case when the attacker does not act. When the price is changed by the attacker, the demand of users will change from $x_{n,k}$ to

$x'_{n,k}$. The attacker can change y_k but will still keep it such that $x'_{n,k}$ is non-negative. It can be calculated by replacing y_k in (21) by $y_k + \delta_y$, which gives

$$x'_{n,k} = \frac{C_n P - \delta_y \beta_n (K-1)P}{(C + \delta_y KP)}. \tag{47}$$

Equation (47) shows that the demand of each user from UC k will decrease if δ_y is positive and it will increase if δ_y is negative. The total demand from UC k from all users will be

$$\sum_{n \in \mathcal{N}} x'_{n,k} = \frac{CP - \delta_y B(K-1)P}{(C + \delta_y KP)}. \tag{48}$$

Now, let us see if the change in the price of UC k affects other UCs. For UC $g \neq k$, the demand from user n will be

$$x'_{n,g} = \frac{C_n}{C} P + \delta_y \beta_n \frac{P}{C}. \tag{49}$$

Using (49), the total demand from UC $g \neq k$ takes the form

$$\sum_{n \in \mathcal{N}} x'_{n,g} = P\left(1 + \delta_y \frac{B}{C}\right). \tag{50}$$

Equation (48) indicates that the total demand from UC k will decrease (increase) if y_k is increased (decreased) by the attacker compared to the case when there is no attack. If the total demand decreases from UC k, a part of the available power will be wasted and it will loose in terms of its revenue. On the other hand, if the demand increases than the available power, serious problems such as black-out may occur to the users side. If the grid attempts to produce the shortage power immediately, the grid might suffer physical damage. There is an interesting observation here. Equations (49), (50) indicate that if the price of UC k is changed, the other UCs will also be affected. If y_k increases, the demand from the other UCs increases. Consequently, because of the excessive demand from the UCs other than k, problems such as grid instability may arise at the generation side while the users will suffer black-out. On the other hand, when y_k decreases, the demand from the other UCs decreases. So, they will suffer economic loss while the generators that supply the demand of the users from UC k may suffer physical damage. In either case ($\delta_y > 0$ or $\delta_y < 0$), the UCs and the grid will suffer both monetary loss and physical damage.

B. Proposed Scheme with Individual Reserve Power

In a scenario where the UCs are aware that an attacker might attempt to create monetary or physical damage, although both kinds of loss are serious, the physical damage can make the grid unstable and it can affect the whole infrastructure on the UC side. Hence, we mainly

focus on avoiding this problem, for maintaining the grid dependable. We propose that each UC should have certain reserve power in addition to the available power to sell. Let us denote the reserve power for UC k by $P_{k,res}$. The reserve power for each UC should be the difference of the available power and the total possible demand from all the users in presence of the attack. Using (48), for UC k, whose price has been changed, the reserve power is

$$P_{k,res} = \frac{-\delta_y P\,(B(K-1)+KP)}{(C+\delta_y KP)}. \tag{51}$$

If $\delta_y > 0$, then $P_{k,res}$ will be negative, since the demand from UC k will be less than the amount of power available. This means that no reserve power is needed. Hence, we have

$$P^{+}_{k,res} = 0. \tag{52}$$

For other UCs $g \neq k$, the reserve power can be calculated as

$$P^{+}_{g,res} = \frac{BP\delta_y}{C}. \tag{53}$$

Equations (52)-(53) indicate that the amount of reserve power for each UC depends on the value of δ_y. Theoretically, each UC can keep a large reserve so that there is always extra power available even if the users demand more than the power desired to be sold. In practice, this is not possible since the UCs have to pay the cost associated to buy or produce the reserve power. Therefore, all UCs prefer to keep as minimum reserve power to save the associated costs, but at the same time, it is extremely important for them to avoid the damage to their infrastructure.

In contrast to communication problems where usually the average performance guarantee measures are taken, we need to consider the worst case for the power-trade scenario. The attacker aims to maximize the harm to the UCs and/or to the grid but without making the demands become out-of-the-range values, e.g., negative demand from the users. The attacker therefore, tries to manipulate y_k in such a way that the demands from all UCs is non-negative even from the user with the lowest budget. Let us examine the following cases.

1) $\delta_y > 0$: If δ_y is positive, $P^{+}_{k,res} = 0$. The maximum harm that the attacker can create in this case would be by letting the user with the lowest C_n buy zero power from UC k, which corresponds to $x^{'}_{n,k} = 0$.

$$\delta_{y_{\max}} = \frac{C_{\min}}{\beta_{\min}(K-1)}. \tag{54}$$

where $C_{\min} = \min(C_n)$ and $\beta_{\min} = \min(\beta_n)$. If $y_{\max}$ is the maximum unit price allowed, the range of δ_y given by $\delta_{y_{\max}}$ should still yield $y_k^{'} \leq y_{\max}$. Thus, $y_k^{'}$ is given by

$$y_k^{'} = \begin{cases} y_k + \delta_y & \text{if } \delta_y \leq y_{\max} - y_k, \\ y_{\max} & \text{otherwise.} \end{cases} \tag{55}$$

Therefore, $\delta_{y_{\max}}$ can be calculated as

$$\delta_{y_{\max}} = \min\left\{ \frac{C_{\min}}{\beta_{\min}(K-1)}, (y_{\max} - y_k) \right\}. \tag{56}$$

Hence, $0 < \delta_y \leq \delta_{y_{\max}}$ and $P_{g,res}^{+}$ can be calculated by substituting $\delta_y = \delta_{y_{\max}}$ into (53).

2) $\delta_y < 0$: When $\delta_y < 0$, the worst impact that the attacker can cause is by setting the prices so low that the user with the lowest budget buys zero power from all UCs other than k. In this case, solving $x_{n,g}^{'} = 0 \ \forall g \neq k$ yields

$$\delta_{y_{\max}} = \frac{C_{\min}}{\beta_{\min}},$$

where n corresponds to the lowest budget user. However, $y_k^{'}$ should not be less than $y_{k,\min}$. If $y_k^{'} = y_k - \frac{C_{\min}}{\beta_{\min}} > y_{k,\min}$, then, substituting δ_y in (48) and (50), and substracting the available power of the UC P, the reserve power for UCs (using $\beta_1 = \beta_2 = ... = \beta_N = \beta$) are

$$\begin{cases} P_{k,res}^{-} = \frac{NC_{\text{n,min}}(K-1)\beta + C_{\text{n,min}}KP^2}{\beta C - C_{\text{n,min}}KP}, \\ P_{g,res}^{-} = 0. \end{cases}$$

If $y_k - \frac{C_{\min}}{\beta_{\min}} \leq 0$ or if $y_k - \frac{C_{\min}}{\beta_{\min}} > 0$ but $y_k^{'} < y_{\min}$, then, the minimum $y_k^{'}$ that can be chosen by the attacker is $y_k^{'} = y_{\min}$. Using this constraint on (47), then summing over $n \in \mathcal{N}$ and subtracting the available power P, we obtain

$$P_{k,res}^{-} = \frac{C + B(K-1)y_d - KPy_{\min}}{Ky_{\min}}.$$

where $y_d = \frac{C}{KP} - y_{\min}$. The UCs should avoid problems even in the worst case, $P_{k,res} = P_{k,res}^{-}$ and $P_{g,res} = P_{g,res}^{+}$. However, the UCs may not know whether the attacker will choose $\delta_y > 0$ or $\delta_y < 0$. So, for each UC, the reserve power is

$$P_{res} = \max\{P_{g,res}^{+}, P_{k,res}^{-}\}. \tag{57}$$

C. Proposed Scheme with Common Reserve Power

The reserve power given by (57) is the worst case reserve power if the UC is attacked and if its price is lowered to the least possible value. Since, each UC needs to have this reserve separately, while the actual number of UCs that will be attacked is limited to 1, there is

definitely a huge wastage of the reserve power from all other UCs who are not attacked. In particular, the UCs have to buy the reserve power from some sources and it is obvious that each UC aims to minimize the reserve power while ensuring that there is no power-outage. The total reserve power needed for the individual reserve scheme is

$$P_{tot,res} = KP_{\text{res}}. \tag{58}$$

From the analysis in section VI-B, we can see that the total demand from all the users from all the UCs will be

$$X_{tot} \leq KP + P_{\text{res}}. \tag{59}$$

From (58) - (59), the total power that will not be used is

$$P_{tot,unused} \geq (K-1)P_{\text{res}}. \tag{60}$$

The incentive for the UCs to use a common reserve is that sharing common reserve power saves a considerable amount of cost for each UC. Furthermore, it is also beneficial from the overall system perspective, as there are increasing concerns towards minimizing the waste of power.

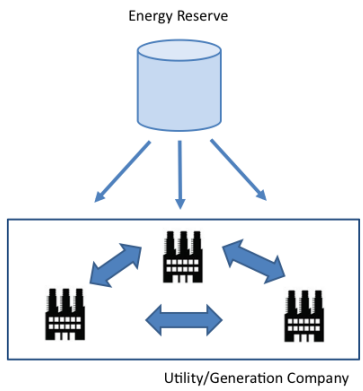


Fig. 3. Common reserve scheme: different utility/generation companies share a common energy reserve

Following the reasoning in Section VI-B, we can show that the total reserve power needed in this case will be

$$P_{tot,res} = P_{\text{res}}. \tag{61}$$

Employing this power sharing, we can see that the UCs can maintain a reliable supply of power to the users in the presence of the attacker with only a fraction of the reserve power

needed for the individual reserve scheme. Thus, the supply side improves its dependability with this reserve power scheme.

D. Discussion

The impact on DRM when one (few) of the generator(s) is (are) unavailable is also important. In our proposed model, the UCs communicate with each other about their available power before the prices are decided. If one of the generators is not available, the prices will be calculated based on the power available with the UCs from the remaining generators. Our model is therefore resilient to the unavailability of one or few generators.

There is another possible type of reliability issue with the generators, even in the absence of an attacker. Even when all the generators have produced the power, one of the generators can be disconnected from the grid after the UCs have transmitted the price information to the users. This kind of problem arises due to physical phenomena e.g., if one of the transmission lines suddenly gets disconnected. In this case, the users would already be using their appliances and thus this kind of failure is critical. The reliability can be improved in this case also by maintaining certain reserve power. This kind of reliability is referred to as $(K-1)$ reliability. On similar grounds as in Section VI, it can be inferred that a common reserve power equal to the available power of one UC (the maximum power in case of heterogeneous UCs) should maintain the system stable in the face of such incidents.

VII. Numerical Results

In this section, we examine how the users choose their optimal power based on the unit prices of the UCs and how the UCs optimize their unit prices based on their available power and the users' cost constraints. We also show the convergence of the distributed algorithm. The last part of this section shows the reserve power needed in the presence of an attacker. We consider 3 UCs and 5 users with parameters $\alpha_n = 1, \beta_n = 1 \ \forall n \in \mathcal{N}$. The cost limits of users are $C_1 = 5, C_2 = 10, C_3 = 15, C_4 = 20, C_5 = 25$ and the available power of the UCs are $P_1 = 10, P_2 = 15, P_3 = 20$, respectively, unless mentioned otherwise.

A. Stackelberg Game

Figs. 4 - 7 show how the change in the prices set by the UCs affects the users' utility, and in turn how the power available and the affordability of the users affect the UCs' equilibrium prices when the budget of user 1 varies from 2 to 42. Fig. 4 shows the total demand of each

user at equilibrium. The demand of user 1 is increasing because its budget is increasing. The budget of other users do not change, but they ultimately demand less power because of the increase in the budget of user 1. Fig. 5 shows the utility of the users at equilibrium. Fig. 6 depicts the equilibrium prices of the UCs. The prices increase linearly as the purchasing capacity of the users increases. UC 1 charges the highest unit price, because it has the lowest amount of power available and the reverse is true for UC 3. Fig. 7 shows the revenue of each UC. UC 1 has the lowest utility despite its high unit price, because its available power is the lowest. UC 3 receives the highest revenue although its unit price is the lowest.

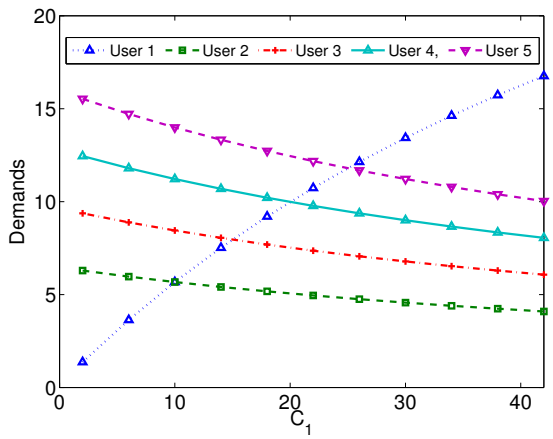


Fig. 4. User demands at equilibrium

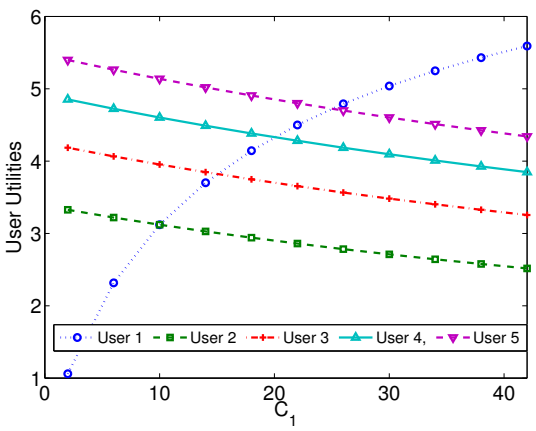


Fig. 5. User utilities at equilibrium

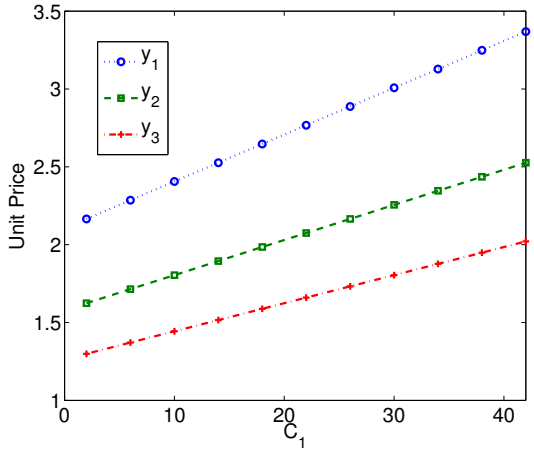


Fig. 6. Unit prices at equilibrium

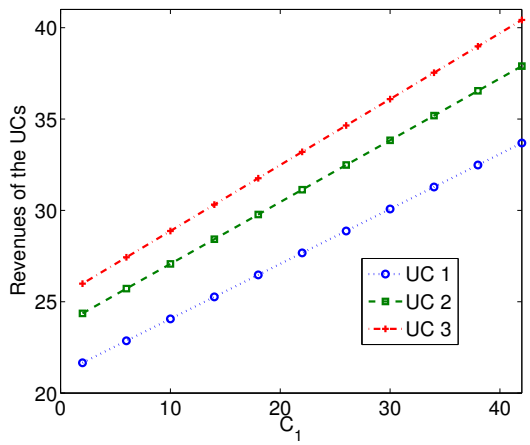
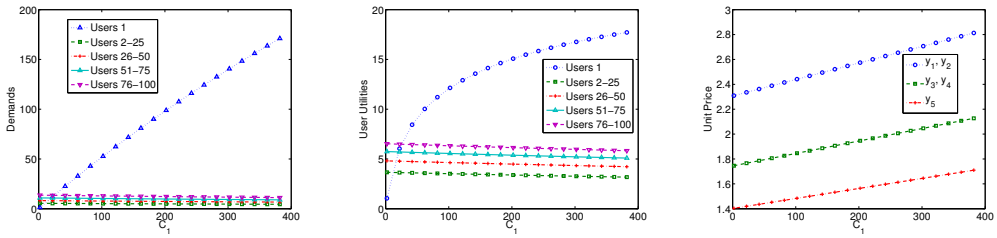


Fig. 7. UC revenues at equilibrium

Next we show the equilibrium of the Stackelberg game for more UCs and a large number of users for a large variation in the budget of one of the users. Figs. 8(a)-8(c) show the user demands, user utilities and unit prices at equilibrium for 5 UCs and 100 electricity users. The budget of user 1 varies from 2 to 400, the budget of users 2 to 25, users 26 to 50, users 51 to 75 and users 76 to 100 are $10, 15, 20$ and 25, respectively. The available power of the UCs used for plots are $P_1 = P_2 = 150$ units, $P_3 = P_4 = 200$ units and $P_5 = 250$ units. The behaviors of the users and the UCs are similar to Figs. 4-6.



(a) User demands (b) User utilities (c) Unit price

Fig. 8. Equilibrium for a large number of users and UCs for a large variation in user budget

B. Distributed Algorithm

Figs. 9 - 12 show the performance of the distributed algorithm for $\sigma_k = 40, \forall k \in \mathscr{K}$. The UCs reach the equilibrium price without communicating with each other. Consequently, the users reach their optimal demands based on the prices from the UCs. The results show that the equilibrium price and demand can be reached very quickly. Comparing Figs. 9 - 12 with Figs. 4 - 7, we can verify that the distributed algorithm converges to the optimal values.

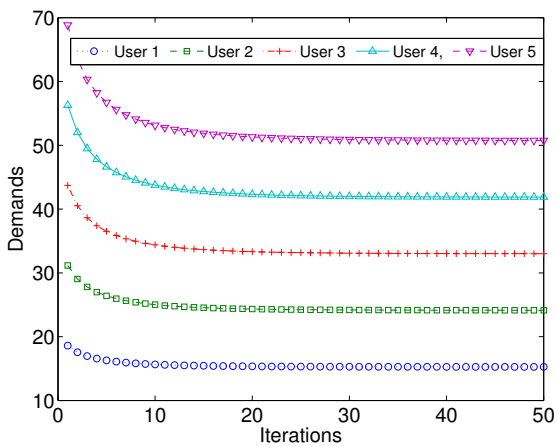


Fig. 9. Reaching equilibrium using distributed algorithm: User demands

Next we evaluate the performance of the distributed algorithm for different values of $y_{k,1}$ and σ_k. Fig. 13(a) shows the performance when $\sigma_k = 10, k = 1, 2, 3, \ldots$. We see from the figure that the algorithm converges much faster (since σ_k is smaller) and the unit prices converge to the same values as in Fig. 11. Figs. 13(b) and 13(c) depict that the algorithm converges to the same values when we start from a different value for $y_{k,1}$, for $\sigma_k = 50$ and 10, respectively

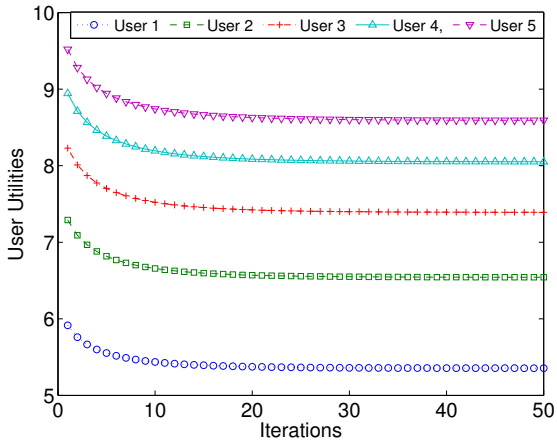


Fig. 10. Reaching equilibrium using distributed algorithm: User utilities

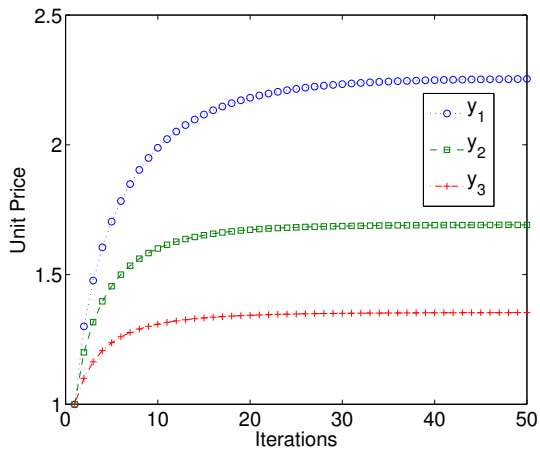


Fig. 11. Reaching equilibrium using distributed algorithm: Unit prices

$,\forall k \in \mathscr{K}$. We see that the convergence speed depends on the value of $\sigma_k, \forall k \in \mathscr{K}$ but the algorithm converges irrespective of different initial points.

C. Dependability of DRM

We show the impact of an attacker in terms of the reserve power needed under the two different schemes discussed in Sections VI-B and VI-C respectively in Fig.14. We consider both cases: when the price is increased and when the price is decreased. We use $P = 10$ and $[y_{\min}, y_{\max}] = [\frac{0.5C}{KP}, \frac{2C}{KP}]$ for the plot. For the sake of illustration, we suppose that the attacker manipulates the price of the third UC. In practice, the attacker can choose any of

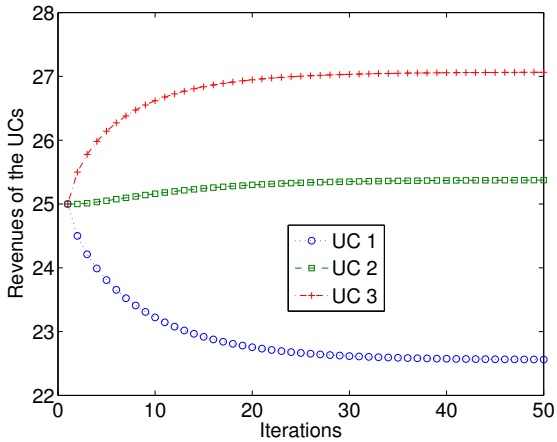
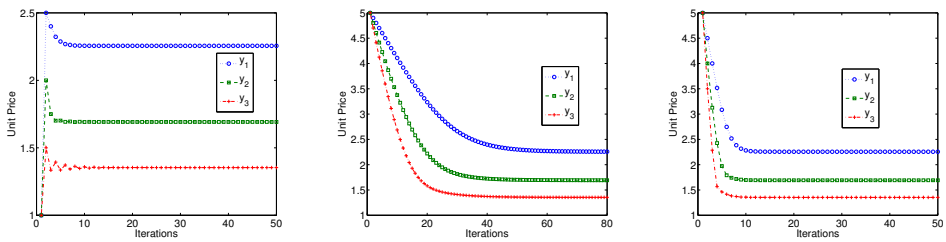


Fig. 12. Reaching equilibrium using distributed algorithm: UC revenues



(a) $y_{k,1} = 1; \sigma_k = 10.$ (b) $y_{k,1} = 5; \sigma_k = 50.$ (c) $y_{k,1} = 5; \sigma_k = 10.$

Fig. 13. The unit price of UCs for different initial starting points $y_{k,1}$ and different values of $\sigma_k, k = 1,2,3$.

the UCs. This figure shows a number of interesting facts. It verifies that the UCs save a lot in terms of the reserve power needed if they share a common reserve. The solid line with square markers shows the extra power needed to cover the total demand of users. The reserve power from a common reserve is sufficient to cover the demands of all the users when the price is increased and also when it is decreased. In fact, this power is extra most of the times and there is a considerable amount of unused power in both cases (individual reserve and common reserve). The unused power is much more when each UC keeps its own reserve as shown by the dotted line with '+' markers. The only case where the reserve power is completely used is when the price is decreased to $y_{\min}$. From this figure, it might appear that the amount of power wasted is still significant even with the common reserve. That is because our analysis is based on the worst case reserve. This figure also indicates how expensive an impact an attacker can create even if it has access to only one UC's price.

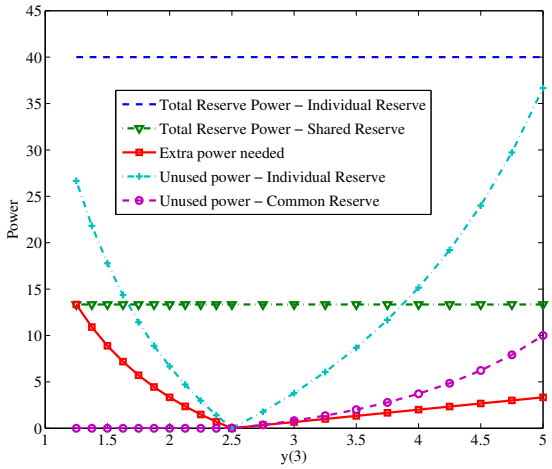


Fig. 14. Reserve power, extra power needed and unused power

The impact will be similar in case of the heterogeneous UCs with the difference in the scale of the power to be reserved.

VIII. Conclusion and Future Work

In this paper, we have proposed a Stackelberg game between the electricity UCs for optimal price setting and the end-users for optimal power consumption. We have derived the SE of the game in closed form and have proved its existence and uniqueness. We have designed a distributed algorithm for convergence to the SE with only local information available to the UCs. We have introduced two types of reliability issues associated with the smart grid: reliability due to physical disturbance and dependability in the face of an attacker. We have investigated the impact of an attacker who manipulates the price information from the UCs. Furthermore, we have proposed a scheme based on the concept of shared reserve power to ensure the reliability and dependability of the grid. We have shown the validity of our concepts through analytical and numerical results.

This work opens the door to some interesting extensions. The DRM analysis incorporating the modeling of instability of the renewable energy sources is a potential direction. Here we have focused on a large time-scale one-period DRM scheme. A higher resolution multi-period scheme with inter-temporal constraints is another possible extension to this work.

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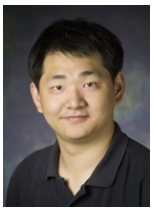

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Paper V

Hierarchical Demand Response Management in the Smart Grid with Large Population Approximations

Sabita Maharjan, Quanyan Zhu, Yan Zhang, Stein Gjessing and Tamer Başar

Hierarchical Demand Response Management in the Smart Grid with Large Population Approximations

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Abstract

Demand Response Management (DRM) is a key component in the smart grid for reducing power generation costs and user bills, and also for optimizing the profits of power generation companies (gencos). However, DRM problem has not been investigated holistically in a network of multiple gencos and consumers, incorporating multiple processes from power generation to market activities and to power consumption. With this motivation, we propose a hierarchical system model that captures the decision process involved in the smart grid, and establish a Stackelberg game between gencos and end-users, where gencos behave as leaders maximizing their profit, and users act as followers maximizing their utility. In the model, we differentiate between industrial users who have strict power requirements and residential users with flexible power requirement but budget constraints, to characterize their reliability and efficiency requirements. We derive analytical results for the Stackelberg equilibrium of the game and prove that a unique equilibrium solution exists for the game. We develop a distributed algorithm which converges to the equilibrium with only price information available to both gencos and end-users. Furthermore, we extend our analysis to a large population regime to obtain deeper insights into the sensitivities of the price and demands on the equilibrium solution. Finally, we provide simulations and numerical examples to illustrate the solutions and to corroborate the results.

Index Terms

Demand response management, large-population regime, reliability, smart grid communications, Stackelberg game, user-differentiation.

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I. Introduction

The demand from electricity consumers has been growing due to new technology and appliances such as plug-in hybrid electric vehicles (PHEVs). Accompanied with the growth are concerns about environment impacts and power system reliability. Traditional power grids, however, are not able to meet these demand and reliability requirements due to the lack of efficient communications between supply and demand sides. Recent blackouts [1] have further indicated these issues, and it is essential to transform the traditional power grid into a more responsive, efficient and reliable system. The smart grid [2] is the future power grid system that incorporates a smart metering infrastructure capable of sensing and measuring power consumption from consumers with the integration of advanced information and communication technologies (ICTs). The bi-directional flow of information provided by the communications infrastructure will enable consumers to control and manage their own electricity usage and allow generation companies (gencos) to effectively control the power generation, achieving instantaneous balance of supply and demand.

Demand Response Management (DRM) is the response system of end-users to changes in electricity prices over time or across different energy sources. In the smart grid, DRM plays a key role in improving different aspects of both supply and demand sides. Effective DRM can yield lower bills and higher utility efficiency for end-users. DRM can also reduce the cost of power generation or improve the revenues of gencos. Existing works have mainly focused on either adjusting the demand side by load shedding schemes when the supply is given, or on improving the profit for the gencos on the supply side when aggregate user demand is given. Efficient DRM relies on both demand and supply sides and an integrated framework is needed to consider DRM in a holistic manner.

Gencos can optimize their profits by producing optimal amount of power and selling at an optimal unit price. End-users, on the other hand, can optimize their utilities by choosing optimally their demand strategies. We establish a multi-layer hierarchical Stackelberg game model to incorporate these strategic behaviors in one single framework. In addition, different types of users have their own reliability and efficiency requirements. Residential users usually consume much less power compared to commercial and industrial ones, while commercial and industrial users often have more strict power reliability and availability requirements. In our game model, we categorize the electricity consumers into two types, i.e., residential and industrial, to capture user-differentiations in the smart grid. The hierarchical Stackelberg

game model consists of three levels:

- top level, where gencos determine optimal power generation,
- middle level, where energy prices are determined through competition among different gencos, and
- lowest level, where end-users strategically respond to price signals (i.e., DRM).

In the three-level hierarchical game model, the gencos and end-users interact through the prices determined at the middle level. The competitions among gencos are captured at the top and middle levels where gencos determine the amount of power to generate as well as the energy prices through a competitive market. The interactions between users are indirectly coupled through the price signal at the lowest level. The coupling between the decisions across the levels makes game theory an appealing approach for cross-level understanding of a multi-player multi-level complex system. Since users respond to prices after gencos announce their level of power production and unit prices, the sequential decision process can be modeled by a Stackelberg game where the gencos behave as the leaders and the users behave as followers. Hence, we adopt a Stackelberg game with multiple leaders as competitors and multiple followers who react to their competition.

The model allows us to provide insights into the outcome of multi-level strategic interactions with different type of players in the power system. We characterize analytically the Stackelberg equilibrium solution to the hierarchical game and provide closed form solutions for homogenous user case. To allow practical implementation of demand response algorithms, we develop distributed algorithms so that both supply and demand side parameters can be determined using only local information.

Another important factor of DRM that we consider in our model is the large population size at the lower level in comparison to the number of gencos. The collective strategic responses of a large number of users will have major impact on the decisions of gencos as compared to one single individual in the large population. To capture this feature, we study the large population approximations of Stackelberg equilibrium of the game for two specific cases: (i) one with large residential population and (ii) one with large industrial population. The distinction between these two cases depends on geographical locations and economic structures. This enables us to gain useful insights into the problem regarding the sensitivities of the cost and budget parameters in the game model. We also derive the necessary conditions on budgets of the residential users to obtain the minimum required power in the large population regime.

Our major contributions in this work can be summarized as follows:

1) We propose a three-level hierarchical Stackelberg game model incorporating gencos from the supply side and the end-users from the demand side. The game-theoretic framework provides a holistic analytical model to study strategic behaviors of the players in DRM from both supply and demand sides.
2) We provide analytical results for characterizing the Stackelberg equilibrium (SE) of the multi-genco and multi-user Stackelberg game, and develop distributed algorithms for the gencos and the users to reach SE with only local information.
3) We differentiate the end-users into residential and industrial users based on their different requirements for reliability and efficiency requirement. We develop large population approximations of the SE and perform sensitivity analysis on parameters in the DRM game model to provide deeper insights into the behavior of the gencos and the users.

The rest of the paper is organized as follows: Related work is described in Section II. We introduce the system model in Section III. In Section IV, we formulate the problem as a Stackelberg game and prove the existence and uniqueness of the SE. We propose a distributed algorithm for the game which converges to the SE, in Section V. In Section VI, we develop large population approximations. We provide numerical results and discuss about them in Section VII. Section VIII concludes the paper. All analytical proofs are summarized in Appendices A-C.

II. Related Work

There have been several studies on demand side management (DSM) and DRM in the smart grid, including [5]-[13]. In [5], the authors have formulated an energy consumption scheduling problem as a non-cooperative game among the consumers for increasing strictly convex cost functions. In [6], the authors have considered a distributed system where price is modeled by its dependence on the overall system load. Based on the price information, the users adapt their demands to maximize their own utility. In [7], a robust optimization problem has been formulated to maximize the utility of a user, taking into account price uncertainties at each hour. In [8], the authors have exploited the awareness of the users and proposed a method to aggregate and manage end-users' preferences to maximize energy efficiency and user satisfaction. In [9], a dynamic pricing scheme has been proposed to provide incentives for costumers to achieve an aggregate load profile suitable for utility companies, and the demand response problem has been investigated for different levels of information sharing among the consumers. In [10], a two-level game framework is established to model distributed

dynamic side management. At the lower level, each household schedules different appliances for energy consumption. At the upper level, a dynamic game is used to capture the interaction among different players in their demand responses through sticky market price. In [11], a multi-resolution two-layer game is studied using mean-field game approach to incorporate inner interactions between users in the region and outer interactions between regions for dynamic distributed demand response in the smart grid. Recent studies [12], [13] on DRM have also incorporated electric vehicles (EVs) or PHEVs into the DRM framework.

We observe that these studies have mainly focused on either only one source or a number of sources/utilities treated as one entity, and the profit maximization of gencos on the supply side has not been incorporated into the study of DRM. In our previous work [14], we have introduced multiple gencos and multiple consumers, who aim to maximize their own revenues and utilities, respectively, under a two-level Stackelberg game framework. In [14], we have assumed that the available power of each genco is given. In this paper, however, we consider the demand side together with the supply side, bringing both sides into one single game framework, in which gencos optimize the revenues and users optimize their demands under budget constraints. The game has three levels. The middle level models the imperfect market where the energy price is determined by the competition between multiple price-setting gencos. This level essentially glues the demand and supply sides together as one problem. Our proposed scheme allows us to investigate the efficiency and reliability issues of differentiate types of users in a holistic manner.

We note that there is rich literature using Stackelberg games for congestion control, revenue maximization and cooperative transmission [17]-[18]. Our approach is similar to those in congestion control and power control, but our game model involves multiple gencos and multiple users. The coupling among followers, i.e., the users, comes indirectly from the price determined by the competitions among the gencos. This special game structure allows us to provide analytical results for the multi-player game with hierarchical structure. Furthermore, our model has a three-level structure, where at the higher level, we treat the amount of power to be produced as a variable to be optimized by each genco. This approach has some similarity to the capacity expansion of the links proportional to the number of users, in the context of congestion control, for a single link in [17], and for multiple links from a single service provider in [18]. However, in the smart grid scenario, we need to take into account the budget limit of the residential users, and the decision on power generation at the higher level depends on the unit price determined through the competition at the middle level. These

characteristics have made our problem distinct from methodologies in existing studies.

III. System Model

Fig. 1 depicts our hierarchical system model. We consider N end-users, which we also call customers, and K electricity gencos, $\mathscr{K} := \{1,2,\ldots,K\}$. Gencos produce and sell electrical energy. Each generator is owned by a genco. The end-user side consists of N_{R} residential users, $\mathscr{N}_{\mathrm{R}} := \{1,2,\ldots,N_{\mathrm{R}}\}$ and N_{I} industrial users, $\mathscr{N}_{\mathrm{I}} := \{1,2,\ldots,N_{\mathrm{I}}\}$, i.e., $N = N_{\mathrm{R}} + N_{\mathrm{I}}$.

The power is dispatched to the consumers through the transmission and distribution networks. The transmission companies (transcos) own transmission assets such as lines, cables, transformers and reactive compensation devices. Distribution companies (discos) own and operate distribution networks. A detailed description about gencos, transcos, market structure and discos in the current power grid can be found in [19].

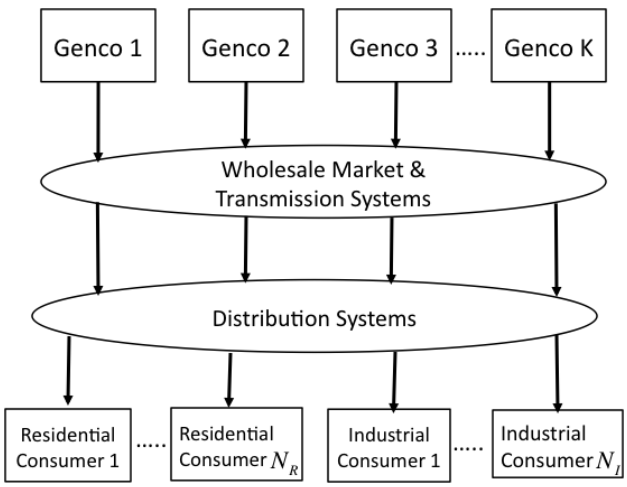


Fig. 1. Illustration of the hierarchical system model composed of multiple processes of power generation, transmission, distribution and consumption. The power generation is at the top level; the price setting game between the gencos is at the middle level, i.e., the market level. The power distribution is assumed to be ideal and the demand decisions from the consumers are made at the bottom level.

Our hierarchical system model consists of three levels: the power generation from the energy sources at the top level, the price determination at the middle level, i.e., the market level, and the demand response to the price signals from the residential consumers at the bottom level. In our model, the gencos and end-users interact through the prices determined through the competitions among the gencos at the market level. Different from a traditional

perfect competitive market, we have an imperfect competition between gencos. In a perfectly competitive market, no market participant has the ability to influence the market price through its individual actions, i.e., the market price is a parameter over which gencos have no control. Consequently, each gencos should increase its production up to the point where the marginal cost is equal to the market price. This is valid when the number of market participants is large and none of them controls a large proportion of the production. However, in this paper, we consider a finite number of market participants (gencos) and each individual genco has non-infintesimal influence in the market. By fixing the demand response from the users, we model the interactions among gencos as an imperfect competition using Cournot-type games.

The gencos and the consumers have bidirectional communications support to exchange price and demand information. The communications between gencos and users for the three-level hierarchical model are shown in Fig. 2. The competitions among gencos are captured at the top and middle levels where gencos deterimine the amount of power to generate as well as the energy prices through a competitive market. The interactions between users are indirectly coupled through the price signal at the lowest level. The gencos can also communicate with each other. The users receive the price information from the gencos and transmit their demand to them. The data communication is carried out through the communication channel using wireless technologies.

The industrial consumers have strict power requirements but do not have severe budget constraints. Let $P_{\mathrm{I},m}$ denote the total power requirement of industrial user $m, (m \in \mathscr{N}_{\mathrm{I}})$. Then, we write

$$\sum_{k \in \mathscr{K}} x_{\mathrm{I},m,k} = P_{\mathrm{I},m}, \ \forall m \in \mathscr{N}_{\mathrm{I}},$$

where $x_{\mathrm{I},m,k}$ is the demand of industrial user m from genco k.

In contrast, residential users have limited budgets $C_{\mathrm{R},n} \in \mathbb{R}_{++}, \forall n \in \mathscr{N}_{\mathrm{R}}$. Let $x_{\mathrm{R},n,k}$ denote the demand of residential user n from genco k. The residential users adjust their demands according to the spot unit price of the gencos to fit their budget. This adjustment does not apply to the industrial users.

IV. Problem Formulation: A Stackelberg Game Approach

The price determination of each genco is affected by the strategies of other gencos. The utility of the residential users and the cost to the industrial users are dependent on the competition among the gencos to determining the unit prices. Moreover, the utility of a

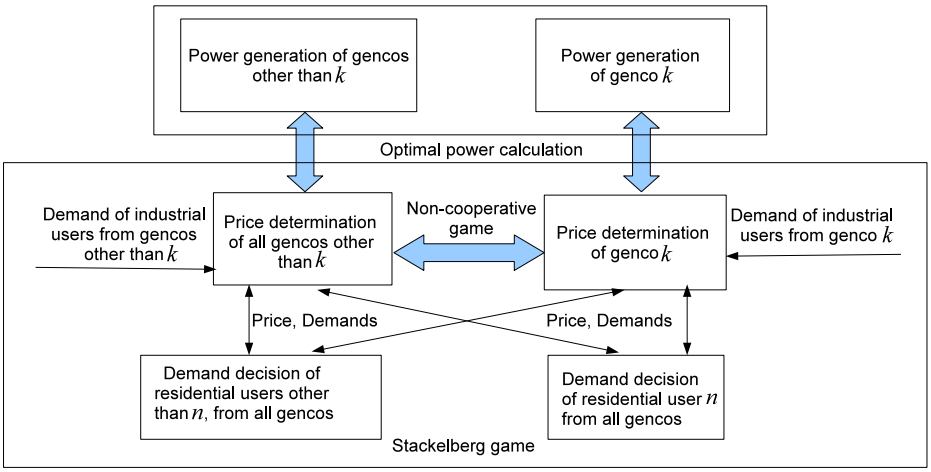


Fig. 2. Illustration of multi-layer interactions among the gencos and the end-users in the 3-level hierarchical DRM. Each genco determines the amount of power to produce and the prices are determined at the market level through the balance of supply and demand. Consumers, consisting of industrial and residential users, determine their consumption by responding to the market prices.

residential user varies based on its own budget and the budget of other users. The strategies of the gencos are directly coupled, while the interactions among users are indirectly coupled through the price signal. These couplings between the decisions across the levels make game theory an appealing approach for cross-level understanding of a multi-player multi-level complex system. Since users respond to prices after gencos announce their strategies, the sequential decision process can be modeled by a Stackelberg game [20] where the gencos behave as the leaders and the users react to their strategies as followers.

The solution to the game is a Stackelberg equilibrium (SE). The SE solutions for the hierarchical game constitute gencos' strategies for power and unit prices, and users' strategies for demands. The unit prices are determined through the competition among gencos. For given unit prices, users play optimal response, and gencos decide on the optimal amount of power to generate, by solving its own optimization problem. We formulate the utility maximization problem for each user and find the optimal demand response from the users for given energy prices in Section IV-A. We formulate the profit maximization problem for each genco in Section IV-B, and obtain the equilibrium unit prices given the optimal user responses. Using this equilibrium solution, we determine the optimal amount of power to generate for each genco. We glue Section IV-A and Section IV-B in Section IV-C, where we characterize the

SE of the game combining parameters from both supply and demand sides. In Section IV-D, we discuss the versatility of the proposed hierarchical Stackelberg game model and discuss its applicability in a broader range of scenarios.

A. Demand Side Analysis

In this section we formulate the problem for the bottom level of the hierarchical model, i.e., the demand response from the residential users. We define the utility of residential user $n, (n \in \mathcal{N}_{\mathrm{R}})$, as

$$U_{\mathrm{user,R},n} = \alpha_n \sum_{k \in \mathcal{K}} \ln\left(\beta_{\mathrm{R},n} + x_{\mathrm{R},n,k}\right), \ \forall k \in \mathcal{K} \tag{1}$$

where α_n and $\beta_{\mathrm{R},n}$ are constants. Typical values of α_n and $\beta_{\mathrm{R},n}$ are 1. The logarithmic function has been widely used in economics for modeling the preference ordering of users and for decision making [13] - [16]. The motivation behind choosing the utility function (1) is that it is closely related to the utility function $\alpha_n \sum \ln(x_{\mathrm{R},n,k})$ that leads to proportionally fair demand response [15] [17].

Let y_k be the unit price set by genco k. For a given set of prices from the generators $\{y_1, y_2, \ldots, y_K\}$, user $n(n \in \mathcal{N}_{\mathrm{R}})$, calculates its optimal demand response by solving the user optimization problem ($\mathrm{OP}_{\mathrm{user}}$)

$$\begin{aligned} &\max_{\mathbf{x}_{\mathrm{R},n} := \{x_{\mathrm{R},n,k}, \forall k \in \mathcal{K}\}} U_{\mathrm{user,R},n} \\ &\text{s.t.} \sum_{k \in \mathcal{K}} y_k x_{\mathrm{R},n,k} \leq C_{\mathrm{R},n}, \\ &x_{\mathrm{R},n,k} \geq 0; \ \forall k \in \mathcal{K}. \end{aligned} \tag{2}$$

$\mathrm{OP}_{\mathrm{user}}$ is a strictly convex optimization problem. Hence, the stationary solution is unique and optimal.

1) The Case of Two Gencos: Let us start the analysis with N_{R} users and 2 gencos. We will later generalize the results to K gencos. The optimization problem for user n in this case is

$$\begin{aligned} &\max_{\mathbf{x}_{\mathrm{R},n} := \{x_{\mathrm{R},n,1}, x_{\mathrm{R},n,2}\}} \alpha_n \sum_{k=1}^{2} \ln(\beta_{\mathrm{R},n} + x_{\mathrm{R},n,k}) \\ &\text{s.t. } y_1 x_{\mathrm{R},n,1} + y_2 x_{\mathrm{R},n,2} \leq C_{\mathrm{R},n}, \\ &x_{\mathrm{R},n,1}, x_{\mathrm{R},n,2} \geq 0. \end{aligned} \tag{3}$$

Using Lagrange multipliers $\lambda_{n,1}, \lambda_{n,2}$ and $\lambda_{n,3}$, we convert the constrained optimization problem (3) to the form

$$L_{user,n} = \alpha_n \sum_{k=1}^{2} \ln\left(\beta_{\mathrm{R},1} + x_{\mathrm{R},n,k}\right) - \lambda_{n,1}\left(\sum_{k=1}^{2} y_k x_{\mathrm{R},n,k} - C_1\right) + \lambda_{n,2} x_{\mathrm{R},n,1} + \lambda_{n,3} x_{\mathrm{R},n,2} \tag{4}$$

with the complementarity slackness conditions

$$\lambda_{n,1}\left(\sum_{k=1}^{2} y_k x_{\mathrm{R},n,k} - C_{\mathrm{R},n}\right) = 0, \tag{5}$$

$$\lambda_{n,2} x_{\mathrm{R},n,1} = 0; \quad \lambda_{n,3} x_{\mathrm{R},n,2} = 0,$$

$$\lambda_{n,1} > 0, \quad \lambda_{n,2}, \lambda_{n,3}, x_{\mathrm{R},n,1}, x_{\mathrm{R},n,2} \geq 0.$$

The first-order optimality condition for the maximization problem is $\nabla L_{\mathrm{user},n} = 0$. Since the only coupling between the users is through y_k, $\nabla L_{\mathrm{user},n} = 0$ leads to

$$\frac{\partial L_{\mathrm{user},n}}{\partial x_{\mathrm{R},n,k}} = 0, \forall n \in \mathscr{N}_{\mathrm{R}}, k \in \mathscr{K}, \text{i.e., },$$

$$\begin{cases} \frac{\alpha_n}{\beta_{\mathrm{R},n} + x_{\mathrm{R},n,1}} - \lambda_{n,1} y_1 + \lambda_{n,2} = 0, \\ \frac{\alpha_n}{\beta_{\mathrm{R},n} + x_{\mathrm{R},n,2}} - \lambda_{n,1} y_2 + \lambda_{n,3} = 0. \end{cases} \tag{6}$$

The optimal demands of the users can take one of the following forms.

Case 1 : $x_{\mathrm{R},n,1}, x_{\mathrm{R},n,2} > 0$. In this case, $\lambda_{n,2} = \lambda_{n,3} = 0$. Substituting $\lambda_{n,2}$ and $\lambda_{n,3}$ into (6) yields

$$x_{\mathrm{R},n,k} = \frac{\alpha_n}{\lambda_{n,1} y_k} - \beta_{\mathrm{R},n}, \qquad \forall n \in \mathscr{N}_{\mathrm{R}}, k = 1, 2. \tag{7}$$

Using (5) and (7) we obtain

$$x_{\mathrm{R},n,k} = \frac{C_{\mathrm{R},n} + \beta_{\mathrm{R},n} \sum_{k=1}^{2} y_k}{2 y_k} - \beta_{\mathrm{R},n}. \tag{8}$$

Case 2 : $x_{\mathrm{R},n,1} > 0, x_{\mathrm{R},n,2} = 0$. This is the case when $\frac{C_{\mathrm{R},n} + \beta_{\mathrm{R},n} \sum_{k=1}^{2} y_k}{2 y_2} = \beta_{\mathrm{R},n}$. Equation (6) implies $\lambda_{n,2} = 0$ and

$$x_{\mathrm{R},n,1} = \frac{\alpha_n}{\lambda_{n,1} y_1} - \beta_{\mathrm{R},n}. \tag{9}$$

Using (5) and (9), we get, $\lambda_{n,1} = \frac{\alpha_n}{C_{\mathrm{R},n} + \beta_{\mathrm{R},n} y_1}$ and

$$x_{\mathrm{R},n,1} = \frac{C_{\mathrm{R},n} + \beta_{\mathrm{R},n} y_1}{y_1} - \beta_{\mathrm{R},n} = \frac{C_{\mathrm{R},n}}{y_1}. \tag{10}$$

$x_{\mathrm{R},n,2} = 0 \Rightarrow \frac{C_{\mathrm{R},n} + \beta_{\mathrm{R},n} \sum_{k=1}^{2} y_k}{2 y_2} = \beta_{\mathrm{R},n}$. Using this and (10), and after simplifying, we obtain

$$x_{\mathrm{R},n,1} = \frac{C_{\mathrm{R},n} + \beta_{\mathrm{R},n}(y_1 + y_2)}{2y_1} - \beta_{\mathrm{R},n}. \tag{11}$$

Case 3 : $x_{1,1} = 0, x_{1,2} > 0$. Similar analysis can be performed as in case 2 to obtain

$$x_{\mathrm{R},n,2} = \frac{C_{\mathrm{R},n} + \beta_{\mathrm{R},n}(y_1 + y_2)}{2y_2} - \beta_{\mathrm{R},n}. \tag{12}$$

Note that in cases 1-3 discussed above, both power constraint and the cost constraint are satisfied as equalities.

Case 4 : $x_{\mathrm{R},n,1} = 0, x_{\mathrm{R},n,2} = 0$

In this case, $\lambda_{n,1} = 0$ and $\lambda_{n,2}, \lambda_{n,3}$ can be any non-negative real values. This is an extreme case, which does not happen unless $C_{\mathrm{R},n} = 0$ or $y_k = \infty,\ \forall k \in \mathscr{K}$.

2) The Case of K Gencos: Analyzing in similar manner, we can obtain the optimal demands for the general case of N_{R} residential users and K gencos can be formulated as

$$x_{\mathrm{R},n,k} = \frac{C_{\mathrm{R},n} + \beta_{\mathrm{R},n} \sum_{k \in \mathscr{K}} y_k}{K y_k} - \beta_{\mathrm{R},n}. \tag{13}$$

B. Supply Side Analysis

In this section we analyze the middle level and the top level of the hierarchical model, i.e., the price and the amount of power to be produced by the gencos. Let $P_{k,\mathrm{b}}$ denote the power demand of all industrial users from generator k. Then, $P_{k,\mathrm{b}} = \sum_{m=1}^{N_I} x_{\mathrm{I},m,k}$. If all industrial users get the same amount of power from genco k, then, let $x_{\mathrm{I},m,k} = x_{\mathrm{I},k},\ \forall n \in \mathscr{N}_{\mathrm{I}}$. In that case, we have, $P_{k,\mathrm{b}} = N_I x_{\mathrm{I},k}$. Let $P_{k,\mathrm{m}} \geq 0$ be the power available for the residential users from genco k, such that the power it generates is $P_k = P_{k,\mathrm{b}} + P_{k,\mathrm{m}}$. We employ a quadratic cost function for all gencos for producing electric power as it has been widely used to model the cost of power generation [5], [21]. Let a_k, b_k and c_k be the coefficients of the cost function for genco k, i.e., if the total power produced by source k is P_k, then, the cost of generation is $a_k P_k^2 + b_k P_k + c_k$. The payoff of a genco is the profit it gains by selling P_k at the unit price of y_k. Thus, we define the utility of genco k as

$$U_{\mathrm{gen},k} = y_k \left(P_{k,\mathrm{b}} + \sum_{n=1}^{N_R} x_{\mathrm{R},n,k} \right) - (a_k P_k^2 + b_k P_k + c_k).$$

The optimization problem for the gencos ($\mathrm{OP}_{\mathrm{gen}}$) is described as follows:

$$\max U_{\mathrm{gen},k} \tag{14}$$

$$\text{s.t.} \sum_{n \in \mathscr{N}_{\mathscr{R}}} x_{\mathrm{R},n,k} \leq P_{k,\mathrm{m}};\quad P_{k,\mathrm{b}} \leq P_k \leq P_{k,\max};\quad y_k \geq 0;\ \forall k \in \mathscr{K},$$

where $P_{k,\max}$ is the maximum power generation capacity of genco k, which is sufficiently large. Although the characterization of the solution to the optimization problem in (14) requires a number of conditions, some of them can be relaxed. Since $P_{k,\max}$ is sufficiently large, $P_{k,\mathrm{b}}, P_k << P_{k,\max}$, the second constraint can be relaxed. Similarly, the third constraint is not a hard constraint, and thus, can be relaxed. Instead, after finding the solution to this problem, we prove that there exists a set of real positive valued solutions: $\{y_k, k \in \mathscr{K}\}$. Relaxing the second and third constraints, we define the Lagrangian $L_{\mathrm{gen},k}$ for $\mathrm{OP}_{\mathrm{gen}}$ as follows:

$$L_{\mathrm{gen},k} = U_{\mathrm{gen},k} - \mu_{k,1}\left(\sum_{n\in\mathscr{N}_{\mathrm{R}}} x_{\mathrm{R},n,k} - P_{k,\mathrm{m}}\right)$$

where $\mu_{k,1}$ is the Lagrange multiplier for the first constraint. Now the constrained optimization problem of genco k can be converted to the unconstrained form as

$$\max L_{\mathrm{gen},k} \tag{15}$$

with the complementarity slackness conditions

$$\mu_{k,1}\left(\sum_{n\in\mathscr{N}_{\mathrm{R}}} x_{\mathrm{R},n,k} - P_{k,\mathrm{m}}\right) = 0; \quad \mu_{k,1}, y_k \geq 0.$$

Next, we decide the optimal amount of total power to be produced by a genco for maximizing its profit. Since $P_{k,\mathrm{b}}$ is fixed, deciding P_k is equivalent to deciding $P_{k,\mathrm{m}}$. For given y_k, since the revenue increases linearly with P_k, the profit will be maximized when $\sum_{n\in\mathscr{N}_{\mathrm{R}}} x_{\mathrm{R},n,k} = P_{k,\mathrm{m}}$. Then, the first constraint of (14) becomes an equality constraint, and $L_{\mathrm{gen,k}} = U_{\mathrm{gen,k}} = y_k P_k - (a_k P_k^2 + b_k P_k + c_k)$, and thus, for given $(\{y_k, k \in \mathscr{K}\})$, (15) can be written as

$$\max_{P_k\in\mathbb{R}_+} y_k P_k - (a_k P_k^2 + b_k P_k + c_k). \tag{16}$$

For given prices $\{y_k, k \in \mathscr{K}\}$, (16) is a convex optimization problem. Using the first order optimality condition for the gencos, i.e., $\nabla U_{\mathrm{gen,k}} = 0$, we obtain $\frac{\partial U_{\mathrm{gen},k}}{\partial P_k} = 0$, for given $y_k, \forall k \in \mathscr{K}$. Hence, using $\frac{\partial U_{\mathrm{gen},k}}{\partial P_k} = 0$, we arrive at the optimal amount of power to be generated by genco k is

$$P_k = \frac{y_k - b_k}{2a_k}; \ \forall k \in \mathscr{K}. \tag{17}$$

With the optimal demand response of residential users (13) and gencos' power generation (13) in response to the energy prices, the objective of each genco is to set prices $y_k, k \in \|$

through competition in the market. The genco's utility function (14) can be rewritten and maximized over the unit price subject to the equality form of the first constraint in (14), i.e.,

$$\sum_{n\in\mathscr{N}_{\mathrm{R}}} x_{\mathrm{R},n,k} = P_{k,\mathrm{m}}. \tag{18}$$

At the middle layer, the gencos play a non-cooperative game in the market to determine their prices through competitions, which can be used to determine power generation at the lop level and user demands at the bottom level. The equality constraint (18) implies that the outcome of the competition among gencos is an equilibrium where the supply and demand of the market are balanced. Since the coupling in (14) among gencos is only through the equality constraint, our equilibrium analysis of the competition can be facilitated by only using the constraints (18) of every $k \in \mathscr{K}$ to obtain the equilibrium values of the prices. By definition, $P_{k,\mathrm{m}} = P_k - P_{k,\mathrm{b}}$. Hence, (18) can be written as

$$\sum_{n\in\mathscr{N}_{\mathrm{R}}} x_{\mathrm{R},n,k} = P_k - P_{k,\mathrm{b}}. \tag{19}$$

Using (13) and (17) in (19) for genco k, we obtain a set of coupled equations

$$Ky_k^2 + \left[2(K-1)B_{\mathrm{R}}a_k - Kb_k - 2KP_{k,\mathrm{b}}\right]y_k - \left[2B_{\mathrm{R}}a_k \sum_{g\in\mathscr{K},g\neq k} y_g + 2C_{\mathrm{R}}a_k\right] = 0. \tag{20}$$

where $C_{\mathrm{R}} = \sum_{n\in\mathscr{N}_{\mathrm{R}}} C_{\mathrm{R},n}, B_R = \sum_{n\in\mathscr{N}_{\mathrm{R}}} \beta_{\mathrm{R},n}$. We assume a certain distribution of the budget of the residential users. Let $\bar{C}$ be the mean budget. Then C_{R} can be calculated as $C_{\mathrm{R}} = N_{\mathrm{R}}\bar{C}$. The gencos set their prices based on C_{R}. The residential users compute their optimal demand based on their local parameters and the unit price set $\{y_k, k \in \mathscr{K}\}$.

Theorem 1: Suppose $P_{k,\mathrm{b}}$ is sufficiently small compared to $P_{k,\max}$. Then, a unique feasible solution of (20) exists. We investigate the following two cases.

1) Case 1*:* When all the gencos have the same parameters, i.e., when $a_k = a, b_k = b, c_k = c, P_{k,\mathrm{b}} = P_b$ and $y_k = y, \forall k \in \mathscr{K}$, the unique positive solution of (20) is

$$y = \frac{K(b+2P_{\mathrm{b}}a) + \sqrt{K^2(b+2P_{\mathrm{b}}a)^2 + 8C_{\mathrm{R}}Ka}}{2K} \tag{21}$$

2) Case 2*:* When the gencos have different parameters (each parameter indexed by k), the unique positive solution of (20) is

$$y_k = \frac{-T_1 + \sqrt{T_2}}{2K} \tag{22}$$

where $T_1 = 2B_{\mathrm{R}}(K-1)a_k - Kb_k - 2KP_{k,\mathrm{b}}a_k$ and $T_2 = \left(2B_{\mathrm{R}}(K-1)a_k - Kb_k - 2KP_{k,\mathrm{b}}a_k\right)^2 + 8a_kK\left(B_{\mathrm{R}}\sum_{g\in\mathscr{K},g\neq k} y_g + C_{\mathrm{R}}\right)$.

Proof: Given in Appendix A. ■

Remark 1: Equation (13) indicates that for $x_{\mathrm{R},n,k} \geq 0$ to hold $\forall n \in \mathscr{N}_{\mathrm{R}}, \forall k \in \mathscr{K}$, the following must be satisfied $\forall n \in \mathscr{N}_{\mathrm{R}}, \forall k \in \mathscr{K}$.

$$C_{\mathrm{R},n} \geq \beta_{\mathrm{R},n} \left[(K-1) y_k - \left(\sum_{g \in \mathscr{K}, g \neq k} y_g \right) \right]. \tag{23}$$

When the gencos are homogeneous, $x_{\mathrm{R},n,k} = x_{\mathrm{R},n} \geq 0, \forall n \in \mathscr{N}_{\mathrm{R}}$ will hold if their budget $C_{\mathrm{R},n} \geq 0$, i.e., the users can always get some power from each genco since $C_{\mathrm{R},n} > 0$. When the gencos are heterogeneous, if $P_{k,\mathrm{b}}$ increases, P_k and thus y_k also increase. However, if y_k keeps on increasing, it may lead to the point where (23) is violated for residential users, i.e., the point at which the demands from the residential users will be negative. The physical meaning of such a point is that $P_{k,\mathrm{b}}$ is so large that the residential users should supply power to the gencos to satisfy the demands from the industrial users. Therefore, $P_{k,\mathrm{b}}$ should be small enough so that (13) is satisfied.

◇

C. Stackelberg Equilibirum: Existence and Uniqueness

Given the behavior of the users, the gencos play a non-cooperative game to determine energy prices and announce them to the users for DRM. In the three-level Stackelberg game depicted in Fig. 2, where the users act as followers and the gencos act as leaders, the Stackelberg equilibrium (SE) strategy constitutes energy generations and prices set by the gencos and the level of energy consumptions of the customers in response to them [20].

Let $\Gamma_{\mathrm{G},k}$, $\Delta_{\mathrm{G},k}$ be the set of pricing and power generation strategies of genco k, respectively. Let $\Gamma_{\mathrm{R},n}$ denote the set of demand strategies of user n. Hence the pricing and power generation strategy spaces of all gencos are $\Gamma_{\mathrm{G}} := \Gamma_{\mathrm{G},1} \times \Gamma_{\mathrm{G},2} \times \ldots.. \times \Gamma_{\mathrm{G},K}$ and $\Delta_{\mathrm{G}} := \Delta_{\mathrm{G},1} \times \Delta_{\mathrm{G},2} \times \ldots.. \times \Delta_{\mathrm{G},K}$ respectively, and let the set of user profiles be defined by $\Gamma_{\mathrm{R}} = \Gamma_{\mathrm{R},1} \times \Gamma_{\mathrm{R},2} \times \ldots.. \times \Gamma_{\mathrm{R},N}$. Let $\Gamma_C \subseteq \Gamma_{\mathrm{R}}$ be the constrained feasible set.

Let $\mathbf{y} = \{y_k, k \in \mathscr{K}\} \in \Gamma_{\mathrm{G}}$ be the energy prices of the gencos, $\mathbf{P} = \{P_k, k \in \mathscr{K}\} \in \Delta_{\mathrm{G}}$ be the energy generations of the gencos and $\mathbf{x} := \{\mathbf{x}_1; \mathbf{x}_2; \ldots..\mathbf{x}_{N_{\mathrm{R}}}\} \in \Gamma_{\mathrm{R}}$ is the strategy of all residential users $1, 2, \ldots.., N_{\mathrm{R}}$. Let $\hat{\mathbf{x}}_n : \Gamma_{\mathrm{G}} \to \Gamma_{\mathrm{R},n}$ be the best response mapping of consumer n to the energy prices given the power generation levels, i.e, $\mathbf{x}_n = \hat{\mathbf{x}}_n(\mathbf{y})$. Let $\hat{P}_k : \Gamma_{\mathrm{G},k} \to \Delta_{\mathrm{G},k}$ be the best response mapping of genco k to the energy price y_k given user demands, i.e., $P_k = \hat{P}_k(y_k)$. More formally, we can define SE solutions as follows.

Definition 1 (Stackelberg Equilibrium): A strategy pair $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{P}^*) \in \Gamma_{\mathrm{R}} \times \Gamma_{\mathrm{G}} \times \Delta_{\mathrm{G}}$ is a SE strategy for the three-level Stackelberg game if the following inequalities hold

$$U_{\mathrm{G},k}(\hat{P}_k(y_k), \mathbf{y}^*, \hat{\mathbf{x}}(\mathbf{y}^*)) \;\; \geq \;\; U_{\mathrm{G},k}(\hat{P}_k(y_k^*), y_k, \mathbf{y}_{-k}^*, \hat{\mathbf{x}}(y_k; \mathbf{y}_{-k}^*)), \quad \forall k \in \mathscr{K},$$

for all $y_k \in \Gamma_{\mathrm{G},k} \cap \Gamma_C(\mathbf{y}_{-k}^*)$ and $\mathbf{x}_n \in \Gamma_{\mathrm{R},n}$, where $\mathbf{y}_{-k}^*$ is the pricing strategies of all gencos other than k at SE, respectively; $\hat{\mathbf{x}}(\mathbf{y}^*)$ is the optimal response of all users for the price vector $\mathbf{y}^*$; $\Gamma_C(\mathbf{y}_{-k}^*)$ is the projected feasible set under coupled constraints when $\mathbf{y}_{-k}^*$ is fixed. The optimal response of user $n \in \mathscr{N}_{\mathrm{R}}$ for given $(y_1, y_2, \ldots.., y_K) \in \Gamma_{\mathrm{G},1} \times \Gamma_{\mathrm{G},2} \times \ldots.. \times \Gamma_{\mathrm{G},K}$ is given by

$$\hat{\mathbf{x}}_n(\mathbf{y}) = \{\mathbf{x}_n, \zeta_{\mathrm{R},n} \in \Gamma_{\mathrm{R},n}; U_{\mathrm{R},n}(\mathbf{y}, \mathbf{x}_n) \geq U_{\mathrm{R},n}(\mathbf{y}, \zeta_{\mathrm{R},n})\}.$$

and the optimal response of the power generation of genco k to the price y_k given user demands is

$$\hat{P}_k(y_k) = \{P_k, \xi_k \in \Delta_{\mathrm{G},k}; U_{\mathrm{G},k}(P_k, y_k, \mathbf{y}_{-k}, \mathbf{x}) \geq U_{\mathrm{G},k}(\xi_k, y_k, \mathbf{y}_{-k}, \mathbf{x})\}$$

To characterize the SE for the game described in Section IV-A and Section IV-B, the optimal response functions $\hat{\mathbf{x}}$ of users is obtained using (13), the best responses $\mathbf{y}^*$ of the gencos are calculated by solving (20), and the optimal solution P_k^* for given $\mathbf{y}^*$ and $\mathbf{x}^*$ is found in (17). Hence, the SE for the three-level game is given by (13) from Section IV-A, (22), (17) from Section IV-B.

Theorem 2: A unique Stackelberg equilibrium exists in the hierarchical Stackelberg game.
Proof: Given in Appendix B. ■

D. Flexibility and applicability of the proposed scheme

Despite fixed demands from industrial users and flexible demands from residential users in our analysis, the model can accommodate industrial users who have some flexible demands and residential users who have a minimum power requirement in addition, with only slight changes to some variables. Let us consider the case when the industrial users have some flexible demands too. We divide the demand of the industrial users into two parts: the strict power requirement (which is usually much higher than the the requirements of the residential users) and the flexible part for which a budget limit is assigned. Then, the problem can be analyzed and solved in the same way as for the case of fixed demands from industrial users, just by replacing $\mathscr{N}_{\mathrm{R}}$ by $\{\mathscr{N}_{\mathrm{R}}, \mathscr{N}_{\mathrm{I,flex}}\}$, where $\mathscr{N}_{\mathrm{I,flex}}$ is the set of industrial users with some flexible power demand, and $N_{\mathrm{I,flex}} = |\mathscr{N}_{\mathrm{I,flex}}|$. The strict demand for the minimum power

requirement from the residential users will add an additional constraint to $\mathrm{OP}_{\mathrm{user}}$. Fortunately, this constraint will not alter the analysis for calculating the optimal demands of the residential users.

For a residential user the minimum power requirement is much less compared to that of the industrial users and thus the budget is the key factor that decides its demands. On the other hand, an industrial user might like to consume more (less) power when it is cheap (expensive). However, the driving factor behind the operation of industrial users is the minimum power requirement, not the budget limit for the flexible power consumption. Therefore, despite the fact that both types of users can have some strict and some flexible power requirement, it is important to note that the user-differentiation is necessary and useful to provide meaningful insights.

Thus, our proposed scheme is very generic and applicable for a large range of scenarios.

V. Distributed Algorithm

In the previous section, finding the SE of the game needs information exchange among gencos about the cost of their power generations, in addition to their communication with consumers. In this section, we design a distributed algorithm that leads to the SE of the game without each genco knowing the parameters of other gencos.

For the distributed algorithm, subscript t is appended to all variables defined in the previous sections, as the time index, i.e., $t \geq 1$ indicates the iteration number. Each genco starts with an arbitrary price $y_{k,1}$ and all of them send their price information to the consumers. This communication is enabled by the advanced metering infrastructure (AMI) between the gencos and the consumers. The power demand of the industrial users is fixed and is known to the gencos. Each residential user decides how much to buy from each genco $\{x_{\mathrm{R},n,k,1}\}$ using (13). The gencos get this demand matrix from all users. Then, one of the generators will calculate the difference between the optimal power generation for the given unit price for iteration t, i.e., $\frac{y_{k,t}-b_k}{2a_k}$ and the total power demanded from it by all users, i.e., $\sum_{n\in\mathcal{N}_{\mathrm{R}}} x_{n,k,t} + P_{k,\mathrm{b}}$. Then, it will update its unit price for the $t+1^{th}$ iteration using

$$y_{k,t+1} = y_{k,t} + \frac{\sum_{n\in\mathcal{N}_{\mathrm{R}}} x_{\mathrm{R},n,k,t} - (\frac{y_{k,t}-b_k}{2a_k} - P_{k,\mathrm{b}})}{\sigma_k}, \tag{24}$$

where t is the iteration number and σ_k is the speed adjustment parameter of genco k, which is a sufficiently large number. Whenever a genco updates its price, it sends this information to the users. The users again, update their demands vectors and inform the gencos. Then, other

gencos will update their prices sequentially at alternative turns between users and gencos. This process continues until the price values converge.

We name this algorithm **Algorithm** 1 and is shown in Table I.

TABLE I

DISTRIBUTED ALGORITHM FOR ITERATIVE UPDATES OF POWER GENERATION, PRICE AND CONSUMER DEMAND (ALGORITHM 1)

1: For t=1, arbitrarily choose $y_{k,1}$ $\forall k \in \mathscr{K}$.
2: Repeat for t=2,3,.....
3: User $n = 1, 2, \ldots, N_{\mathrm{R}}$
4: **Do**
5: Find $\mathbf{x}_{\mathrm{R},n,t}$ by solving (2) for given $\mathbf{y}_t := \{y_{1,t}, y_{2,t}, \ldots, y_{K,t}\}$ and $\mathbf{x}_{\mathrm{R},n,t} := \{x_{\mathrm{R},n,1,t}, x_{\mathrm{R},n,2,t}, \ldots x_{\mathrm{R},n,K,t}\}$.
6: Transmit $x_{\mathrm{R},n,k,t}$ to each genco.
7: **end**
8: Generator $k \in \mathscr{K}$, which has not updated the price for iteration $t+1$, calculate $y_{k,t+1}$ using (24).
9: **If** $y_{k,t+1} - y_{k,t} = 0$,
10: Send a no-change signal to all consumers.
11: **If** any genco has not updated the price for iteration $t+1$,
12: Go to 8.
13: **end** 14: **else**
15: Send the new value of price to all consumers.
16: Go to 3.
17: **end**
18: **If** $y_{k,t+1} = y_{k,t}$ $\forall k \in \mathscr{K}$,
19: stop.
20: **else**
21: Go to 2.
22: **end**.

Theorem 3: Let $D_1 := 2a_k B_{\mathrm{R}} \sum_{g \in \mathscr{K}, g \neq k} y_g$ and $D_2 := \left[2a_k C_{\mathrm{R}} - K(b_k - 2a_k(K-1)B_{\mathrm{R}} - 2a_k P_{k,\mathrm{b}})\right] y_k$. Provided that

$$\sigma_k > \frac{K y_{k,t}^2 - D_2 y_k + D_1}{2a_k K y_{k,t}^2}, \tag{25}$$

Algorithm 1 converges to the optimal solutions for both the users and the gencos if the individual strategies are updated sequentially.

Proof: Given in Appendix C. ■

VI. Large Population Regime

We have derived the closed form solutions for the unit price of the gencos. However, since there are many variables involved, it may be hard to relate the prices with the variation of the important parameters in all situations. In addition, the behavior of the involved entities can be very different in different regions as the dominating parameters change. For instance, in residential zones, either residential consumers are the only electricity users, or their number is dominant over the number of commercial or industrial consumers. On the other other hand, in industrial zones, either there are no residential users, or there are few residential users, making the impact of the change in residential users' parameters negligible. It is simpler to relate and easier to understand the interdependencies if we separate the regions with noticeably different behavioral patterns of the involved entities. Such analysis is of paramount importance for practical implementation of the proposed schemes in different types of zones. In this section we introduce large-population approximations which simplify the expressions and provide some more insights into the behavior dynamics of the entities involved.

A. Large Population Approximations

For the case of homogeneous generators, $y_k = y, \forall k \in \mathscr{K}$. Consequently, (20) can be written as

$$Ky^2 - K\left(b + 2P_{\mathrm{b}}\right)y - 2a\bar{C}N_{\mathrm{R}} = 0 \tag{26}$$

where $\bar{C} = \sum_{n\in\mathscr{N}_{\mathrm{R}}} \frac{C_{\mathrm{R},n}}{N_{\mathrm{R}}} = \frac{C_{\mathrm{R}}}{N_{\mathrm{R}}}$. The unique feasible solution of (26) is

$$y = \frac{T_3}{2} + \frac{1}{2}\sqrt{T_3^2 + T_4}, \tag{27}$$

where $T_3 := (b + 2aP_{\mathrm{b}})$ and $T_4 := \frac{8aN_{\mathrm{R}}\bar{C}}{K}$. The generator coefficients a and b are small numbers. For given $a, b, \bar{C}$, the value of T_3 depends on P_{b}. P_{b} can be written as $P_{\mathrm{b}} = N_{\mathrm{I}}D_I$, where D_I represents the average power demand of an industrial user from one genco. Therefore, P_{b} is large either if N_{I} is too large or if N_{I} is low or moderate but if D_I is too large. On the other hand, for given a and $\bar{C}$, the value of T_4 is governed by N_{R}. Thus, N_{I} and N_{R} are two important parameters in understanding the price-setting behavior of the gencos. We further divide our analysis into the following cases.

1) Case 1: When N_R is very large: This scenario is applicable either in completely residential areas or in bigger areas for instance, a district or a county, where the density of residential users is usually much larger than the density of industrial and commercial users. We consider an oligopolistic market scenario i.e., $K \ll N_{\mathrm{R}}$. When the number of residential users is very high, since $K \ll N_{\mathrm{R}}$, $\frac{N_{\mathrm{R}}}{K}$ is also arbitrarily large. If $N_{\mathrm{R}} \gg \frac{K(b+2aN_{\mathrm{I}}D_{\mathrm{I}})^2}{8a\bar{C}}$, $T_3^2 \ll T_4$. In this case, $T_4 + T_3^2 \approx T_4$. Hence, (27) can be written as

$$y \approx \frac{1}{2}\sqrt{T_4} = \sqrt{\frac{2aN_{\mathrm{R}}\bar{C}}{K}}, \tag{28}$$

i.e., for given power generation cost parameters a, b and users' parameter $\bar{C}$, the unit price y is a function of $\sqrt{N_{\mathrm{R}}}$.

When the gencos are homogeneous, we replace y_k and $x_{\mathrm{R},n,k}$ in (13) by y and $x_{\mathrm{R},n}$, respectively. Then, the demand of residential user n from a genco can be expressed as

$$x_{\mathrm{R},n} = \frac{C_{\mathrm{R},n}}{Ky}, \ \forall n \in \mathscr{N}_{\mathrm{R}}, \ \forall k \in \mathscr{K}. \tag{29}$$

Substituting y from (28) into (29) yields

$$x_{\mathrm{R},n} = \frac{C_{\mathrm{R},n}}{\sqrt{2aKN_{\mathrm{R}}\bar{C}}}, \ \forall n \in \mathscr{N}_{\mathrm{R}}. \tag{30}$$

For the user utility maximization problem defined in section IV, we introduced a positivity constraint on the demand of the residential users i.e., $x_{\mathrm{R},n,k} \geq 0, \forall k \in \mathscr{K}, \forall n \in \mathscr{N}_{\mathrm{R}}$. In practice, even though the residential users are willing to adjust their electric power consumption to reduce their electricity bills, they have certain minimum requirement for their non-shiftable appliances. Let $x_{\mathrm{th},n}$ be the threshold power of user n i.e., the minimum power needed from each genco. Now, we derive a necessary condition on the budget of residential users to satisfy their minimum demand.

User n needs to get at least $x_{\mathrm{th},n}$ from all gencos, i.e.,

$$\sum_{k \in \mathscr{K}} x_{\mathrm{R},n} \geq x_{\mathrm{th},n} \Rightarrow C_{\mathrm{R},n} \geq \sqrt{\frac{2aN_{\mathrm{R}}\bar{C}}{K}} x_{\mathrm{th},n}. \tag{31}$$

When all users are homogeneous, i.e., when they all have the same budget and the same minimum power requirement, the demand of all residential users is $x_{\mathrm{R}} = \sqrt{\frac{\bar{C}}{2aKN_{\mathrm{R}}}}$, $\forall n \in \mathscr{N}_{\mathrm{R}}$. Subsitituting $C_{\mathrm{R},n} = \bar{C}, x_{\mathrm{th},n} = x_{\mathrm{th}}, \forall n \in \mathscr{N}_{\mathrm{R}}$ into (31), the minimum required budget of each residential user can be written as

$$\bar{C} \geq \frac{2aN_{\mathrm{R}}}{K} x_{\mathrm{th}}^2$$

2) Case 2:$N_I \gg N_R$ or D_I is arbitrarily large : This scenario is applicable in heavily industrialized areas, where the number of industrial and/or commercial users is many folds higher than the number of residential users, or the number of industrial users is few/moderate but their power requirements are very high. In this case, P_b is arbitrarily large (but less than P_max). If $N_\mathrm{I}D_\mathrm{I} \gg \frac{\sqrt{8N_\mathrm{I}a\bar{C}}-b\sqrt{K}}{2a\sqrt{K}}$, $T_4 \ll T_3^2$. This scenario is not likely to be the case where small or medium scale gencos are involved. However this can be the case if some big gencos with extremely large capacity are serving large or many industrial users. In this case $T_4+T_3^2 \approx T_3^2$. Thus the solution of (27) takes the form

$$y \approx T_3 = (b+2aP_\mathrm{b}). \tag{32}$$

The industrial users' parameters will dominantly decide the unit price. This is a very interesting case. Although the industrial users' demands are fixed and they do not participate in the game directly, when there is a large population of industrial users or when there are few but large industries, their demands will decide the unit price of the gencos.

In this case, the demand of residential user n from a genco can be obtained by substituting y from (32) into (29), giving

$$x_{\mathrm{R},n} = \frac{C_{\mathrm{R},n}}{K(b+2aP_\mathrm{b})}, \ \forall n \in \mathscr{N}_\mathrm{R}.$$

With a similar approach as in section VI-A1, the minimum budget for user n to get at least $x_{\mathrm{th},n}$ amount of power, is

$$C_{\mathrm{R},n} \geq (b+2aP_\mathrm{b})x_{\mathrm{th},n}, \ \forall n \in \mathscr{N}_\mathrm{R}.$$

In this case, for the case of homogeneous users too, the required minimum budget is $\bar{C} \geq (b+2aP_\mathrm{b})x_\mathrm{th}$.

B. Sensitivity Analysis

When the gencos are homogeneous, they all set the same unit price and produce the same amount of power i.e., $P_k = P, y_k = y, \ \forall k \in \mathscr{K}$. The power available for residential users is $P_\mathrm{R} = P - P_\mathrm{b}$. We investigate the following two cases:

1) Case 1: $N_R \gg N_I$: Let us start our analysis with the impact of different parameters on the supply side. For a given small value of P_b, using approximation (28), the variation of y w.r.t. the residential users' parameter $\bar{C}$, is

$$\frac{\partial y}{\partial \bar{C}} = \sqrt{\frac{aN_\mathrm{R}}{2K\bar{C}}}.$$

This implies that if the average budget of residential consumers $\bar{C}$ increases, the gencos will increase their unit price also. Similarly, the variation of y due to a change in the generator parameter a is

$$\frac{\partial y}{\partial a} = \sqrt{\frac{N_{\mathrm{R}}\bar{C}}{2Ka}}.$$

This implies that if the quadratic cost factor of the generators increases, the gencos will increase their unit price. Differentiating (30) w.r.t. $\bar{C}$ and a gives

$$\frac{\partial x_{\mathrm{R},n}}{\partial \bar{C}} = -\frac{C_{\mathrm{R},n}}{(aKN_{\mathrm{R}})^{\frac{1}{2}}(2\bar{C})^{\frac{3}{2}}}, \frac{\partial x_{\mathrm{R},n}}{\partial a} = -\frac{C_{\mathrm{R},n}}{(KN_{\mathrm{R}}\bar{C})^{\frac{1}{2}}(2a)^{\frac{3}{2}}}.$$

The demand of a residential user shows an inverse relationship to the average budget of the residential population if its own budget is not equal to the average budget. Thus, the increase in $\bar{C}$ makes a user demand less, and vice versa. The demands also vary inversely w.r.t. the power generation cost coefficient a. It is natural that if the cost of production increases, the commodity becomes more expensive. Hence a user with the same budget will be able to buy less.

When the residential users are homogeneous, the variations in their demand x_{R} w.r.t. $\bar{C}$ and a are as follows:

$$\frac{\partial x_{\mathrm{R}}}{\partial \bar{C}} = \frac{1}{2\sqrt{2aKN_{\mathrm{R}}\bar{C}}}, \frac{\partial x_{\mathrm{R}}}{\partial a} = -\frac{1}{(2a)^{\frac{3}{2}}}\sqrt{\frac{\bar{C}}{KN_{\mathrm{R}}}}. \tag{33}$$

From (33), we observe that the demands will increase if $\bar{C}$ increases. This difference in the pattern of variation of $x_{\mathrm{R},n}$ and x_{R} w.r.t. $\bar{C}$ is an interesting fact. In the case of homogeneous residential users, an increase (decrease) in the average budget of the residential population actually means the increase (decrease) in every user's budget. While in the case of heterogeneous consumers, the increase in the average budget when one's own budget has not changed, implies a relative decrease in its budget w.r.t. the average residential budget. Because of this difference, the influence on users' demands in the two cases are different. The influence of the power generation cost factor a in the case of homogeneous users, is similar to the case of heterogeneous users.

2) Case 2:$N_I \gg N_R$ or D_3 is arbitrarily large : For a given small value of N_{R}, when $N_{\mathrm{I}} \gg N_{\mathrm{R}}$ or D_3 is arbitrarily large, using approximation (32), the variation of y w.r.t. the generator parameter b is

$$\frac{\partial y}{\partial b} = 1.$$

This implies that the unit price of the gencos varies linearly with the cost coefficient b. Note that y does not change with a in this case. If the average budget $\bar{C}$ changes, the impact will not be reflected in the unit price i.e., when the industrial users dominate, the system equilibrium is not influenced by the changes in the residential users' parameters. It is to be noted that when the population of residential users is large, the unit price will still be affected by the demand from the industrial users, P_{b}. If P_b starts increasing, the unit price and hence the demand from the residential users will also be affected even for very large N_{R}.

The variation of $x_{\mathrm{R},n}$ w.r.t. b is

$$\frac{\partial x_{\mathrm{R},n}}{\partial b} = -\frac{C_{\mathrm{R},n}}{K(b+2aP_{\mathrm{b}})^2}. \tag{34}$$

Equation (34) indicates that the demand from residential user n will decrease (increase) if b becomes higher (lower).

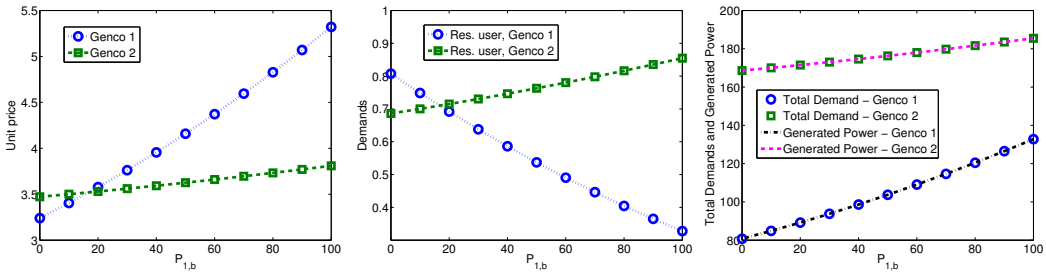
VII. Numerical Results

In this section, we examine how the users determine their optimal demands based on the unit prices of the gencos and how the gencos optimize their unit prices and the power to sell based on the users' budgets. This section is divided into three parts. In the first part, we show the results for the SE of the game for heterogeneous gencos. The second section depicts the convergence of the distributed algorithm to the equilibrium. The third part illustrates some results for the large population regime.

Although the results are affected by the choice of the variables, for illustration purpose, we have chosen 2 gencos and 100 residential users with parameters $\alpha_n = 1, \beta_{\mathrm{R},n} = 1,\ \forall n \in \mathcal{N}_{\mathrm{R}}$, and 2 industrial users. Note that, since, we have derived closed form solutions for the optimal demands, unit prices and power, using $K > 2$ is trivial. The reason behind choosing $K = 2$ is so that we can show different curves for different gencos. Other parameters used were $\bar{C} = 5,\ a_1 = 0.02, a_2 = 0.01, b_1 = 0.01, b_2 = 0.1, c_1 = 0.001, c_2 = 0.002, P_{2,\mathrm{b}} = 100)$, unless mentioned otherwise.

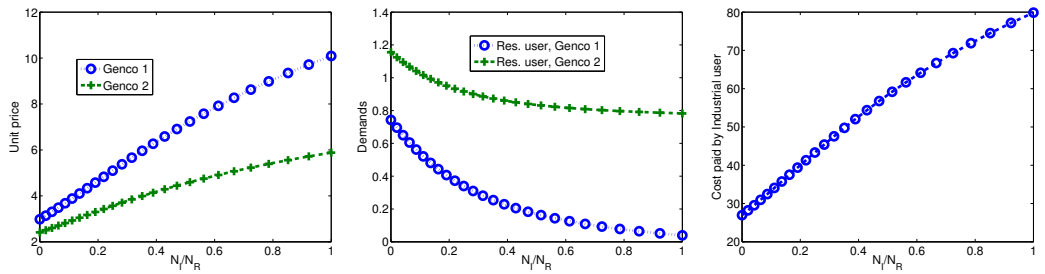
A. Stackelberg Game

Figs. 3(a)-3(c) show the unit price set by the two generators, user demands and the total power sold by the gencos, respectively, at SE when the power demand of the industrial users from genco 1 varies as indicated by the x-axis in these figures. From Fig. 3(a), we see that when the power demand of the industrial users from genco 1 increases, the unit price of



(a) Unit price (b) Demand from the residential users (c) Total Demand, Generated Power

Fig. 3. Unit price, Demands and Power Generated ($a_1 = 0.02, a_2 = 0.01, b_1 = 0.01, b_2 = 0.1, c_1 = 0.001, c_2 = 0.002, P_{2,\mathrm{b}} = 100$)



(a) Unit price (b) Demand from the residential users (c) Cost paid by industrial users

Fig. 4. Unit price, demands and cost when the population of industrial users increases ($a_1 = 0.02, a_2 = 0.01, b_1 = 0.01, b_2 = 0.1, c_1 = 0.001, c_2 = 0.002$)

that company goes up. Although, the demand from genco 2 has not increased, the unit price charged by it rises as well because of the competition between the gencos. Consequently, the demand of the residential users from genco 1 decreases. Even though the unit price of genco 2 has also increased, the increment is relatively less, thus the demand of the residential users from genco 2 increases, as illustrated in Figure 3(b). Fig. 3(c) shows that the total demands from the gencos (of both residential and industrial consumers) and the power generated are exactly equal, indicating that the companies can generate an optimal amount of power and gain an optimal profit by selling all of it. This figure indicates that the scheme helps to balance the total supply and the total demand in the market, and also the supply and the

demand of each individual genco.

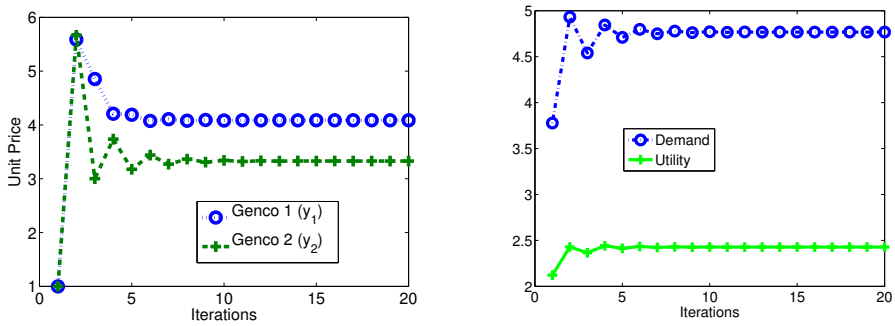
Figs. 4(a)-4(c) illustrates the effect of the residential and industrial users on the genco strategies and on each other. For these plots we used $N = 50$, N_{R} and N_{I} vary from 100 to 50 and 0 to 50, respectively. Each industrial user consumes 10 units of power. From Fig. 4(a) we observe that the unit prices of both the gencos increase as the ratio $\frac{N_{\mathrm{I}}}{N_{\mathrm{R}}}$ increases, despite the total number of consumers, budget of residential users and the demand of each industrial user being constants. The reason is that the reduction in unit price caused by the decreasing number of residential users is far too little to compensate the increase in price caused by the increased demand from the total number of industrial users. The demands of the residential users are lowered as illustrated in Fig. 4(b), which is the impact of the industrial users on them. In Fig. 4(c) we observe that the cost paid by each of industrial user increases if their number increases, despite the demand of each industrial user being fixed.

B. Convergence of the Distributed Algorithm

Fig. 5(a) shows the convergence of Algorithm 1 for determining the parameters of the gencos. Fig. 5(a) shows the unit price of the gencos. The convergence is very quick, within 10 iterations. Since y_k is the key parameter based on which other generator parameters can be computed, it is clear that the total power generated and the utilities of the gencos will also converge. Fig. 5(b) depicts the convergence of the residential users' parameters using the distributed algorithm. From this figure it is clear that both the average demands of residential users and their average utility converge within about 10 iterations.

C. Large Population Regime

We compare the actual prices and the approximations for the case of homogeneous gencos in Figs. 6(a)-6(c), when $P_b = 100$, $P_b = 5000$ and $P_b = 100000$, respectively. The x-axis represents the number of residential users in log scale ($\log_{10} N_{\mathrm{R}}$) and the y-axis represents the unit price set by the gencos in log scale ($\log_{10} y$). As indicated by Figure 6(a), when the demand from industrial users is not heavy, the approximation $T_4 >> T_3^2$ (dash-dash curve) overlaps with the actual curve (solid line) for $\log_{10} N_{\mathrm{R}} > 3$. The dashed-dotted line is the approximation where the influence of the demand from the industrial users is the main factor in deciding the price. But when $P_{\mathrm{b}} = 100$, the large population of residential users have a significant impact in the price setting game. In Figure 6(b), we see that the dashed-dashed curve gets closer to the solid curve after $\log_{10} N_{\mathrm{R}} > 5$. From the point where $\log_{10} N_{\mathrm{R}} \leq 3$, the

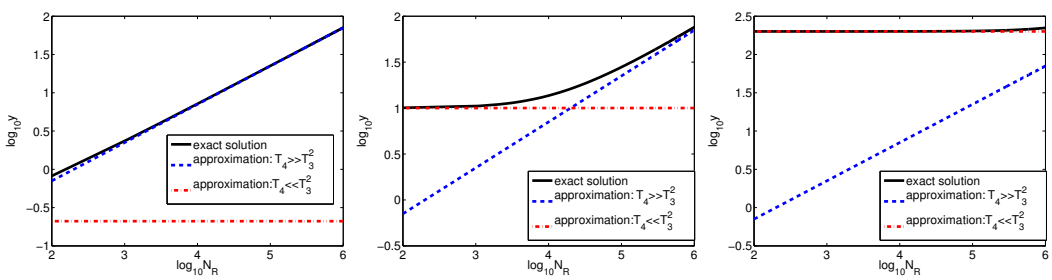


(a) Unit price of the gencos

(b) [Demand and utility of the residential users

Fig. 5. Parameters of the gencos and the residential users ($a_1 = 0.02, a_2 = 0.01, b_1 = 0.01, b_2 = 0.1, c_1 = 0.001, c_2 = 0.002, P_{1,\mathrm{b}} = 50, P_{2,\mathrm{b}} = 75, \sigma = 60$)

dashed-dotted curve and the exact solution overlap. This is the area where the impact of the residential population is not comparable to the given demand from the industrial consumers. In Figure 6(c), we observe similar behavior but it is evident that the dashed-dotted curve now covers a large part of the solid curve and the dashed-dashed curve does not come close to the actual price. This is an interesting fact. When the population of industry users becomes very large or the demand from the existing industrial users increases heavily, then parameters of the residential users will not impact the unit price setting of the gencos, even for a fairly large residential population.



(a) $P_\mathrm{b} = 100$

(b) $P_\mathrm{b} = 5000$

(c) $P_\mathrm{b} = 100000$

Fig. 6. The unit price for three different values of $P_b (a = 0.001, b = 0.01, c = 0.01)$

Figure 7 depicts the minimum required budget of the residential users for getting a supply

of x_{th}, which is represented by the x-axis of the plot. The figure consists of four lines: The solid line and the dashed line with 'o' markers, represent the exact and approximated required budgets, respectively, when $P_{\text{b}} = 100$ for a residential population of 1000 users. The dotted line and the dashed-dashed line with '+' markers are the exact and approximated budget requirements, when $P_{\text{b}} = 10000$. The figure clearly shows that the approximations are very close to the exact values. The interesting point in this figure is the difference in minimum $\bar{C}$ for the two cases. When the minimum power requirement of a residential user is 1 unit, the minimum budget required in the two cases are 4.472 and 40.02. The big difference is a result of the difference in P_{b}, and the number of residential users N_{R}. When $P_{\text{b}} = 10000$, the power to be produced by each genco P is very large. Consequently the unit price increases drastically, which explains why the users need to have such a large budget even for a small power requirement, in this case. Furthermore, the difference in the amount of power allocations may seem huge. However, it should be noted that P_b is the total amount of power for all industrial users from one genco, while x_{th} is the power requirement of one out of 1000 residential users.

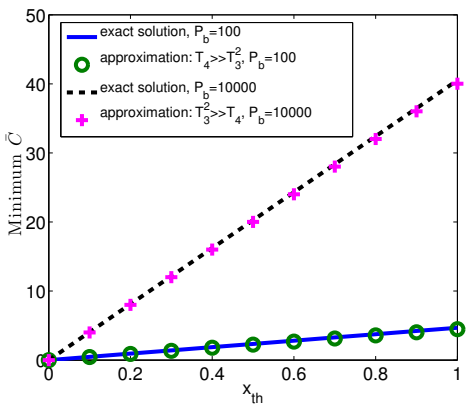


Fig. 7. Minimum budget required for residential users ($a = 0.001, b = 0.01, c = 0.001, N_{\text{R}} = 10^3$)

VIII. Conclusion

In this paper, we have proposed a hierarchical system model incorporating multiple processes from power generation to market activities and to power consumption, into one framework. We have modeled the decision processes of both supply and demand sides

as a multi-leader multi-follower Stackelberg game, where the gencos act as the leaders to maximize their profits, and the residential users are the followers, who optimally respond to the strategies of the gencos. In our model, we also integrated differentiation to residential and industrial users. We have developed analytical results for the Stackelberg equilibrium of the game and have proved that a unique Stackelberg equilibrium exists. We have also designed a distributed algorithm for the gencos to determine their unit prices without communicating with other gencos, and for the users to determine their optimal demand response with only price information available to them. Furthermore, we have extended our analysis to obtain large-population approximations for specific cases and presented insights into the behavior dynamics of the gencos and the users. Finally, we have provided extensive numerical results to illustrate the performance of the proposed solutions, algorithms and approximations.

APPENDIX A
PROOF OF THEOREM 1

When $a_k = a, b_k = b, c_k = c, P_{k,\mathrm{b}} = P_b$ and $y_k = y, \forall k \in \mathscr{K}$, (20) takes the form

$$Ky^2 - K\left(b + 2aP_{\mathrm{b}}\right)y - 2C_{\mathrm{R}}a = 0.$$

Solving for y, we get

$$y = \frac{K(b+2P_{\mathrm{b}}a) \pm \sqrt{K^2(b+2P_{\mathrm{b}}a)^2 + 8C_{\mathrm{R}}Ka}}{2K}. \tag{35}$$

Here, $K(b+2P_{\mathrm{b}})a > 0$ and $\sqrt{K^2(b+2P_{\mathrm{b}}a)^2 + 8C_{\mathrm{R}}Ka} > K(b+2aP_{\mathrm{b}})$. Therefore it is clear that y has a unique positive solution given by (21).

When the gencos have different parameters, the solution of (20) is $y_k = \frac{-T_1 \pm \sqrt{T_2}}{2K}$. Suppose the unit prices of all generators other than k ($\mathbf{y}_{-k}$) are positive. Then, we have the following cases.

When $\frac{2B_{\mathrm{R}}(K-1)a_k - Kb_k}{2Ka_k} \leq P_{k,\mathrm{b}} \leq P_{k,\max}$, $T_1 \leq 0$. Since $-T_1 < \sqrt{T_2}$, the root $y_k = \frac{-T_1+\sqrt{T_2}}{2K}$ yields a real positive value for y_k. The root $y_k = \frac{-T_1-\sqrt{T_2}}{2K}$ gives a real negative value for y_k, which is not a valid solution.

When $P_{k,\mathrm{b}} \leq \frac{2B_{\mathrm{R}}(K-1)a_k - Kb_k}{2Ka_k} \leq P_{k,\max}$, $T_1 \geq 0$. Since $-T_1 < \sqrt{T_2}$, the root $y_k = \frac{-T_1+\sqrt{T_2}}{2K}$ yields a real positive value for y_k. The root $y_k = \frac{-T_1-\sqrt{T_2}}{2K}$ gives a real negative value for y_k, which is not a valid solution.

Therefore, a unique positive solution always exists for y_k and is given by (22). ■

APPENDIX B
PROOF OF THEOREM 2

Given the prices $\mathbf{y}$ and power generations $\mathbf{P}$, the optimal responses $\hat{\mathbf{x}}$ to $\mathbf{y}$ can be determined by solving the strictly convex problem (2) in Section A. Given these unique responses, an equilibrium exists for the non-cooperative game between price setting gencos if the following conditions hold.

C1 $\mathbf{y}$ is a non-empty, convex, and compact subset of some Euclidean space $\mathscr{R}^K$.

C2 $U_{\text{gen},k}(\mathbf{y})$ is continuous in $\mathbf{y}$ and concave in $y_k, \forall k \in \mathscr{K}$.

In addition to positivity, there are tighter limits on y_k such that $y_k \in [y_{k,\min}, y_{k,\max}]$. The lower limit $y_{k,\min}$ is due to the associated generation costs. The gencos will not reduce their price below $y_{k,\min}$. The upper bound $y_{k,\max}$ can be fixed according to government standards. In the price selection game for the gencos, the strategy space is $\Gamma_{\text{gen}} = \Gamma_{\text{gen},1} \times \Gamma_{\text{gen},2} \times ..... \times \Gamma_{\text{gen},K}$ where $y_k \in \Gamma_{\text{gen},k} := [y_{k,\min}, y_{k,\max}]$, $\forall k \in \mathscr{K}$. Thus the strategy set is a nonempty, convex and compact subset of the Euclidean space $\mathscr{R}^K$. From (14), we see that $U_{\text{gen},k}$ is continuous in y_k. Next, the second order derivative of $U_{\text{gen},k}$ w.r.t. y_k is

$$\frac{\partial^2 U_{\text{gen},k}}{\partial y_k^2} = -2a_k < 0, \ \forall k \in \mathscr{K}. \tag{36}$$

Hence, $U_{\text{gen},k}(\mathbf{y})$ is concave in y_k. Note that the genco utility function is not dependent on the generation of other gencos when $\mathbf{y}$ and $\mathbf{x}$ are fixed, and it is strictly concave in P_k, which leads to a unique solution in (17). Since the optimal response of users are given by (13), using (C1) and (C2), we can conclude that a SE exists for the game.

As proven in Section IV-B, there exists only one positive set of solutions for the price selection game given by (22). Therefore the equilibrium for the gencos price setting game is unique and hence the solutions (22),(13) is the unique Stackelberg equilibrium of the hierarchical DRM scheme. ■

APPENDIX C
PROOF OF THEOREM 3

For iteration $t+1$, if $\sum_{n \in \mathscr{N}_{\text{R}}} x_{\text{R},n,k,t} + P_{k,\text{b}} < (\frac{y_{k,t} - b_k}{2a_k})$, then $|\sum_{n \in \mathscr{N}_{\text{R}}} x_{\text{R},n,k,t} - (\frac{y_{k,t} - b_k}{2a_k} - P_{k,\text{b}})| < y_{k,t}$ is the sufficient condition for $y_{k,t+1}$ not to acquire a negative value. Upon simplification, this condition takes the form (25).

The users' response (13) is the optimal response to given $\{y_k, \forall k \in \mathscr{K}\}$. The demands of the residential users, and the power generation of the gencos will converge to a fixed

set once the price set converges to a fixed point. Consequently, it is sufficient to show the convergence of the price vector to prove the convergence of Algorithm 1. Eqn. (24) implies that the price y_k of generator k decreases if $\sum_{n\in\mathscr{N}_{\mathrm{R}}} x_{n,k,t} - (\frac{y_{k,t}-b_k}{2a_k} - P_{k,\mathrm{b}})$ is negative and the price increases if $\sum_{n\in\mathscr{N}_{\mathrm{R}}} x_{n,k,t} - (\frac{y_{k,t}-b_k}{2a_k} - P_{k,\mathrm{b}})$ is positive. Equation (24) shows that when $\sum_{n\in\mathscr{N}_{\mathrm{R}}} x_{n,k,t} - (\frac{y_{k,t}-b_k}{2a_k} - P_{k,\mathrm{b}}) = 0$, the price will remain unchanged. This is the fixed point to which Algorithm 1 converges. The prices corresponding to stock clearance are the prices that maximize the generator profit. Therefore, if $\{y_{k,T}, \forall k \in \mathscr{K}\}$ be the prices at iteration T such that $\sum_{n\in\mathscr{N}_{\mathrm{R}}} x_{\mathrm{R},n,k,T} - P_k = 0$, then, $U_{\mathrm{gen},k}(y_{k,T}, \mathbf{y}_{-k,T}) \geq U_{\mathrm{gen},k}(y'_{k,T}, \mathbf{y}_{-k,T})$ where $U_{\mathrm{gen},k}(y_{k,T}, \mathbf{y}_{-k,T})$ and $U_{\mathrm{gen},k}(y'_{k,T}, \mathbf{y}_{-k,T})$ are the utilities of genco k at price $y_{k,T}$ and $y'_{k,T}$ respectively. Thus the fixed point to which Algorithm 1 converges is the NE of the game among the gencos i.e., SE of the game between the gencos and the consumers. Thereafter, the gencos will not deviate from this point. ■

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Paper VI

User-Centric Demand Response Management in the Smart Grid with Multiple Providers

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User-Centric Demand Response Management in the Smart Grid with Multiple Providers

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Abstract

The smart grid is the next generation power grid with bidirectional communications between the electricity users and the providers. Demand Response Management is vital in the smart grid to reduce power generation costs as well as to lower the users' electricity bills. In this paper, we introduce multiple fossil-fuel and multiple renewable energy sources on the supply side, and propose an end-user oriented generator selection scheme to minimize user costs. We formulate the problem as a non-cooperative game, incorporating the uncertainty associated with the power supply of the renewable sources, and prove that there exists a Nash equilibrium for the game. We develop a joint scheme, to further reduce users' costs by integrating shiftable load scheduling with generator selection. We model the joint scheme also as a non-cooperative game, and prove the existence of a Nash equilibrium for the game. For both schemes, we propose distributed algorithms for the users to find the equilibrium of the game using only local information. We evaluate our schemes and compare their performances to two other approaches. The results show that our joint generator selection and load scheduling scheme incurs the least cost to the users.

I. Introduction

A smart grid is an intelligent electricity network that makes use of advanced information, control, and communication technologies to save energy, reduce cost and enhance reliability and transparency [1] of the grid. Demand response (DR) is the response system of the end-users to manage their consumption of electricity in response to supply conditions. In the smart grid, bidirectional communications between energy providers and end users, will greatly enhance demand response capabilities of the whole system. E.g., consumers will be able to control and manage their electricity usage, and the providers can plan and manage the generation and supply of power in a more structured and efficient manner. Thus, power

generation, distribution and consumption are envisaged to be more efficient, more economical and more reliable in the smart grid.

With the internationalization of energy market and the incorporation of new renewable energy sources (e.g., wind and solar power) in the smart grid, users will have more options in terms of choosing providers or the type or energy source they use. With this motivation, we include multiple renewable energy generators on the supply side, in addition to the traditional fossil fuel based generators. However, incorporating renewable energy sources brings uncertainty to the supply side. While existing literature has focused mainly on minimizing the power generation cost of a single provider by scheduling the shiftable load of the end-users to different time-slots, we introduce a generator selection approach for demand response management (DRM), to minimize the costs of the users, considering the uncertainty about the states of the renewable energy sources. To this end, in order to reduce the costs of the users further, we develop a joint scheme by combining generator selection and conventional load scheduling schemes. This joint scheme is very generic, and is useful for residential, commercial as well as industrial electricity consumers.

The cost of a user depends not only on its own demand and scheduling, but also on the demands and scheduling of other users, who prefer the same provider. Moreover, in a multiple provider scenario which we focus on, the cost of each user varies according to the cost of power generation of many sources, and the selection of the sources by other users, thus making the interactions between users and sources, and among users, complicated. This complicated coupling makes game theory an appealing approach to model users' costs in this scenario. In particular, since all users are selfish and their aim is to reduce their own costs, we model these interactions as a non-cooperative game among the users to schedule their demands by optimally choosing the generators and by optimally scheduling their shiftable appliances in order to minimize their payments to the providers, and we characterize the Nash equilibrium (NE) solution of the games.

Our contributions in this work are as follows.

1) We study DRM with multiple energy sources with a user-centric approach, incorporating uncertainty regarding the availability of renewable energy sources.
2) We propose a non-cooperative generator selection game among users, and develop a distributed algorithm to find the NE of the game using only local information.
3) We introduce a joint generator selection and load scheduling scheme as a non-cooperative game, to further improve the users' costs, and propose a distributed algorithm to reach

the NE of the scheme.

The rest of the paper is organized as follows: Related work is described in Section II. The system model that includes the generator availability model and the cost model, is decribed in Section III. In Section IV, we introduce a non-cooperative generator selection game among the users and design a distributed algorithm for the game to converge to the NE. In Section V, we develop a joint scheme by integrating shiftable load scheduling with the generator selection scheme, and propose a distributed algorithm to find the NE of the game using only local information. In Section VI we present and discuss our results and Section VII concludes the paper.

II. Related Work

There are several studies on DRM in the smart grid [3]-[9]. In [3], the authors formulated the energy consumption scheduling problem as a non-cooperative game among the consumers for increasing and strictly convex cost functions. In [4], the authors considered a distributed system where price is modeled by its dependence on the overall system load. Based on the price information, the users adapt their demands to maximize their own utilities. In [5], a robust optimization problem was formulated to maximize the utility of a consumer, taking into account price uncertainties at each hour. In [6], the authors have exploited the awareness of the end-users and proposed a method to aggregate and manage end-users' preferences to maximize energy efficiency and user satisfaction. In [9], a dynamic pricing scheme was proposed to incentivize costumers to achieve an aggregate load profile suitable for providers, and the demand response problem was investigated for different levels of information sharing among the consumers in the smart grid. We observe that [3]-[9] mainly focus on either only one source or a number of sources/providers treated as one entity, and the cost minimization problem was solved from the provider's perspective. Differently in our study, we include multiple renewable and non-renewable energy sources. We notice that there are few studies exploring DRM in the context of multiple energy sources [11] [13]. These two studies are based on the cost minimization of the supply side. In our study, we focus on minimizing the cost for each user. In our previous work [10], we introduced multiple providers who aim to maximize their revenues, and multiple consumers who aim to maximize their utilities. In [10], we focused our analysis on a static one time slot game. In this paper, however, we deploy a pricing scheme that is closely related to the cost of power generation, and we also introduce the temporal aspect of DRM in the joint scheme. Recently there has been

a drastic increase in the volume of work considering bateries as storage devices, or solar panels at many buildings to reduce the peak-to-average ratio of power demand, and thus the cost for the users [15], [16]. The problem we address is different. When each user has a battery or a solar panel, these alternatives are local for each user. In our study, the renewable energy sources are on the supply side and the same renewable sources can supply multiple users, which makes the problem more complicated. Moreover, there has been an increasing interest in integrating renewable energy sources into the grid with storage units [17], [18]. Our focus in this paper is different than the analyses in [17], [18]. While[17], [18] focus on the integration issues of renewable sources into the grid, and provide simulation and/or experiment based performance evaluation in the presence of storage units, we concentrate our analysis on the market level and provide user-centric approaches to get economic benefits by making optimal choices.

III. System Model

Fig. 1 shows the system model. There are N energy users, $\mathcal{N} := \{1, 2, ..., N\}$, and K different providers, $\mathcal{K} := \{1, 2, ..., K\}$. In our system model, each provider consists of one energy source. Thus, we use the terms providers, generators and sources interchangeably. Some of the energy sources are fossil-fuel based (which are the non-renewable sources), and some are renewable energy sources. Then, we have, $K = K_f + K_r$, where K_f and K_r are the number of fossil fuel generators and renewable energy sources, respectively. The daily energy consumption schedule of each user is divided into different time slots. Each user intends to select the source of energy that minimizes the cost it pays to the providers. In each time slot, each user can select only one generator, but in case of deficit power from a renewable source, the demand will be served by a fossil-fuel generator. There is bidirectional communication between the providers and the consumers.The consumers form a Neighborhood Area Network (NAN). The data from the users' side (demands and generator preferences), and from the providers' side (price) is exchanged through the Neighborhood Area Network Gateway (NAN-GW) as shown in Figure 1.

The fossil-fuel based energy sources are available all the time but the price they charge to the users will include the cost due to their pollution emission. On the other hand, the renewable energy sources are pollution free but they cannot provide a stable supply of power. There are two components associated with a generator model: the cost of power generation and the generator availability.

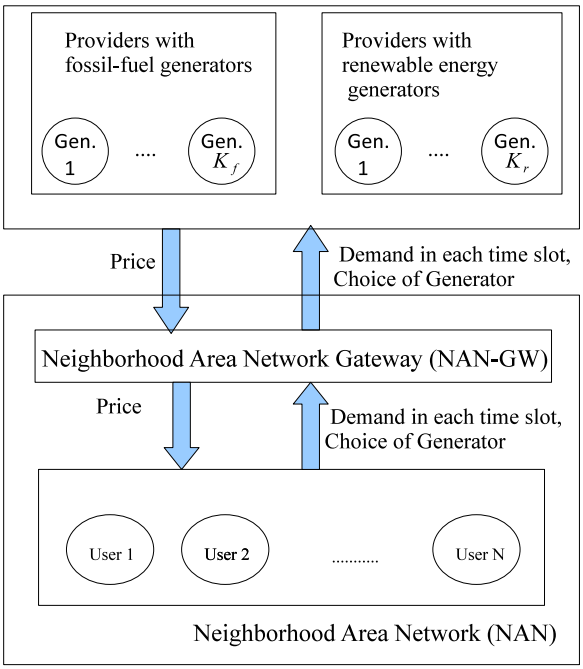

Fig. 1. Communication between the providers and the users

A. Cost of Power Generation

We choose the cost function of each generator such that the following properties are satisfied for both types of sources.

Property 1. *The cost functions and hence the prices charged to the users are increasing with higher demand for any time slot, i.e., if* $\overline{d}_{k,\text{tot}}, \tilde{d}_{k,\text{tot}} \geq 0$*, are the total demands from generator* k*,* $(k \in \mathcal{K})$ *such that* $\overline{d}_{k,\text{tot}} < \tilde{d}_{k,\text{tot}}$*, and if* $C_{gen,k}(\overline{d}_{k,\text{tot}})$ *and* $C_{gen,k}(\tilde{d}_{k,\text{tot}})$ *are the corresponding costs of power generation, then,* $C_{gen,k}(\overline{d}_{k,\text{tot}}) < C_{gen,k}(\tilde{d}_{k,\text{tot}}),\ \forall\ \overline{d}_{k,\text{tot}} < \tilde{d}_{k,\text{tot}},\ \forall k \in \mathcal{K}$*.*

Property 2. *The cost functions and hence the prices charged to the users are strictly convex in any time slot, i.e.,* $C_{gen,k}(\theta\overline{d}_{k,\text{tot}} + (1-\theta)\tilde{d}_{k,\text{tot}}) < \theta C_{gen,k}(\overline{d}_{k,\text{tot}}) + (1-\theta)C_{gen,k}(\tilde{d}_{k,\text{tot}})$*,* $\forall k \in \mathcal{K}$*, where* $\overline{d}_{k,\text{tot}}, \tilde{d}_{k,\text{tot}} \geq 0$*, and* $0 < \theta < 1$ *is any real number.*

Examples of cost functions that satisfy these properties can be the two-step conservation

rate model, quadratic cost functions, etc. These types of cost functions are of significant importance and are realistic too, since they have been deployed by many big power plants such as BC Hydro [3], and they can be closely related to the cost of power generation. In addition, when a pricing scheme closely related to the cost of generation is deployed, the pricing scheme provides incentives to the end-users to adjust their demands such that the peak-to-average ratio of demands from the generator is reduced. Thus the demand response and demand side management can be autonomously related. This motivated us to employ a quadratic cost function for the generators.

Let the coefficients of provider k for the cost of power generation be $\beta_{k,2,\text{prod}}, \beta_{k,1,\text{prod}}$ and $\beta_{k,0,\text{prod}}$ respectively . Let the pollution coefficients be $\alpha_{k,2}, \alpha_{k,1}$ and $\alpha_{k,0}$, respectively. Then $\beta_{k,2} := \beta_{k,2,\text{prod}} + \alpha_{k2}, \beta_{k,1} := \beta_{k,1,\text{prod}} + \alpha_{k,1}$ and $\beta_{k,0} := \beta_{k,0,\text{prod}} + \alpha_{k,0}$ are the coefficients of the cost function of generator k. The users are charged proportional to the cost of producing the electricity. Note that for a renewable energy generator, k, $\alpha_{k,2} = \alpha_{k,1} = \alpha_{k,0} = 0$. Let d_n^h denote the demand of user n at time slot h, $h \in \mathcal{H}$, where $\mathcal{H} := \{1, 2, \ldots, H\}$. Let $\mathbf{g}_n := \{g_{n,k}^h, k \in \mathcal{K}, h \in \mathcal{H}\}$, denote the generator selection strategy of user n, where $g_{n,k}^h$ is the decision of user n for generator k in hour h, i.e.,

$$g_{n,k}^h = \begin{cases} 1 & \text{if user } n \text{ selects generator } k \text{ in hour h,} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

We use vectors $\mathbf{g}_n^h$ to represent the strategy of user n for hour h, i..e., $\mathbf{g}_n^h := \{g_{n,k}^h, k \in \mathcal{K}\}$.

Suppose user n prefers generator k in time-slot h. Let $\mathcal{N}_k^h := \{1, 2, \ldots, N_k^h\}$, denote the set of users who select generator k, where $N_k^h := |\mathcal{N}_k^h|$, is the total number of users who prefer buying power from generator k in time-slot h. Then, the total demand from generator k in time-slot h is $\sum_{m \in \mathcal{N}_k^h} d_m^h$. The cost of generating this amount of power is

$$C_{\text{gen},k} = \beta_{k,2} \left(\sum_{m \in \mathcal{N}_k^h} d_m^h \right)^2 + \beta_{k,1} \left(\sum_{m \in \mathcal{N}_k^h} d_m^h \right) + \beta_{k,0}. \tag{2}$$

B. Availability of Renewable Energy Generators

When renewable energy sources are incorporated into the system, we add uncertainty to the supply side. We employ a discrete time Markov chain (DTMC) to model the power available with the renewable sources. The users do not know the exact state of the generators, but they have knowledge about their statistical behavior, i.e., the only information the users have is the steady state probabilities of the DTMC.

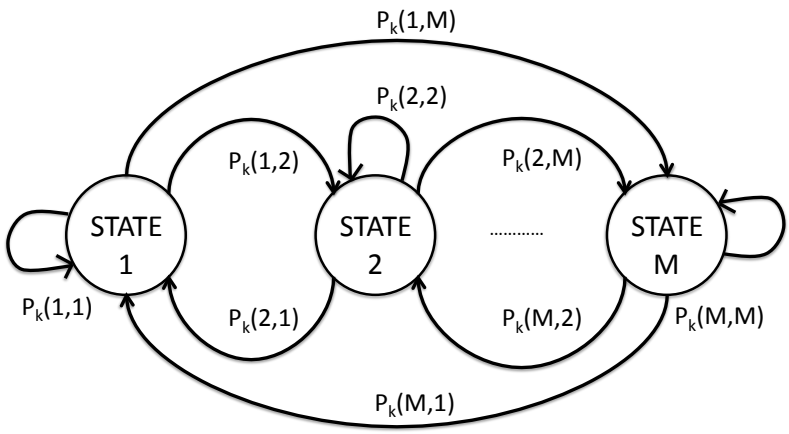

Fig. 2. DTMC for renewable energy generator k

The availability of the renewable energy generators is represented as an M state DTMC as shown in Fig. 2. Let the state of the renewable energy generator k ($k \in \mathcal{K}_r, \mathcal{K}_r := \{1, 2, \ldots, K_r\}; K_r := |\mathcal{K}_r|$) be s_k^h at time slot h. Each state of a generator represents its power supply level. The power supply state space can be divided into M discrete levels i.e., $s_k^h = m$, $(m \in \mathcal{M}, \mathcal{M} := \{1, 2, \ldots, M\}; M := |\mathcal{M}_r|)$. Let the $1 \times K_r$ vector $\mathbf{s}^h := (s_1^h, s_2^h, \ldots.., s_{K_r}^h)$ represent the power supply state of all renewable energy generators, i.e., the system state in time slot h. Then, the state space of this vector is $\Omega^{\mathbf{s}} = \{(\omega_1, \omega_2, \ldots.., \omega_{K_r}) | \omega_k = m, \quad k \in \mathcal{K}\}$. If the generators are independent, the dynamics of $\mathbf{s}^h$ follows DTMC with transition probability from state vector ω to $\omega^{'}$, given by

$$\begin{aligned} P(\omega, \omega^{'}) &= Pr(\mathbf{s}^{h+1} = \omega^{'} | \mathbf{s}^h = \omega) \\ &= \prod_{k=1}^{K_r} P_k(\omega_k, \omega_k^{'}) \ \forall \omega, \omega^{'} \in \Omega^{\mathbf{s}}, \end{aligned} \tag{3}$$

where ω_k and $\omega_k^{'}$ are the kth element of the state vector ω and $\omega^{'}$, respectively, and $P_k(\omega_k, \omega_k^{'})$ represents the state transition probability matrix of generator k.

For a time homogeneous DTMC model, the steady state probabilities of each renewable energy source k, ($k \in \mathcal{K}_r$) being in state m can be obtained as follows. Let us define

$$\pi_k^h(\omega') = Pr(s_k^h = \omega'),\ \forall k \in \mathcal{K}_{\mathrm{r}}. \tag{4}$$

Then,

$$\pi_k^{h+1} = \pi_k^h P, \tag{5}$$

where π_k is a row vector. The stationary distribution is

$$\pi_k = \lim_{h\to\infty} \pi_k^h,\ \text{if the limit exists.} \tag{6}$$

If π_k exists, it can be obtained by solving

$$\pi_k = \pi_k P,\ \sum_{\omega'} \pi_k(\omega') = 1. \tag{7}$$

C. Interactions between the providers and the users

Each user has some appliances whose power requirements are non-shiftable, and some flexible appliances, whose power requirement shiftable in time. Each residential user has a smart meter that, with the aid of the bidirectional communication, can perform load scheduling and generator selection optimally in order to minimize the cost for the user. The data communication among the users can be carried out through a NAN-GW as shown in Fig. 1, or through a Local Area Network (LAN), as described in [3].

IV. Generator Selection: A Non-cooperative Game Theoretical Approach

Different users choose the energy generators independently. However, the aim of each user is to minimize the cost that it pays to the providers. Next, we discuss the pricing scheme and the cost model for the users.

A. Cost model for each user

In order to motivate the users to help in the demand side management, the providers employ a pricing scheme such that the consumers are charged according to the cost of generating the required power.

The electricity delivery process can be divided into two phases. In the first phase, the power generation units are committed for a certain amount of demand from users; this phase is called unit commitment. In the second phase, the electricity is dispatched i.e., the end-users start using the energy; this phase is called economic dispatch. If a user chooses a renewable

energy source but the source is not available or if the power supply level of the source is not adequate to meet the demand, the total available power is supplied proportionally to the users' demands. The amount of power that can not be supplied by the source (deficit power) will have to be generated by and bought from one of the fossil-fuel based sources. Since the deficit power has be generated immediately, it is costlier than the power reserved from the fossil-fuel based generator beforehand. We associate a scaling factor $\delta_k \geq 1$ with the price charged to the users if deficit occurs for source k.

If a user chooses a renewable energy source, its cost will be affected by the type of the renewable material. For instance, if the energy source is a wind generator, the price of energy will depend on the wind speed, weather conditions of the place and other parts of the costs of production. This dependency can be captured by the state transition matrix of the source.

Let us $p_{k,m}$ denote the steady state probability of the renewable energy source $k, (k \in \mathcal{K}_r)$, being in state m. The steady state probabilities are obtained from $\pi_k = \{p_{k,m}, m \in \mathcal{M}\}$. Let $S^h_{\text{avl},k,m}$ be the available power of the renewable generator in time slot h in state m. Then, the amount of power supplied by the renewable energy generator k is

$$S^h_{\text{supp},k,m} = \min\left(S^h_{\text{avl},k,m}, \sum_{l \in \mathcal{N}^h_k} d^h_l\right).$$

If a user chooses renewable energy generator $k, k \in \mathcal{K}_r$, in time-slot h i.e. if $g^h_{n,k} = 1$, then, the amount of power that it gets from generator k is

$$S^h_{\text{user},n,k} = \min\left(d^h_n, \frac{d^h_n}{\sum_{l \in \mathcal{N}^h_k} d^h_l} S^h_{\text{avl},k,m}\right).$$

The total amount of deficit power for renewable energy source k in time-slot h is

$$S^h_{\text{def},k,m} = \max\left(\sum_{l \in \mathcal{N}^h_k} d^h_l - S^h_{\text{avl},k,m}, 0\right)$$

If this deficit power is supplied by the fossil-fuel generator $k^{'}$, then, the total amount of power to be supplied by generator $k^{'}$ is

$$S^h_{\text{tot},k'} = \left(\sum_{l \in \mathcal{N}^h_{k'}} d^h_l + \sum_{l \in \mathcal{K}^h_{r,k'}, \omega \in \mathbf{\Omega}^s} \pi^h_l(\omega) S^h_{\text{def},l,\omega}\right)$$

where $\sum_{l \in \mathcal{N}^h_{k'}} d^h_l$ is the total demand from the fossil-fuel generator, $\sum_{l \in \mathcal{K}^h_{r,k'}, \omega \in \mathbf{\Omega}^s} S^h_{\text{def},l,\omega}$ is the sum of the deficit powers of all renewable energy sources which has to be supplied by fossil-fuel generator $k^{'}$ when the system state is ω, $\mathcal{K}^h_{r,k'} := \{1, 2, \ldots, K^h_{r,k'}\}$ is the set of

the users who choose renewable energy sources but whose demands are at least partially supplied by fossil-fuel generator $k^{'}$, and $K^{h}_{r,k^{'}}$ is the total number of these users. Therefore, if user n chooses renewable energy generator k in time-slot h, the price charged to the user (averaged over the possible states of the generator) will be

$$C^{h}_{\text{user},n} := \sum_{m\in\mathcal{M}} p_{k,m}\left(C^{h}_{\text{user},n,\text{r}} + \delta_k C^{h}_{\text{user},n,\text{f}}\right) \tag{8}$$

where

$$C^{h}_{\text{user},n,\text{r}} := \beta_{k,2} S^{h}_{\text{supp},k,m} S^{h}_{\text{user},n,k} + \beta_{k,1} S^{h}_{\text{user},n,k} + \frac{\beta_{k,0}}{N^{h}_{k}},$$

is the cost to be paid to the renewable source $k, k \in \mathcal{K}_r$, and

$$C^{h}_{\text{user},n,\text{f}} = \beta_{k^{'},2} S^{h}_{\text{tot},k^{'}} \max\left(d^{h}_{n} - \frac{d^{h}_{n}}{\sum\limits_{l\in\mathcal{N}^{h}_{k}} d^{h}_{l}} S^{h}_{\text{avl},k,m}, 0\right)$$
$$+ \beta_{k^{'},1} \max\left(d^{h}_{n} - \frac{d^{h}_{n}}{\sum\limits_{l\in\mathcal{N}^{h}_{k}} d^{h}_{l}} S^{h}_{\text{avl},k,m}, 0\right) + \frac{\beta_{k^{'},0}}{N^{h}_{k^{'}}},$$

is the cost to be paid to the fossil-fuel based source $k^{'} \in \mathcal{K}$. Since the price charged to the users by each generator is closely related to its quadratic cost function, every additional unit of energy will cost more. Consequently, we can also take $\delta_k = 1, \forall k \in \mathcal{K}_r$. Note that even if $\delta_k > 1$, the users will choose renewable energy generator $k, k \in \mathcal{K}_r$, only if the average cost that it pays for selecting k is the least, for given generator selection of other users.

If consumer n chooses the fossil-fuel based generator $k^{'}$, i.e., $g^{h}_{n,k^{'}} = 1$, the price charged to it for time slot h will be

$$C^{h}_{\text{user},n} = a_{k^{'},2} S^{h}_{\text{tot},k^{'}} d^{h}_{n} + a_{k^{'},1} d^{h}_{n} + \frac{a_{k^{'},0}}{N^{h}_{k^{'}}} \tag{9}$$

The centralized cost minimization problem can be written as

$$\min_{\{\mathbf{g}_n, n\in\mathcal{N}\}} \sum_{n\in\mathcal{N}} C_{\text{user},n} = \min_{\{\mathbf{g_n}, n\in\mathcal{N}\}} \sum_{n\in N} \sum_{h\in\mathcal{H}} C^{h}_{\text{user},n}. \tag{10}$$

Since, each user is selfish, the objective of each user is to minimize its own cost. The optimization problem for user n given the strategies of other users, ($\text{OP}_{\text{user},n}$) is

$$\min_{\mathbf{g}_n} \sum_{h\in\mathcal{H}} C^{h}_{\text{user},n}. \tag{11}$$

For the generator selection game, the power consumptions and thus the costs are decoupled in time. Hence, minimizing the total price over a time span of H time slots is equivalent to minimizing the price in each time slot, i.e., ($\text{OP}_{\text{user},n}$) takes the form

$$\min_{\{g_{n,k}^h, k\in\mathcal{K}\}} C_{\text{user},n}^h, \ \forall h \in \mathcal{H}. \tag{12}$$

B. A Non-Cooperative Game Among Users

The choice of generators by consumers depends not only on their own demands but also on the demands and generator selection strategies of the other consumers. This implies that the strategy of each user is coupled with the strategies of the other users. As each user is concerned about maximizing its own payoff, a non-cooperative game [12] is a natural fit to model the behavior of the users in this case.

The components of the game are as follows:

- *Players*: Users in the set $\mathcal{N}$.
- *Strategies*: The strategy set of user n is the selection of energy source for all time slots in a day, $\mathbf{g}_n$. The strategy space is defined as $\mathbf{g} := \{\mathbf{g}_1 \times \mathbf{g}_2 \times \ldots \mathbf{g}_N\}$.
- *Payoffs*: $u_{\text{user},n}(\mathbf{g}_n, \mathbf{g}_{-n})$, for each user $n \in \mathcal{N}$, where $u_{\text{user},n}(\mathbf{g}_n, \mathbf{g}_{-n}) = -C_{\text{user},n}$, and $\mathbf{g}_{-n}$ is the strategy of all users other than n.

C. Existence of Nash equilibrium

Definition 1. *For a finite N-user game in strategic form $\mathcal{G} := \{\mathcal{N}, (A_n)_{n\in\mathcal{N}}, (u_{user,n})_{n\in\mathcal{N}}\}$, where $u_{user,n}$ is the utility function of user n, the strategy set of user n is denoted by $\mathbf{A}_n$ and the strategy space for all users other than n is denoted by $\mathbf{A}_{-n}$. Let us represent the actions taken by user n and the rest of the users as a_n and $\mathbf{a}_{-n}$, respectively. Then, for all $n \in \mathcal{N}$, the strategy $a_n^* \in \mathbf{A}_n$ is a Nash equilibrium (NE) for user n if for every alternative strategy $a_n \in \mathbf{A}_n$, $(a_n^* \neq a_n)$, the following relation holds:*

$$u_{user,n}(a_n^*, \mathbf{a}_{-n}^*) \geq u_{user,n}(a_n, \mathbf{a}_{-n}^*). \tag{13}$$

Equation (13) indicates that at the NE, a player cannot improve its payoff by deviating alone from the equilibrium, given the strategies of all other players.

For the generator selection game, the NE is any strategy set $(\mathbf{g}_n^*, \mathbf{g}_{-n}^*)$ at which

$$u_{\text{user},n}(\mathbf{g}_n^*, \mathbf{g}_{-n}^*) \geq u_{\text{user,n}}(\mathbf{g}_n, \mathbf{g}_{-n}^*),$$

where $\mathbf{g}_n^*$ is the optimal generator selection strategy of user n, for given optimal strategies of other users $\mathbf{g}_{-n}^*$, and $u_{\text{user},n}(.) = -C_{\text{user},n}(.)$.

Theorem 1. *The generator selection game between the consumers is a concave N-user game and a Nash equilibrium exists for the game.*

Proof: For the generator selection game for one whole day, since there is no coupling in time, it is sufficient to prove that an NE exists for the game for each time slot.

For time-slot h, the demand of user n from generator k can be written as $d_{n,k}^h = g_{n,k}^h d_n^h = \{0, d_n^h\}$. If k is a renewable energy source, user n will get $S_{\text{user},n,k}^h$ from the renewable energy source, and $[d_{n,k}^h - S_{\text{user},n,k}^h]^+$ from the corresponding fossil-fuel generator, where $[\theta]^+ := max[\theta, 0]$. The demand of users other than n who also prefer generator k, can be represented as $\mathbf{d}_{-n,k}^h = g_{m,k}^h d_m^h, \forall m \in \mathcal{N}_k^h, m \neq n$. Suppose $\mathcal{N}_k^{h'} := \{1, 2, \ldots, n, \ldots N_k^{h'}\}$ is the set of all users who get power from generator k. Note that $N_k^{h'}$ is the total number of users who buy power from generator k, while N_k^h is the total number of users who preferred generator k. If k is a fossil fuel based generator, $\mathcal{N}_k^{h'}$ includes those users who had chosen renewable energy generators, but whose demands are at least partially supplied by generator k. If k is a renewable energy generator, $\mathcal{N}_k^{h'} = \mathcal{N}_k^h$. Let us write the payoff of user n, who gets power from generator k, as $u_{\text{user},n}(\mathbf{d}_{\text{all},k}^h) = u_{\text{user},n}(d_{1,k}^h, \ldots, d_{n,k}^h, \ldots, d_{N_k^{h'},k}^h)$. Since the cost functions are increasing and strictly convex for each generator, the payoff function $u_{\text{user},n}(d_{n,k}^h, \mathbf{d}_{-n,k}^h)$ is strictly concave w.r.t. $d_{n,k}^h$ for given $(d_{1,k}^h, \ldots, d_{n-1,k}^h, d_{n+1,k}^h, \ldots, d_{N_k^{h'},k}^h)$ and is continuous in $\mathbf{d}_{\text{all},k}^h$. This holds for all users $n \in \mathcal{N}$, who prefer a different generator $g \in \mathcal{K}, g \neq k$, too. Thus, the generator selection game is a concave N-user game. An NE exists for a concave N-user game [14]. The game will be a concave game even if a part of the demand of user n from $k \in \mathcal{K}_r$, is supplied by a fossil-fuel generator, since the sum of the convex functions is also convex. Consequently, an NE exists for this game. ■

D. Distributed algorithm to converge to NE

Equation (10) can be solved in a centralized manner by using integer programming. However, with increasing N, the computation overhead will increase exponentially for a centralized solution. Thus, for a practical implementation of the scheme, we need a distributed algorithm such that the end-users can reach their optimal strategies using only local information. Notably, (11) or (12) can be solved by each user distributedly provided the users can select their strategies sequentially. Exploiting this fact, next, we propose a distributed

algorithm to implement the generator selection scheme.

Let, $\mathbf{g}_{n,t} = \{\mathbf{g}_{n,t}^h,\ h \in \mathcal{H}\}$ denote the generator selection strategy of user n. Each user starts by randomly selecting a generator for each time slot. In the next iteration, one of the consumers solves the local optimization problem (12) and updates its strategy for the whole day. Then, another user gets its turn to solve its local optimization problem and to update its choice. If in one complete round of updating, the selection of generators from all users do not change, the point is an equilibrium. We call this algorithm, **Algorithm 1** and the detailed algorithm is shown in Table I. The strategy update iterations should be performed until the strategies of all users do not change compared to their strategies in the previous iterations. Note that strategy update should be done sequentially by each user. The NAN-GW can be used to send the control signal (e.g., a 1-bit signal) for asynchronously giving turn to each user to update their strategies.

Note that the term *iteration* 't' and the term *time-slot* 'h' are used to represent different time scales. An optimal generator selection game for one time slot needs many iterations for the users' strategy to converge (see results in Section VI).

TABLE I

DISTRIBUTED ALGORITHM FOR NE

1: t=1; Arbitrarily choose $\mathbf{g}_{n,t} = \{\mathbf{g}_{n,t}^h,\ h \in \mathcal{H}\},\ \forall n \in \mathcal{N}$, and announce the starting strategy, i.e., $\{\mathbf{g}_{n,t}, n \in \mathcal{N}\}$.
2: $t = t + 1$
3: Randomly choose user $n, n \in \mathcal{N}$.
4: User n: Solve the optimization problem (12), $\forall h \in \mathcal{H}$ using integer programming to find optimal $\mathbf{g}_{n,t}$.
5: **If** $\mathbf{g}_{n,t}$ changes compared to $\mathbf{g}_{n,t-1}$,
6: Send a control message to announce the new $\mathbf{g}_{n,t}$ to other smart meters.
7: **end**
8: **If** any user has not updated its strategy for iteration t,
9: Send a control message to user l who has not updated its strategy for iteration t, $l \in \mathcal{N}, l \neq n$.
10: Update $n = l$. User n, go to Step 4.
11: **end**
12: **If** $\mathbf{g}_{n,t}$ changes compared to $\mathbf{g}_{n,t-1}$ for at least one user,
13: Go to Step 2.
14: **end**

Theorem 2. ***Algorithm*** **1** *converges to an NE as long as the individual strategies are updated sequentially.*

Proof: Line 4 in Table I finds the best response. If users play the best responses using this algorithm in a sequential manner, the cost of each user decreases or remains unchanged every time a user updates its strategy. However, the cost cannot decrease below a certain value since $C_{\text{user},n} \geq 0$. This implies that the algorithm converges to a fixed point, beyond which the cost can not be improved, for given strategies of other users. To prove that this fixed point is an NE, let us denote $C^*_{\text{user},n}(\mathbf{g}^*_n, \mathbf{g}^*_{-n})$ as the cost for user n at the fixed point, which corresponds to the strategy set $(\mathbf{g}^*_n, \mathbf{g}^*_{-n})$ and let $C^{'}_{\text{user},n}(\mathbf{g}^{'}_n, \mathbf{g}^*_{-n})$ be the cost for the same user for the strategy set $(\mathbf{g}^{'}_n, \mathbf{g}^*_{-n})$ where $\mathbf{g}^*_n \neq \mathbf{g}^{'}_n$. By definition, if $(\mathbf{g}^*_n, \mathbf{g}^*_{-n})$ is a fixed point of the generator selection game, then,

$$C^*_{\text{user},n}(\mathbf{g}^*_n, \mathbf{g}^*_{-n}) \leq C^{'}_{\text{user},n}(\mathbf{g}^{'}_n, \mathbf{g}^*_{-n}),$$

$$\Rightarrow u^*_{\text{user},n}(\mathbf{g}^*_n, \mathbf{g}^*_{-n}) \geq u^{'}_{\text{user},n}(\mathbf{g}^{'}_n, \mathbf{g}^*_{-n}).$$

Thus, the fixed point $(\mathbf{g}^*_n, \mathbf{g}^*_{-n})$ is an NE of the game. If the NE is unique, the algorithm converges to the unique NE. If there exist multiple Nash equilibria, the algorithm converges to one of them depending on the initially selected strategies. ■

V. Load Scheduling and Generator Selection: A Joint Approach

Scheduling in time is a conventional approach used to reduce the Peak to Average Ratio (PAR) of a single generator/provider which consequently reduces the cost of generation for the (single) generator and also the user bills. However, it should be noted that not all users prefer saving on their bills by shifting their flexible load. Such time-scheduling schemes are useful only for those users who have a considerable amount of flexible load. In fact, many consumers may not even have any shiftable load, or they may have some flexible appliances but they may prefer to use them at certain hours rather than scheduling them at any time for the sake of reducing costs. In addition, the electricity requirements of the commercial and the industrial consumers are usually rather strict, and scheduling in time may not be a preferable option for them. On the contrary, if the users have shiftable appliances, their costs can be reduced significantly by scheduling their shiftable appliances in different time slots. We accommodate the needs and goals of all these types of consumers together into one framework in our proposed DRM solution. To this end, we introduce a joint scheme where users schedule their shiftable appliances in different time slots and also choose generators in each time slot. While the generator selection game is played in a similar manner for each

time slot to schedule the total load of the NAN among providers, the load scheduling in time has to be done in advance for the whole day.

As mentioned in Section I, we would like to emphasize that the generator selection scheme incorporating the uncertainty in the supply side associated with the renewable energy resources, is one of our main contributions in this paper. The joint scheme is another contribution of this work which further reduces the cost for the users. The joint scheme is not a straightforward extension of the generator selection game. The joint scheme is comprised of two levels of non-cooperative games: the optimal load scheduling in time for the whole day and the optimal generator selection for all time slots in a day. It is worth noting that the two scenarios have rather different application domains. The generator selection game finds its extensive use in a scenario where the end-users have relatively strict power requirements such as commercial and/or industrial users who have either no or very little flexible demands. On the other hand, the scheduling in time is more useful for users with higher proportion of shiftable appliances, such as residential users. The joint scheme is very generic and is suitable for residential, commercial as well as industrial users.

In addition to a non-shiftable demand in each time slot, we now incorporate some shiftable power requirement for each consumer. Let $d^h_{n,\text{ns}}, d^h_{n,\text{s}}$ be the non-shiftable demand in time slot h, and part of the shiftable demand of user n, scheduled in time slot h, respectively, for a given scheduling in time. Here, the subscript s represents *shiftable*, n is the *user* index and ns indicates *non-shiftable*. The total amount of shiftable power per day is fixed but its scheduling across time and the selection of generators is the strategy of each consumer in this case. We incorporate the coupling across different time slots for the shiftable load of consumer n as

$$\sum_{h=\tau_{n,1}}^{\tau_{n,2}} d^h_{n,\text{s}} = \bar{d}_{n,\text{s}};\ d^h_{n,\text{s}} \geq 0,\ \forall h \in [\tau_{n,1}\ \tau_{n,2}].$$

For this scheme, the strategy of each consumer is $\{\mathbf{x}_n, \mathbf{g}_n\}$, where $\{\mathbf{x}_n\} := \{x^h_n, h \in \mathcal{H}\}$, is a demand scheduling vector that incorporates its shiftable load $\bar{d}_{n,\text{s}}$, i.e., $x^h_n = d^h_{n,\text{ns}} + d^h_{n,\text{s}}$, is the power requirement of user n in time slot h, and $\mathbf{g}_n := \{g^h_{n,k}, h \in \mathcal{H}, k \in \mathcal{K}\}$ is the generator selection strategy of user n for one complete day, for a given schedule of the shiftable appliances. If a user chooses renewable energy generator k in time slot h, the price charged on user n is calculated as follows. The amount of power supplied by the renewable

energy generator k is

$$S^h_{\text{supp},k,m} = \min\left(S^h_{k,m}, \sum_{l\in\mathcal{N}^h_k} x^h_l\right).$$

If a user chooses renewable energy generator k in time slot h i.e. if $g^h_{n,k} = 1$, then, the amount of power that it gets from generator k is

$$S^h_{\text{user},n,k} = \min\left(x^h_n, \frac{x^h_n}{\sum_{l\in\mathcal{N}^h_k} x^h_l} S^h_{\text{avl},k,m}\right).$$

The total deficit amount of power for renewable energy source k in time slot h in this case is

$$S^h_{\text{def},k,m} = \max\left(\sum_{l\in\mathcal{N}^h_k} x^h_l - S^h_{\text{avl},k,m}, 0\right)$$

If this deficit power is supplied by the fossil-fuel generator $k^{'}$, then, the total amount of power to be supplied by generator $k^{'}$ is

$$S^h_{\text{tot},k'} = \left(\sum_{l\in\mathcal{N}^h_{k'}} x^h_l + \sum_{l\in\mathcal{K}^h_{r,k'},\omega\in\Omega^s} \pi^h_l(\omega) S^h_{\text{def},l,\omega}\right)$$

where $\sum_{l\in\mathcal{N}^h_{k'}} x^h_l$ is the total demand from the fossil-fuel generator, $\sum_{l\in\mathcal{K}^h_{r,k'},\omega\in\Omega^s} S_{\text{def}^h,l,\omega}$ is the sum of the deficit power of all renewable energy generators which has to be supplied by fossil-fuel generator $k^{'}$ when the system state is ω. Therefore, if user n chooses renewable energy generator k in time slot h, the price charged to the user will be

$$C^h_{\text{user},n} := \sum_{m\in\mathcal{M}} p_{k,m}\left(C^h_{\text{user},n,\text{r}} + \delta_k C^h_{\text{user},n,\text{f}}\right) \tag{14}$$

where

$$C^h_{\text{user},n,\text{r}} := \beta_{k,2} S^h_{\text{supp},k,m} S^h_{\text{user},n,k} + \beta^h_{k,1} S^h_{\text{user},n,k} + \frac{\beta^h_{k,0}}{N^h_k},$$

is the cost to be payed to the renewable source and

$$C^h_{\text{user},n,\text{f}} = \beta_{k',2} S^h_{\text{tot},k'} \max\left(x^h_n - \frac{x^h_n}{\sum\limits_{l\in\mathcal{N}^h_k} x^h_l} S^h_{\text{avl},k,m}, 0\right)$$

$$+ \beta_{k',1} \max\left(x^h_n - \frac{x^h_n}{\sum\limits_{l\in\mathcal{N}^h_k} x^h_l} S^h_{\text{avl},k,m}, 0\right) + \frac{\beta_{k',0}}{N^h_{k'}},$$

is the cost to be payed to the fossil-fuel based generator $k^{'} \in \mathcal{K}, k^{'} \neq k$. If consumer n chooses the fossil-fuel based generator $k^{'}$, i.e., $g^{h}_{n,k^{'}} = 1$, the price charged to it for time slot h will be

$$C^{h}_{\text{user},n} = a_{k^{'},2} S^{h}_{\text{tot},k^{'}} x^{h}_{n} + a_{k^{'},1} x^{h}_{n} + \frac{a_{k^{'},0}}{N^{h}_{k^{'}}} \tag{15}$$

The objective for a centralized approach would be to minimize the total daily cost of all users i.e.,

$$\min_{\{\mathbf{g}_n, \mathbf{x}_n, n \in \mathcal{N}\}} \sum_{n \in \mathcal{N}} C_{\text{user},n} = \min_{\{\mathbf{g}_n, \mathbf{x}_n, n \in \mathcal{N}\}} \sum_{n \in \mathcal{N}} \sum_{h \in \mathcal{H}} C^{h}_{\text{user},n} \tag{16}$$

$$\text{s.t.} \sum_{h=\tau_{n,1}}^{\tau_{n,2}} d^{h}_{n,\text{s}} = \bar{d}_{n,\text{s}}, \forall n \in \mathcal{N}.$$

The objective of each user, however, is to minimize its own cost given the strategy of other users, i.e., the optimization problem for user n ($\text{OP}_{\text{user},n}$) in this case, is

$$\min_{\{\mathbf{g}_n, \mathbf{x}_n\}} \sum_{h \in \mathcal{H}} C^{h}_{\text{user},n}, \tag{17}$$

$$\text{s.t.} \sum_{h=\tau_{n,1}}^{\tau_{n,2}} d^{h}_{n,\text{s}} = \bar{d}_{n,\text{s}}. \tag{18}$$

A. Existence of NE

For the joint generator selection and load scheduling game, the NE is defined as the strategy set $\{\mathbf{x}^* := \{\mathbf{x}^*_n, \mathbf{x}^*_{-n}\}, \mathbf{g}^* := \{\mathbf{g}^*_{\mathbf{n}}, \mathbf{g}^*_{-\mathbf{n}}\}\}$ such that

$$C_{\text{user},n}(\mathbf{x}^*_n, \mathbf{x}^*_{-n}, \mathbf{g}^*_n, \mathbf{g}^*_{-n}) \leq C_{\text{user},n}(\mathbf{x}_n, \mathbf{x}^*_{-\mathbf{n}}, \mathbf{g}_n, \mathbf{g}^*_{-\mathbf{n}}),$$

Theorem 3. *The joint generator selection and load scheduling game is a concave N-user game and a Nash equilibrium exists for this game.*

Proof:

The payoff function is $u_{\text{user},n}(\mathbf{x}, \mathbf{g}) = u_{\text{user},n}(\{\mathbf{x}_1, \ldots, \mathbf{x}_n, \ldots, \mathbf{x}_{N_k}\}, \{\mathbf{g}_1, \ldots, \mathbf{g}_n, \ldots, \mathbf{g}_{N_k}\})$ for user n, where $(\mathbf{x}_1, \ldots, \mathbf{x}_n, \ldots \mathbf{x}_{N_k})$ and $(\mathbf{g}_1, \ldots, \mathbf{g}_n, \ldots \mathbf{g}_{N_k})$ are the vectors that represent the demands and the generator selection of users $1, \ldots, n, \ldots, N_k$, all of whom get power from generator $k, (k \in \mathcal{K}$, here k may indicate different generators at different time slots).

For a given scheduling of the shiftable load of the users, an NE exists for the generator selection game for each time slot, as shown in Section IV-C. Thus we need to prove that an NE exists only for the scheduling game.

Since the cost function of each generator is increasing and strictly convex, the payoff function $u_{\text{user},n}(\mathbf{x}_n, \mathbf{x}_{-n})$ is strictly concave w.r.t. $\mathbf{x}_n$. For $\mathbf{x}_n \in \mathbf{x}$, where $\mathbf{x} = \mathbf{x}_1 \times \mathbf{x}_2 \times \ldots \times \mathbf{x}_{N_k}$, $u_{\text{user},n}(\mathbf{x})$ is continuous in $\mathbf{x}$ and is concave in $\mathbf{x}_n$ for given $(\mathbf{x}_1, \ldots, \mathbf{x}_{n-1}, \mathbf{x}_{n+1}, \ldots, \mathbf{x}_{N_k})$. This holds for all users $n \in \mathcal{N}$ selecting different time slots $h \in \mathcal{H}$ for their load scheduling from the same generator. Hence, the load scheduling game is a concave N-user game and the existence of an NE for such a game is straightforward [14].

■

B. Distributed algorithm to find NE of the game

Problem (16) can be solved by using integer programming. The computation overhead, however, can be quite high for a centralized implementation of this scheme since the computation in this case is quite intensive. In particular, as the users have to choose both the time slots and the generators optimally, the higher the number of generators and time slots, the number of possible combinations of strategies is huge, thus drastically increasing the computational complexity. Therefore, we need a distributed algorithm so that the optimal scheduling can be done by the end-users, with only local information.

Equations (17), (18) represent a local optimization problem for each consumer given the strategies of other users. Therefore, each user can reach the NE of the game starting from some arbitrary initial selection vectors $\{\mathbf{x}_n, \mathbf{g}_n\}$. We devise a distributed algorithm for each user to solve (17),(18) locally in order to find its optimal strategy. We name this algorithm, **Algorithm 2**, and is described next.

Each user starts with a random scheduling of the shiftable appliances and by randomly selecting a generator, for each time slot. To start the next iteration, a 1-bit control information is sent to one of the users by the NAN-GW to update its strategy. The user who gets the control signal solves the local optimization problem (17) for given $\{\mathbf{x}_{-n}, \mathbf{g}_{-n}\}$ and updates its strategy $\{\mathbf{x}_n, \mathbf{g}_n\}$. Then, another user is chosen randomly to update its strategy. The process stops when the strategies of all users ($\{\mathbf{x}_n, \mathbf{g}_n, n \in \mathcal{N}\}$) do not change. The detailed algorithm is shown in Table II.

Theorem 4. ***Algorithm 2*** *converges to an NE as long as the updating of the individual strategies is done sequentially.*

Proof: Line 4 in Table II finds the best response. If users play the best responses sequentially using this algorithm in an asynchronous manner, the cost of each user decreases

TABLE II

DISTRIBUTED ALGORITHM TO FIND THE NE OF THE JOINT SCHEME

1: $t = 1$; Arbitrarily choose $\mathbf{x}_{n,t} = \{\mathbf{x}_{n,t}^h\}, \mathbf{g}_{n,t} = \{\mathbf{g}_{n,t}^h\},\ \forall h \in \mathcal{H}$, such that (18) is satisfied $\forall n \in \mathcal{N}$, and announce the starting strategies, i.e., $\{\mathbf{x}_{n,t}, \mathbf{g}_{n,t}, n \in \mathcal{N}\}$.
2: $t = t + 1$
3: Randomly choose user $n, n \in \mathcal{N}$.
4: User n: Solve the optimization problem (17), (18) to find optimal $\{\mathbf{x}_{n,t}, \mathbf{g}_{n,t}\}$ for given strategies of other users.
5: **If** $\{\mathbf{x}_{n,t}, \mathbf{g}_{n,t}\}$ changes compared to $\{\mathbf{x}_{n,t-1}, \mathbf{g}_{n,t-1}\}$,
6: Send a control message to announce the new $\{\mathbf{x}_{n,t}, \mathbf{g}_{n,t}\}$ to other smart meters.
7: **end**
8: **If** any user has not updated its strategy for iteration t,
9: Send a control message user l who has not updated its strategy for iteration t, $l \in \mathcal{N}, l \neq n$.
10: Update $n = l$. User n, go to Step 4.
11: **end**
12: **If** $\{\mathbf{x}_{n,t}\mathbf{g}_{n,t}\}$ changes compared to $\{\mathbf{x}_{n,t-1}, \mathbf{g}_{n,t-1}\}$ for at least one user,
13: Go to Step 2.
14: **end**

or remains unchanged every time a user updates its strategy. However, the cost cannot decrease below a certain value because $C_{\text{user},n} \geq 0$. This implies that the algorithm converges to a fixed point. Let us denote $C^*_{\text{user},n}(\mathbf{x}^*_n, \mathbf{x}^*_{-n}, \mathbf{g}^*_n, \mathbf{g}^*_{-n})$ as the cost for user n at the fixed point, which corresponds to the strategy space $\{\mathbf{x}^*_n, \mathbf{x}^*_{-n}, \mathbf{g}^*_n, \mathbf{g}^*_{-n}\}$ and let $C'_{\text{user},n}(\mathbf{x}'_n, \mathbf{x}^*_{-n}, \mathbf{g}'_n, \mathbf{g}^*_{-n})$ be the cost for the same user for the strategy space $\{\mathbf{x}'_n, \mathbf{x}^*_{-n}, \mathbf{g}'_n, \mathbf{g}^*_{-n}\}$ where $\{\mathbf{x}^*_n \neq \mathbf{x}'_n$, $\mathbf{g}^*_n \neq \mathbf{g}'_n\}$. By definition, if $(\mathbf{x}_n, \mathbf{x}^*_{-n}, \mathbf{g}^*_n, \mathbf{g}^*_{-n})$ is a fixed point of the game, then,

$$C^*_{\text{user},n}(\mathbf{x}^*_n, \mathbf{x}^*_{-n}, \mathbf{g}^*_n, \mathbf{g}^*_{-n}) \leq C'_{\text{user},n}(\mathbf{x}'_n, \mathbf{x}^*_{-n}, \mathbf{g}'_n, \mathbf{g}^*_{-n}).$$

This implies that the fixed point $(\mathbf{x}^*_n, \mathbf{x}^*_{-n}, \mathbf{g}^*_n, \mathbf{g}^*_{-n})$ is an NE of the game. If the NE is unique, the algorithm converges to the unique NE. If there exists multiple Nash equilibria, the algorithm converges to one of them depending on the initially selected strategies. ■

VI. NUMERICAL RESULTS

In this section we evaluate the performance of the proposed algorithms. This section is divided into two parts: convergence of **Algorithms 1** and **2**, and performance evaluation and comparison of the two algorithms. The equilibrium of the game will certainly depend on the values of the parameters. For the purpose of illustration, we use a certain set of parameters in

all analysis in this section. Note that the value of the results may change with a different set of parameters, but the pattern of the results will be similar. We consider 10 users ($N = 10$) and 2 generators ($K = 2$). Generator 1 is fossil-fuel based and generator 2 is renewable. We divide the daily consumption of users into 24 hours. The renewable generator has three states: state 1 is the OFF state i.e. no power available, state 2 means it has a power of 15 KW and state 3 means it has a power of 30 KW.

Each user has some non-shiftable appliances such as refrigerators, bulbs, heating/cooling units, electric stoves, televisions, computers, etc., and some shiftable appliances such as dishwashers, laundry machines, electrical vehicles, Plug-in hybrid electric vehicles (PHEVs), etc. The power requirement for these appliances are typical standard values e.g., 1.32 - 3.96 KWh for refrigerators (depending on the number of refrigerators a household has), 1-2 KWh for bulbs, 6.4-9.6 KWh for heating, 3.9 KWh for electric stoves, 1.1-2.0 KWh for televisions/computers, 1.44 KWh for a dish-washer, 3.43 KWh for a laundry machine and 9.9 KWh for PHEVs. The power requirement for the non-shiftable appliances was generated using a uniform random distribution over the typical values mentioned above. Other parameters used are $\beta_{1,2} = 0.1875, \beta_{1,1} = 0.02, \beta_{1,0} = 0.05, \alpha_{1,2} = 0.1875, \alpha_{1,1} = 0.02, \alpha_{1,0} = 0.05, \beta_{2,2} = 0.02, \beta_{2,1} = 0.3, \beta_{2,0} = 0.5, \delta_2 = 1$, unless otherwise stated. Note that we have chosen $\beta_{2,2} \ll \beta_{1,2}, \beta_{2,0} \gg \beta_{1,0}$ assuming that the production cost is dominant for fossil-fuel based generators and the initial installation cost is higher for renewable energy sources. The transition probability matrix of the renewable energy generator used for the numerical analysis is

$$P = \begin{bmatrix} 0.3 & 0.4 & 0.3 \\ 0.2 & 0.4 & 0.4 \\ 0.1 & 0.6 & 0.3 \end{bmatrix}.$$

A. Convergence of Generator Selection Algorithm

Fig. 3 shows that Algorithm 1 converges. This figure represents the convergence of the cost of each user to the NE for a 24 hours period without using shiftable load. The convergence is within 10 iterations. Fig. 4 depicts the total hourly cost for 24 hours i.e., the cost paid by all users in the NAN every hour. We have used the abbreviation GS in the graphs to indicate generator selection. The upper part of the figure shows the cost without employing the generator selection scheme and the lower graph displays the performance when the generator selection scheme is employed. It is evident that the cost decreases in every hour when the

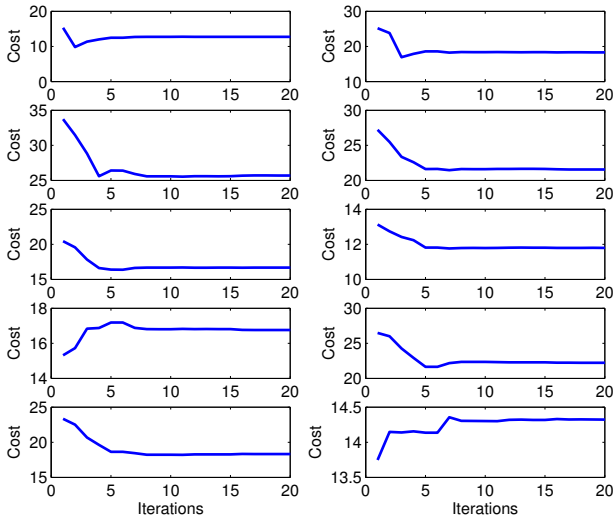

Fig. 3. Convergence of the costs of each user to NE for a 10-users game

generator selection scheme is used. E.g., at hour 20, the total cost reduces from 29.5 to 17, i.e., the generator selection scheme results in an improvement of upto about 40%.

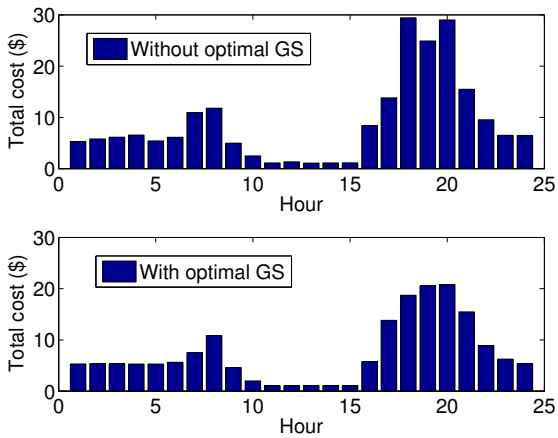

Fig. 4. Comparison of hourly costs

B. Generator Selection and Load Scheduling: Joint Cost Minimization

We added shiftable demands of 20 KWh to all the users such that each user needs this amount of energy in 5 hours. Each user schedules this demand in time. Fig. 5 shows the total cost of each user for a 24-hour period calculated using Algorithm 2. The figure shows that the algorithm converges very fast, in about 20 iterations. Fig. 6 depicts the total cost of all users for a 24-hour period. The figure shows four curves. The curve with '$+$' markers is the cost when neither the optimal generator selection nor the optimal load scheduling is employed. The dashed line shows the scheme where only optimal load scheduling is performed. The dotted line with square markers shows the performance of the generator selection scheme. The solid line shows the performance of our joint scheme where both optimal load scheduling and optimal generator selection are employed. The fourth curve (solid line) shows that Algorithm 2 converges within 20 iterations. We observe that the load scheduling and the generator selection schemes reduce the cost compared to the case when no scheduling is done, from about $256\$$ to about $220\$$, and $180\$$, respectively, i.e., by 14.71% and 25.7%, respectively. Our joint scheme incurs the least cost to the users compared to the remaining three schemes. It reduces the cost paid by the users to $154\$$, i.e., by 3.2% compared to the generator selection scheme, by 14.19% compared to the load scheduling scheme and by 28.9% compared to the scheme without using generator selection and load scheduling.

Fig. 7 shows the total daily cost paid by each user. The total cost paid by all users may not be reduced in every hour but the total daily cost is reduced for every user.

VII. Conclusion

We have formulated a generator selection problem as a non-cooperative game, incorporating multiple renewable and non-renewable energy sources. We have proved that there exists a Nash equilibrium for the game and have proposed a distributed algorithm that converges to the Nash equilibrium. Next, we improved the performance of our proposed scheme by integrating optimal load scheduling with it, using a two-stage non-cooperative game framework for analysis. This is a novel approach exploiting both spatial and temporal dimensions for demand response management in the smart grid. We have proven the existence of a Nash equilibrium for this game also, and have developed a distributed algorithm to converge to the equilibrium. We have verified the convergence of the proposed algorithms and have shown that while the generator selection scheme improves the costs of the users compared to the case without any scheduling, the joint scheme yields the lowest costs compared to three other schemes.

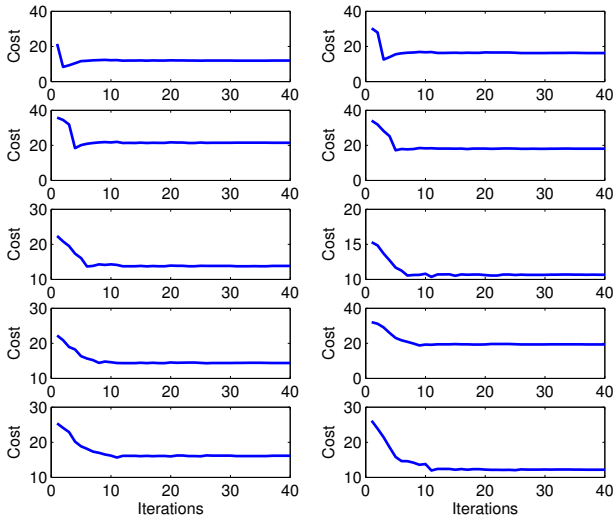

Fig. 5. Convergence of the cost of each user (for one day)

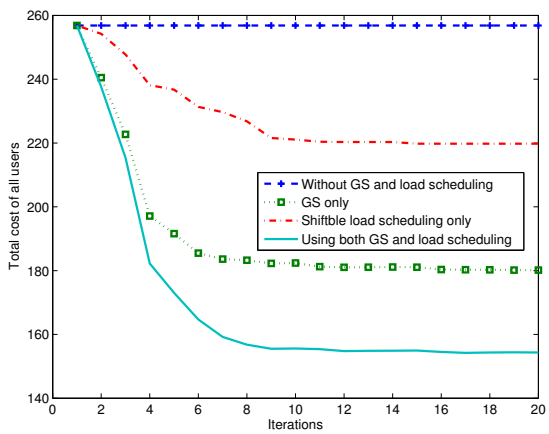

Fig. 6. Convergence of the total cost for all users (for one day)

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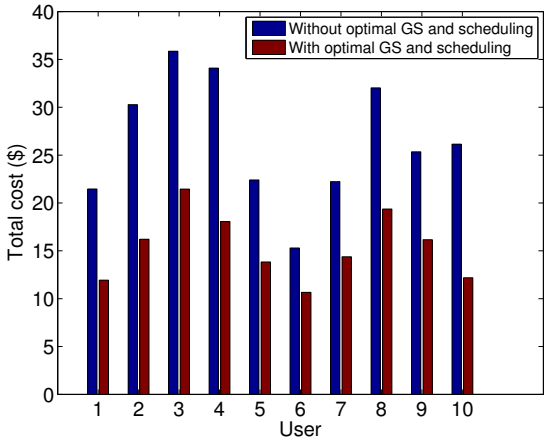

Fig. 7. Comparison of daily costs per user

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Paper VII

Sensing-Performance Tradeoff in Cognitive Radio enabled Smart Grid

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Sensing-Performance Tradeoff in Cognitive Radio enabled Smart Grid

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Abstract

Smart grid is widely considered to be the next generation of power grid, where power generation, management, transmission, distribution and utilization are fully upgraded to improve agility, reliability, efficiency, security, economy and environmental friendliness. Demand Response Management (DRM) is recognized as a control unit of smart grid, with the attempt to balance the real-time load as well as to shift the peak-hour load. Communications are critical to the accuracy and optimality of DRM, and hence at the core of the control performance of smart grid. In this paper, we introduce cognitive radio into smart grid to improve the communication quality. By means of spectrum sensing and channel switching, smart meters can decide to transmit data on either an original unlicensed channel or an additional licensed channel, so as to reduce the communication outage. Considering the energy cost taxed by spectrum sensing together with the control performance degradation incurred by imperfect communications, we formulate the sensing-performance tradeoff problem between better control performance and lower communication cost, paving the way towards green smart grid. The impact of the communication outage on the control performance of DRM is also analyzed, which reduces the profit of power provider and the social welfare of smart grid, although it may not always

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decrease the profit of power consumer. By employing the energy detector, we prove that there exists a unique optimal sensing time which yields the maximum tradeoff revenue, under the constraint that the licensed channel is sufficiently protected. Numerical results are provided to validate our theoretical analysis.

Index Terms

Demand response management, cognitive radio enabled smart grid, sensing-performance trade-off

I. Introduction

Power grid is a large interconnected infrastructure for delivering electricity from power plants to end users. New challenges are emerging in the traditional grid, e.g., rising demand, aging infrastructure, and increasing greenhouse gas emission, which have become an urgent global concern. As widely considered to be the next generation of power grid, smart grid fully upgrades power generation, management, transmission, distribution and utilization to improve agility, reliability, efficiency, security, economy and environmental friendliness [2] [3] [4]. It is envisioned as a promising technology to integrate with renewable green energy resources such as wind and solar power. Besides, it is an open market for electricity providers and consumers with flexible pricing strategies and load shifting capabilities. For example, the widespread use of plug-in hybrid electric vehicles with vehicle-to-grid capacity in smart grid can help alleviate overload during peak hours [5].

Advances in smart metering and digital communications promote smart grid to be an intelligent closed-loop system where power plants and end users interact closely to achieve efficient and economical power generation, distribution and utilization. As shown in Fig. 1, input of the system is supply provided by power plants, while feedback is demand of end users measured by smart meter. Demand Response Management (DRM) acts as a control unit to balance and shape the real-time load. Output of the system is electricity delivered to each user through transmission and distribution. The forward path is *power flow*, and the bidirectional path is *information flow*, which provides two-way communications in smart grid.

There are several studies on DRM, with the attempt to reduce and shift the peak-hour load [6] [7] [8] [9] [10] [11]. Real-time pricing is known as one of the most common tools that can encourage power utilization in an efficient and economical way. A distributed and iterative algorithm is proposed in [8] [9] to balance and shape the real-time load. In [10]

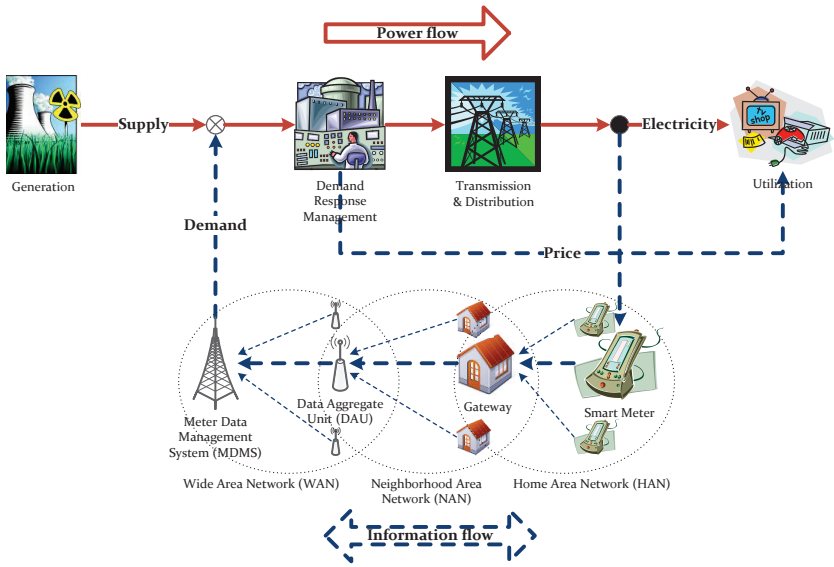

Fig. 1. A closed-loop system scheme of smart grid

[11], game theory is used to address DRM via price predication and energy consumption scheduling. However, most of these studies assume perfect two-way communications which is too strong for practical applications. Whereas, the impact of communication unreliability on the control performance of DRM has not been revealed in the literature.

Communications are at the core of realization and performance of smart grid. Due to low installation cost and high flexibility, wireless communications are prevalent in smart gird. As depicted in Fig. 1, there are three main types of wireless networks different in size and location, i.e., Home Area Networks (HAN), Neighborhood Area Networks (NAN), and Wide Area Networks (WAN) [12]. In HAN, smart meters from home are connected to the gateway based on wireless protocols such as ZigBee. Gateways transmit meter data to Data Aggregate Unit (DAU) through NAN using WiFi. WAN connects all NANs in smart grid to Meter Data Management System (MDMS) by employing broadband wireless communication technologies (e.g., 3G and WiMAX) [13].

With the rapid development of smart grid, more and more smart meters are applied, requiring tremendously increasing amount of meter data to be transmitted. Therefore, more frequency bands are required to support wireless communications in smart grid, which poses a significant challenge on originally scarce spectrum resources. According to the report by

Federal Communications Commission of the United States, the allocated spectrum, however, is heavily under-utilized in vast temporal, spatial and spectral dimensions [14]. The main reason is that under the current regulatory policy, frequency bands are statically restricted to the licensed users (Primary Users (PU)) and no reutilization is permitted for the unlicensed users (Secondary Users (SU)). A novel technology to tackle the conflict between spectrum scarcity and under-utilization is cognitive radio, which enables SU to opportunistically utilize the channel when PU is absent, and to vacate it instantly when PU returns in order to avoid interfering with PU [15] [16].

Spectrum sensing, which enables SU to detect the state of channel occupancy, is a core technique for cognitive radio [17]. Since spectrum sensing and data transmission cannot be performed at the same time, several works have been devoted to sensing time optimization to tradeoff between interference avoidance and sensing efficiency [18] [19]. It has been widely recognized that cognitive radio can dramatically improve channel utilization as well as SU's communication quality. Considering the energy consumption by spectrum sensing, the authors in [20] [21] propose an energy-efficient algorithm to reduce the taxed cost.

The pioneering work of applying cognitive radio to smart grid communications is proposed in [22] [23]. By means of spectrum sensing and channel switching, the smart meter can decide to transmit its data on either an original unlicensed channel or an additional licensed channel, in order to improve communication quality [1]. However, the sensing-dependent energy consumption has not been taken into account. In [24], the impact of communication unreliability on an independent price-demand model is considered. However, electricity price and power supply/demand are always coupled with each other in smart grid.

Considering the cost taxed by spectrum sensing together with the performance degradation incurred by imperfect communications, we attempt to achieve better control performance with lower communication cost, paving the way towards green smart grid. Our contributions are summarized as follows.

1) We introduce cognitive radio with spectrum sensing and channel switching into smart grid to improve communication reliability;
2) We assess the impact of communication quality on the control performance of DRM;
3) We formulate the sensing-performance tradeoff problem in view of the sensing cost, and provide both theoretical analysis and simulation verification.

The remainder of this paper is organized as follows. The sensing-performance tradeoff problem is formulated in Section II. We analyze in Section III how cognitive radio improves

the communication quality, and hence the control performance of DRM. The existence of the optimal solution is proved in Section IV, followed by numerical results in Section V to validate our theoretical analysis. We conclude this paper with future work in Section VI.

II. Problem Formulation

A. Demand Response Management

We consider a smart grid consisting of one power provider, a total of N power consumers, and a control unit. The cycle of a day is divided into several time slots. In each slot, let s denote the supply of power provider, and d_i be the demand of power consumer i (where $i = 1, 2, \cdots, N$). We consider the cost function $C(s)$ indicating the expense of supplying power s by the provider, which is increasing and convex [11]. The power demand of each consumer depends on electricity price and consumer type. Specifically, for each consumer i, the gain function $G_i(d_i)$ represents the obtained satisfaction as a function of its power demand d_i, which is non-decreasing and concave [8].

For the power provider, under electricity price p, the profit by supplying power s is defined as $P_p(s) = ps - C(s)$. Its goal is to adjust its supply to maximize its profit, i.e.,

$$\max_{s} \; ps - C(s) \tag{1}$$

For each consumer i, under electricity price p, the profit by demanding power d_i is calculated as $P_c^i(d_i) = G_i(d_i) - pd_i$. Its goal is to adjust its demand to maximize its profit, i.e.,

$$\max_{d_i} \; G_i(d_i) - pd_i \tag{2}$$

From the social perspective, it is desirable that the expense of provider is minimized and the aggregate satisfaction of consumers is maximized. Therefore, the social welfare can be defined as $\Phi(s, \{d_i\}) = \sum_{i=1}^{N} G_i(d_i) - C(s)$ with $\sum_{i=1}^{N} d_i \leq s$, which can be considered as the control performance of DRM. Therefore, under the constraint that supply should satisfy the total demand, the global optimization problem is:

$$\begin{aligned} \max_{s, \{d_i\}} \; & \sum\nolimits_{i=1}^{N} G_i(d_i) - C(s) \\ \text{s.t. } & s \geq \sum\nolimits_{i=1}^{N} d_i \end{aligned} \tag{3}$$

Although (3) can be solved by convex optimization in a centralized manner [25], the challenge is that the control unit needs to know the exact cost function of the provider and

the gain function of each consumer. However, since the information is private, the control unit may not be able to solve the problem.

In [8], the authors have shown it is possible to approach the optimal solution of (3) in a distributed and iterative way as follows.

1) The control unit begins with any initial electricity price $p_1 \geq 0$ and announces it to the provider and each consumer.
2) On receiving p_k (where $k \in \mathbb{N}^+$ denotes the number of iterations), the provider updates its supply s_k^* by solving (1) and feeds back to the control unit; at the same time, each consumer also updates its demand $d_{i,k}^*$ by solving (2) and feeds back too.
3) On receiving the local optimal supply s_k^* and demand $\left\{d_{i,k}^*\right\}$, the control unit updates the electricity price p_{k+1} for next iteration using a gradient projection method, i.e., $p_{k+1} = \left[p_k - \theta\left(s_k^* - \sum_{i=1}^N d_{i,k}^*\right)\right]^+$, where $\theta > 0$ is the step size which adjusts the convergence rate, and $[x]^+ \triangleq \max\{x, 0\}$.
4) Repeat steps 2) - 3) until the electricity price remains unchanged.

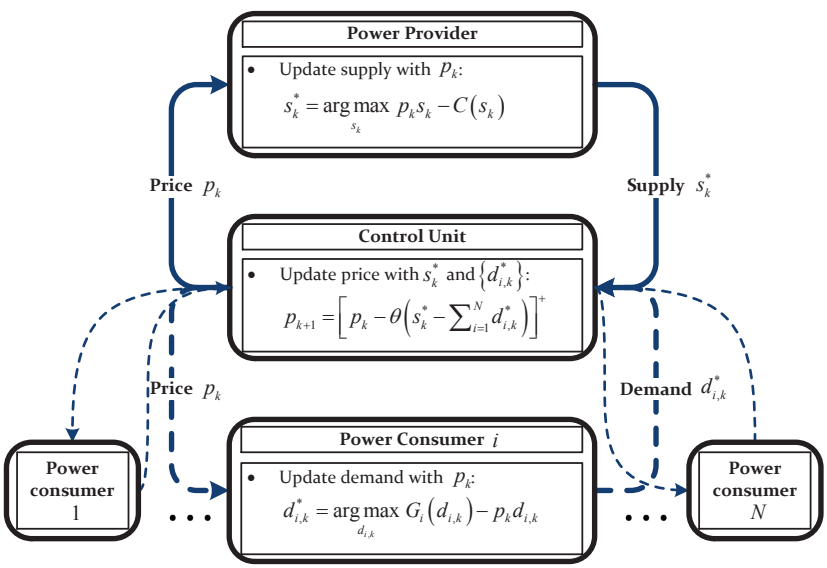

Fig. 2. Interactions in demand response management

The interactions in DRM are shown in Fig. 2. The above control process is based on perfect two-way communications. Usually the control unit is located at the supply side, and reliable communications are often assumed. However, the control unit is far away from the demand

side, and power consumers are connected to it through wireless communications. Due to the unreliability of wireless communication (e.g., packet loss and delay), outage events may occur in the two-way communications between consumers and the control unit, as shown by the dashed lines in Fig. 2. Let ζ denote the average outage, which is assumed to be uniform and independent for all consumers. The control performance with outage ζ is defined as $\tilde{\Phi}(\zeta)$. We show in Section III-B that the outage is critical to the accuracy and optimality of DRM, and analyze the control performance degradation incurred by the outage.

B. Cognitive Radio

In order to improve the communication quality and reduce the outage, we integrate cognitive radio into DRM. In HAN, smart meters periodically transmit data to the gateway. In this paper, we consider that the data can be transmitted through two channels: one is from the unlicensed spectrum, referred to as the original channel Ch_1; the other lies in the licensed spectrum, referred to as the cognitive channel Ch_2, which is randomly occupied by PU. The smart meter can opportunistically switch data transmission to Ch_2. Let H_1 denote the state that Ch_2 is busy (PU is in operation), and H_0 be that Ch_2 is idle (PU is absent). The traffic pattern of PU can be modeled as a two-state independent and identically distributed process [26]. Let P_1 denote the probability of H_1, and P_0 be that of H_0, where $P_1 + P_0 = 1$.

Spectrum sensing, incorporated into the transceiver of smart meters, is able to detect the state of Ch_2 for opportunistic reutilization. A collection of spectrum sensing techniques are proposed in [27], among which the energy detector is popular and optimal for detecting a weak unknown signal from a known zero-mean constellation [28].

With the energy detector, by integrating signal in bandwidth W over sensing time τ, the smart meter compares the collected energy with a predefined threshold ε to decide whether Ch_2 is occupied by PU or not. There are two important metrics used to evaluate the performance of spectrum sensing. The detection probability P_d is that the channel is detected busy when PU is in operation; while the false alarm probability P_f is that the channel is detected busy when PU is absent. They can be calculated in terms of $\mathbb{Q}$-function as [19]

$$P_d = \mathbb{Q}\left(\frac{\varepsilon}{2\sqrt{W\tau}(\gamma+1)\sigma_n^2} - \sqrt{W\tau}\right) \tag{4}$$

$$P_f = \mathbb{Q}\left(\frac{\varepsilon}{2\sqrt{W\tau}\sigma_n^2} - \sqrt{W\tau}\right) \tag{5}$$

where $\mathbb{Q}(z) \triangleq \frac{1}{\sqrt{2\pi}} \int_z^{+\infty} e^{-\frac{\beta^2}{2}} d\beta$, σ_n^2 is the variance of Additive White Gaussian Noise (AWGN), and γ is Signal to Noise Ratio (SNR) at the smart meter. The higher the P_d, the better the PU is protected; while the lower the P_f, the more efficiently Ch_2 can be reutilized by the smart meter.

***Proposition** 1:* With any detection and false alarm probabilities, spectrum sensing before channel switching reduces collision probability on Ch_2.

Proof: See Appendix A. ■

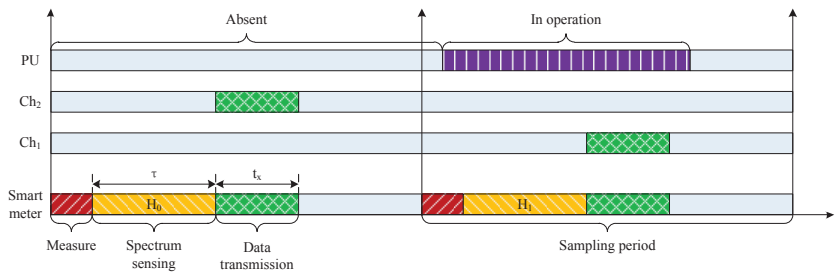

Fig. 3. Cognitive radio framework for two-way communications

Proposition 1 leads to two advantages: i) less collision with PU's transmission guarantees that PU is better protected; ii) lower collision probability means lower outage. As shown in Fig. 3, in each sampling period, before data transmission, the smart meter performs spectrum sensing on Ch_2 to detect whether it is occupied by PU. If it is detected idle, the smart meter can switch data transmission to Ch_2 for lower outage; otherwise, the smart meter remains on Ch_1 to avoid interfering with PU.

C. Sensing-Performance Tradeoff Problem

Though cognitive radio is able to improve communication quality, it involves energy cost for spectrum sensing. Therefore, the system should control the outage requirement using a sensing-dependent cost, assumed to be an increasing and convex function $\varphi(\tau)$ of sensing time τ. Although the energy consumed for channel switching is rather small compared with that for spectrum sensing, we incorporate it into $\varphi(\tau)$. The control performance of DRM is dependent on outage, while the outage is dependent on sensing time. Thus control performance is a function of sensing time, which can be rewritten as $\tilde{\Phi}(\tau)$. Considering the cost taxed by spectrum sensing together with the performance degradation incurred by outage, our aim is

to achieve better control performance with lower communication cost, towards green smart grid.

Intuitively, the longer the sensing time, the lower the outage. The control performance degradation incurred by outage will decrease, but the communication cost taxed by spectrum sensing will increase. Thus there exists a fundamental tradeoff between sensing time and system performance in cognitive radio enabled smart grid. The net revenue can be defined as

$$R(\tau) = \tilde{\Phi}(\tau) - \kappa\varphi(\tau) \tag{6}$$

where the parameter $\kappa > 0$ is used to indicate the importance of $\tilde{\Phi}(\tau)$ and $\varphi(\tau)$ in the revenue. Besides, the application of cognitive radio should not violate the protection of PU. The objective of sensing-performance tradeoff is to find the optimal sensing time such that the total revenue is maximized while the PU is sufficiently protected, i.e.,

$$\max_{\tau} \; R(\tau) \tag{7}$$

$$\text{s.t. } P_d \geq \overline{P}_d \tag{8}$$

where $\overline{P}_d$ is the low bound of detection probability with which the interference incurred to PU is tolerable. In practice, $\overline{P}_d$ is chosen close to but less than 1, because if PU requires full protection, the smart meter is no longer allowed to transmit data on Ch_2. Besides, we suppose P_1 is small, such that it is economically advisable for the smart meter to switch channels.

III. Theoretical Analysis

A. Cognitive Radio Improves Communication Quality

Under cognitive radio framework for two-way communications, the smart meter performs spectrum sensing before channel switching. However, channel switching probability P_{sw} is not exactly equal to P_0, due to the inaccuracy of spectrum sensing. P_{sw} can be evaluated considering two cases: i) when H_1 but the smart meter misses to detect it, i.e., $P_1(1 - P_d)$; ii) when H_0 and no false alarm is generated, i.e., $P_0\left(1 - P_f\right)$.

$$P_{sw} = 1 - P_1 P_d - P_0 P_f \tag{9}$$

If spectrum sensing is perfect, i.e., $P_d = 1$ and $P_f = 0$, then $P_{sw} = P_0$.

Let ζ_1 and ζ_2 denote the outages of Ch_1 and Ch_2 respectively. The resultant outage ζ with cognitive radio can be evaluated considering two cases: i) when Ch_2 is detected busy, the

smart meter remains data transmission on Ch_1, i.e., $(1 - P_{sw})\zeta_1$; ii) when Ch_2 is detected idle, the smart meter switches data transmission to Ch_2, i.e., $P_{sw}\zeta_2$.

$$\zeta = (\zeta_2 - \zeta_1) P_{sw} + \zeta_1 \tag{10}$$

If $\zeta < \zeta_1$, it is economically advisable for the smart meter to perform spectrum sensing and channel switching. Thus we obtain $\zeta_2 < \zeta_1$, so it makes sense to assume the communication quality of Ch_2 to be better than that of Ch_1.

Using the energy detector, since the collected energy depends on sensing time τ, it makes sense to have the energy threshold ε also dependent on τ. We keep $P_d = \overline{P}_d$ to satisfy the constraint of (8). From (4), the relationship between ε and τ can be formulated as

$$\varepsilon = 2\sqrt{W\tau}\left(\mathbb{Q}^{-1}\left(\overline{P}_d\right) + \sqrt{W\tau}\right)(\gamma + 1)\sigma_n^2 \tag{11}$$

Substituting (11) into (5), we obtain the false alarm probability

$$P_f = \mathbb{Q}\left(\gamma\sqrt{W\tau} + \mathbb{Q}^{-1}\left(\overline{P}_d\right)(\gamma + 1)\right) \tag{12}$$

Since $\mathbb{Q}$-function is decreasing, P_f decreases with an increase of τ, which means that increasing sensing time can improve spectrum sensing in terms of P_f.

Furthermore, keeping $P_d = \overline{P}_d$ and substituting (12) into (9), we obtain the channel switching probability

$$P_{sw} = 1 - P_1\overline{P}_d - P_0\mathbb{Q}\left(\gamma\sqrt{W\tau} + \mathbb{Q}^{-1}\left(\overline{P}_d\right)(\gamma + 1)\right) \tag{13}$$

It is observed that P_{sw} increases with an increase of τ, which is because longer sensing time can make spectrum sensing more accurate and provide the smart meter more chances to switch data transmission to Ch_2.

From the analysis above, the longer the sensing time, the lower the false alarm probability, and then the higher the channel switching probability. We see from (10) that ζ decreases with an increase of τ, which indicates that cognitive radio can reduce the communication outage.

B. Communication Quality Affects Control Performance

First assuming the two-way communications in smart grid are perfect, DRM converges to the global optimal price p^*, which balances between supply and demand

$$s^* = \sum_{i=1}^{N} d_i^* \tag{14}$$

By jointly solving (1), (2) and (14), we can obtain the optimal electricity price, supply and demand as $p^* = f^{-1}(1)$, $s^* = \left(C'\right)^{-1}(p^*)$, and $d_i^* = \left(G_i'\right)^{-1}(p^*)$, where $f \triangleq \frac{\left(C'\right)^{-1}}{\sum_{i=1}^{N}\left(G_i'\right)^{-1}}$.

However, when the two-way communications in smart grid suffer from outage ζ, the power demand of each consumer i estimated at the control unit becomes $(1-\zeta)\, d_{i,k}^*$. The electricity price updated at the control unit becomes $p_{k+1} = \left[p_k - \theta\left(s_k^* - (1-\zeta)\sum_{i=1}^{N} d_{i,k}^*\right)\right]^+$ and the iteration converges to a suboptimal price $\tilde{p}$ when

$$\tilde{s} = (1-\zeta)\sum_{i=1}^{N} \tilde{d}_i \tag{15}$$

By jointly solving (1), (2) and (15), we can obtain the suboptimal electricity price, supply and demand as

$$\tilde{p} = f^{-1}(1-\zeta) \tag{16}$$

$$\tilde{s} = \left(C'\right)^{-1}(\tilde{p}) \tag{17}$$

$$\tilde{d}_i = \left(G_i'\right)^{-1}(\tilde{p}) \tag{18}$$

Note that the cost function C is increasing and convex, hence $\left(C'\right)^{-1}$ is positive and increasing; while the gain function G_i is non-decreasing and concave, thus $\left(G_i'\right)^{-1}$ is non-negative and decreasing. Therefore f^{-1} is positive and increasing, and it is clear that $\tilde{p} < p^*$, $\tilde{s} < s^*$ and $\tilde{d}_i > d_i^*$.

Usually the operational decision of smart grid is made in two stages [29]. In the first stage called *unit commitment*, the provider reserves power from power plants according to the estimated demand. Then, in the second stage called *economic dispatch*, the reserved power is supplied to each consumer. However, if the reserved power is not enough, additional power will be bought from the spot market to meet the demand [30]. Since the decision in unit commitment stage is made in advance, consumers can choose and buy power with a cheaper *forward price* p_f than that in economic dispatch stage with an *option price* p_o.

Firstly, the profit of provider is calculated as

$$\tilde{P}_p = \tilde{p}\tilde{s} - C(\tilde{s}) \tag{19}$$

Next, all consumers demand a total of $\sum_{i=1}^{N}\tilde{d}_i$ power, but only get $\tilde{s}$ from the provider with a cheaper forward price $p_f = \tilde{p}$. The additional $\sum_{i=1}^{N}\tilde{d}_i - \tilde{s} = \zeta\sum_{i=1}^{N}\tilde{d}_i$ power will be bought from the spot market with a more expensive optional price $p_o > \tilde{p}$. Therefore the aggregate profit of consumers is calculated as

$$\tilde{P}_c = \sum_{i=1}^{N} G_i\left(\tilde{d}_i\right) - \tilde{p}\tilde{s} - p_o\zeta\sum_{i=1}^{N}\tilde{d}_i \tag{20}$$

Finally, the social welfare (control performance) is calculated as

$$\tilde{\Phi} = \tilde{P}_p + \tilde{P}_c \tag{21}$$

***Proposition** 2:* 1) The outage in two-way communications reduces the profit of power provider;

2) The imperfect communication may not always decrease the aggregate profit of power consumers;

3) The control performance of DRM drops with an increase of outage.

Proof: See Appendix B. ■

IV. Existence of Optimal Solution

When the energy detector is employed, substituting (13) into (10), we have

$$\begin{aligned}\zeta(\tau) \quad &= (\zeta_1 - \zeta_2) P_0 \mathbb{Q}\left(\gamma\sqrt{W\tau} + \mathbb{Q}^{-1}\left(\overline{P}_d\right)(\gamma+1)\right) \\ &+ (\zeta_1 - \zeta_2) P_1 \overline{P}_d + \zeta_2\end{aligned} \tag{22}$$

which indicates that outage is a function of sensing time.

***Theorem** 1:* There exists a unique optimal sensing time τ^* which yields the maximum revenue.

Proof: It can be verified from (22) that $\frac{\partial\zeta}{\partial\tau} = (\zeta_1 - \zeta_2) P_0 \frac{\partial\mathbb{Q}}{\partial z}\frac{\partial z}{\partial\tau}$, where $z = \gamma\sqrt{W\tau} + \mathbb{Q}^{-1}\left(\overline{P}_d\right)(\gamma+1)$ is an increasing function of τ. Recall that the derivative of $\mathbb{Q}$-function is $\frac{\partial\mathbb{Q}}{\partial z} = -\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$, so

$$\frac{\partial\zeta}{\partial\tau} = -\frac{A}{\sqrt{\tau}}e^{-\frac{z^2}{2}} \tag{23}$$

where $A = \frac{(\zeta_1-\zeta_2)P_0\gamma\sqrt{W}}{2\sqrt{2\pi}}$ is a positive constant. Therefore,

$$\frac{\partial R}{\partial\tau} = \frac{\partial\tilde{\Phi}}{\partial\zeta}\left(-\frac{A}{\sqrt{\tau}}e^{-\frac{z^2}{2}}\right) - \kappa\varphi' \tag{24}$$

1) if $\tau \to 0$, that is, $z \to 0$, we have

$$\lim_{\tau\to 0}\frac{\partial R}{\partial\tau} \to \frac{\partial\tilde{\Phi}}{\partial\zeta}\left(-\frac{A}{\sqrt{\tau}}\right) > 0$$

2) if $\tau \to +\infty$, that is, $z \to +\infty$, we have

$$\lim_{\tau\to +\infty}\frac{\partial R}{\partial\tau} \to -\kappa\varphi' < 0$$

In Appendix C, we further prove that $R(\tau)$ is concave. $R(\tau)$ first increases when τ is small and then decreases when τ is large, which makes the maximum point to be unique.

■

To solve (7)-(8), we can take the constraint into the objective by Lagrangian multiplier. The Lagrangian is defined as $L(\tau, P_d, \lambda) = \tilde{\Phi}(\tau) - \varphi(\tau) + \lambda\left(P_d - \overline{P}_d\right)$, where λ is Lagrangian multiplier associated with the constraint of (8). By calculating $\frac{\partial L}{\partial \tau} = \frac{\partial L}{\partial P_d} = \frac{\partial L}{\partial \lambda} = 0$, we can obtain the unique optimal sensing time τ^*.

V. Numerical Results

A. Demand Response Management

In order to evaluate the distributed and iterative approach to DRM, a simple one-provider and one-consumer case is considered. However, the result can be trivially extended to the one-provider and multi-consumer case. We consider a quadratic cost function $C(s) = as^2 + bs + c$, and a quadratic gain function $G(d) = \begin{cases} \omega d - \frac{\alpha}{2}d^2, & 0 \le d \le \frac{\omega}{\alpha} \\ \frac{\omega^2}{2\alpha}, & d > \frac{\omega}{\alpha} \end{cases}$. The parameters are set as $a = 0.1$, $b = 0.5$, $c = 0$ and $\omega = 3$, $\alpha = 0.5$. The social welfare is defined as $\Phi(s, d) = G(d) - C(s)$ when $s \ge d$, and $\Phi(s, d) = G(s) - C(s)$ when $s < d$.

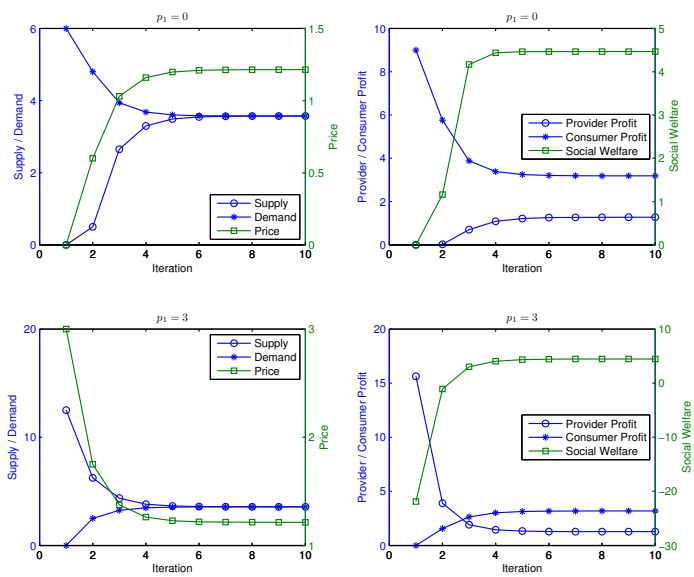

Fig. 4. Demand response management

In Fig. 4, we fix step size at $\theta = 0.1$, and set the initial price at $p_1 = 0$ and 3 respectively.

It is shown that in both cases the electricity price converges to the global optimum which balances between supply and demand. The social welfare achieves the best, where the locally optimal solution of the provider and consumer becomes globally optimal at the converged price.

B. Cognitive Radio Improves Communication Quality

We conduct extensive simulations to understand the capability of cognitive radio in terms of improving communication quality. PU on Ch_2 has bandwidth $W = 6$MHz. The noise is AWGN. We are interested in a low SNR scenario where $\gamma = -15$dB. The lower bound of the detection probability is chosen to be $\overline{P}_d = 0.9$ in order to protect PU.

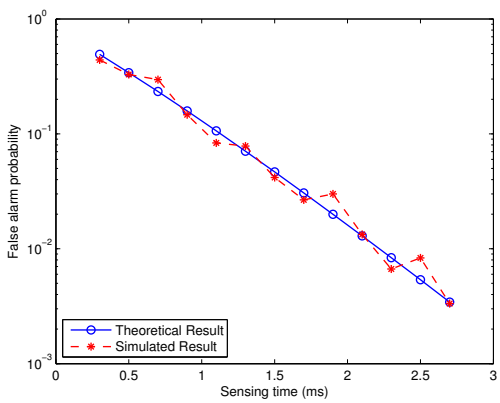

Fig. 5. Simulated and theoretical false alarm probability P_f

Firstly, we compare the false alarm probability between simulation and theoretical results from (12). In the simulation, for each sample under H_1, we find out the energy threshold ε to achieve the target detection probability based on 600 test statistics. Then ε is applied under H_0 to obtain the false alarm probability. As shown in Fig. 5, the false alarm probability decreases with an increase of sensing time.

Next, we compare the channel switching probability between simulation and theoretical results from (13). For the sake of simplicity, we assume that the durations of H_1, H_0 are exponentially distributed with the means of t_1, t_0 respectively. The value of t_0 is chosen to be three times t_1, then $P_1 = \frac{t_1}{t_0+t_1} = 0.25$. As shown in Fig. 6, the channel switching probability increases with an increase of sensing time, which is because the false alarm probability decreases as sensing time increases.

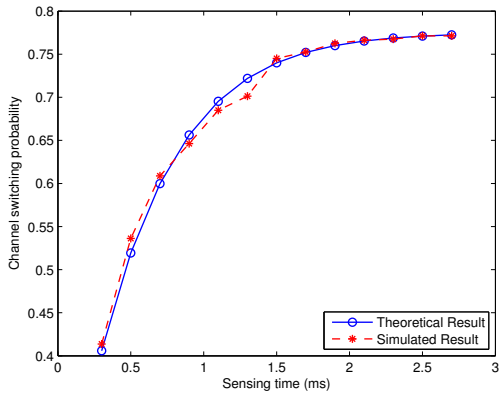

Fig. 6. Simulated and theoretical channel switching probability P_{sw}

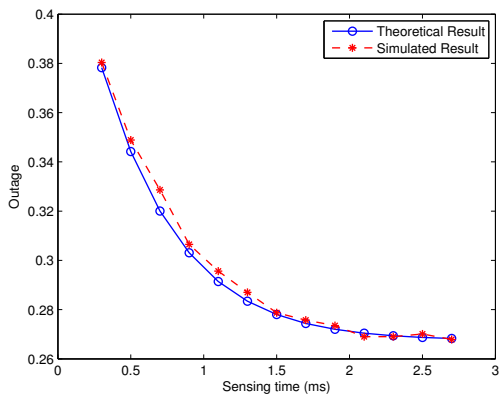

Fig. 7. Simulated and theoretical outage ζ

Finally, we compare outage between simulation and theoretical results from (22). We have assumed that $\zeta_2 < \zeta_1$, for instance, $\zeta_1 = 0.5$ and $\zeta_2 = 0.2$. Fig. 7 shows that outage decreases with an increase of sensing time.

C. Communication Quality Affects Control Performance

We conduct simulations to verify the performance degradation when DRM suffers from the communication outage. We first compare the electricity price, provider supply and consumer demand under perfect communications with those suffering from outage. As shown in Fig. 8, $\tilde{p} < p^*$, $\tilde{s} < s^*$ and $\tilde{d} > d^*$, which is the same as our theoretical analysis.

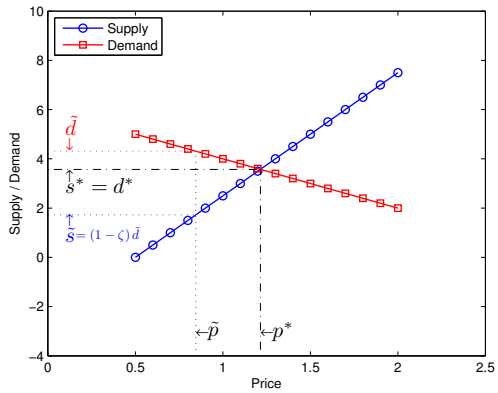

Fig. 8. Price and supply/demand with outage ζ

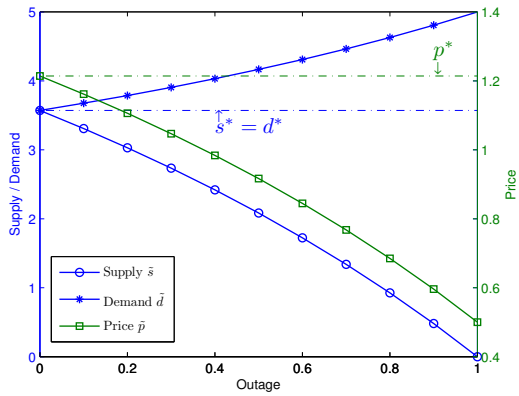

Fig. 9. Outage affects price and supply/demand

Next, we investigate how the outage will affect the control performance of DRM. The optional price is chosen to be $p_o = p^*$. The outage ζ grows from 0 to 1, with the step size of 0.1. As shown in Fig. 9, the electricity price and supply decrease, while the demand increases, with an increase of the outage. From Fig. 10, we can draw such conclusion that the outage in two-way communications reduces the profit of power provider and the social welfare of smart grid, however it may not always decrease the profit of power consumer.

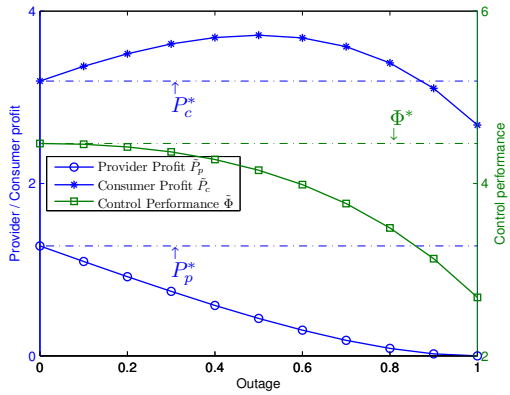

Fig. 10. Outage affects profit and control performance

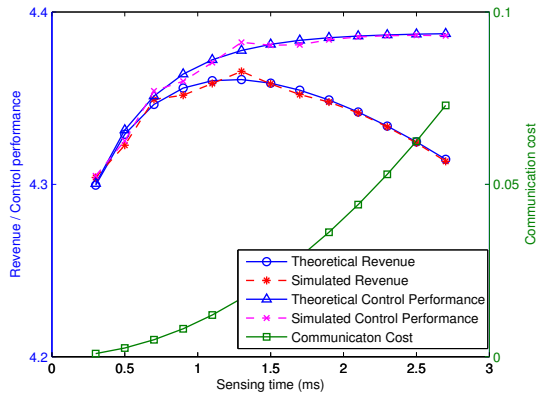

Fig. 11. Simulated and theoretical tradeoff

D. Sensing-Performance Tradeoff

Numerical results are provided to demonstrate the sensing-performance tradeoff in cognitive radio enabled smart grid. We consider the communication cost function to be quadratic as $\varphi(\tau) = \tau^2$. The parameter κ is set to be 10^4. Sensing time varies from 0.3ms to 2.7ms, with the step size of 0.2ms. Fig. 11 shows the comparison between simulated and theoretical tradeoff. It is shown that both the control performance and communication cost increase with sensing time. The simulation results match with the theoretical analysis, where the revenue first increases when sensing time is small and then decreases when it becomes large. There

exists a unique optimal sensing time τ^* which yields the maximum revenue.

VI. Conclusion

By introducing cognitive radio with spectrum sensing and channel switching into smart grid, we show that the communication outage can be reduced. How the communication quality affects the control performance of DRM is also analyzed. We claim that the outage in two-way communications reduces the profit of power provider and the social welfare of smart grid, however it may not always decrease the profit of power consumer. Given two channels, there exists a unique optimal sensing time which achieves the best sensing-performance tradeoff, while the PU on the licensed channel is sufficiently protected. This paper provides the guidelines of achieving better control performance with lower communication cost, paving the way towards green smart grid. In our future work, the impact of both supply uncertainty and communication unreliability on the control performance of DRM will be considered.

Appendix A
Proof of Proposition 1

Let Y denote the duration of H_0, and the meter data transmission time be t_x. If the smart meter switches data transmission to Ch_2 without spectrum sensing in advance, the collision probability P_{ns} can be evaluated considering two cases: i) when H_1, collision occurs; ii) when H_0, the smart meter's transmission collides with PU if and only if PU returns during t_x.

$$P_{ns} = P_1 + P_0 \Pr\{Y < t_x\}$$

However, if the smart meter switches data transmission to Ch_2 only when it is detected idle, the collision probability P_s can be evaluated considering two cases: i) when H_1 but the smart meter misses to detect it, collision occurs; ii) when H_0 and no false alarm is generated, the smart meter's transmission collides with PU if and only if PU returns during t_x.

$$P_s = P_1 (1 - P_d) + P_0 \left(1 - P_f\right) \Pr\{Y < t_x\}$$

The difference of the collision probability on Ch_2 by spectrum sensing before channel switching is calculated as

$$\Delta = P_{ns} - P_s = P_1 P_d + P_0 P_f \Pr\{Y < t_x\} > 0$$

where the inequality always holds because P_1, P_0, P_d, P_f, $\Pr\{Y < t_x\} > 0$.

Appendix B

Proof of Proposition 2

Firstly, the derivative of (19) is $\frac{\partial \tilde{P}_p}{\partial \zeta} = \frac{\partial \tilde{p}}{\partial \zeta}\tilde{s} + \tilde{p}\frac{\partial \tilde{s}}{\partial \zeta} - \frac{\partial C(\tilde{s})}{\partial \tilde{s}}\frac{\partial \tilde{s}}{\partial \zeta}$. Note that from (16) and (17) we have $\frac{\partial \tilde{p}}{\partial \zeta} = -\left(f^{-1}\right)' = -\frac{1}{f'}$ and $\frac{\partial C(\tilde{s})}{\partial \tilde{s}} = \tilde{p}$, where $f' = \frac{\left(\left(C'\right)^{-1}\right)' \sum_{i=1}^{N}\left(G_i'\right)^{-1} - \left(C'\right)^{-1}\sum_{i=1}^{N}\left(\left(G_i'\right)^{-1}\right)'}{\left(\sum_{i=1}^{N}\left(G_i'\right)^{-1}\right)^2} > 0$, and then $\frac{\partial \tilde{p}}{\partial \zeta} < 0$. Therefore

$$\frac{\partial \tilde{P}_p}{\partial \zeta} = \frac{\partial \tilde{p}}{\partial \zeta}\tilde{s} < 0$$

which means that the profit of power provider decreases with an increase of outage.

Next, the derivative of (20) is $\frac{\partial \tilde{P}_c}{\partial \zeta} = \sum_{i=1}^{N}\frac{\partial G_i(\tilde{d}_i)}{\partial \tilde{d}_i}\frac{\partial \tilde{d}_i}{\partial \zeta} - \frac{\partial \tilde{p}}{\partial \zeta}\tilde{s} - \tilde{p}\frac{\partial \tilde{s}}{\partial \zeta} - p_o\sum_{i=1}^{N}\left(\tilde{d}_i + \zeta\frac{\partial \tilde{d}_i}{\partial \zeta}\right)$. Note that from (18) and (15) we have $\frac{\partial G_i(\tilde{d}_i)}{\partial \tilde{d}_i} = \tilde{p}$ and $\frac{\partial \tilde{s}}{\partial \zeta} = -\sum_{i=1}^{N}\tilde{d}_i + (1-\zeta)\sum_{i=1}^{N}\frac{\partial \tilde{d}_i}{\partial \zeta}$. Therefore

$$\frac{\partial \tilde{P}_c}{\partial \zeta} = (\tilde{p} - p_o)\sum_{i=1}^{N}\left(\tilde{d}_i + \zeta\frac{\partial \tilde{d}_i}{\partial \zeta}\right) - \frac{\partial \tilde{p}}{\partial \zeta}\tilde{s}$$

where $\frac{\partial \tilde{d}_i}{\partial \zeta} = \left(\left(G_i'\right)^{-1}\right)'\frac{\partial \tilde{p}}{\partial \zeta} > 0$.

1) When $\zeta \to 0$, then $\tilde{p} \to p^*$ and $\tilde{s} \to \sum_{i=1}^{N}\tilde{d}_i$. If choose $p_o = p^*$, we have

$$\lim_{\zeta \to 0}\frac{\partial \tilde{P}_c}{\partial \zeta} \to -\frac{\partial \tilde{p}}{\partial \zeta}\tilde{s} > 0$$

2) When $\zeta \to 1$, then $\tilde{s} \to 0$, and we have

$$\lim_{\zeta \to 1}\frac{\partial \tilde{P}_c}{\partial \zeta} \to (\tilde{p} - p_o)\sum_{i=1}^{N}\left(\tilde{d}_i + \frac{\partial \tilde{d}_i}{\partial \zeta}\right) < 0$$

from which we can never infer that the aggregate profit of power consumers always decreases with an increase of outage.

Finally, the derivative of (21) is

$$\frac{\partial \tilde{\Phi}}{\partial \zeta} = \frac{\partial \tilde{P}_p}{\partial \zeta} + \frac{\partial \tilde{P}_c}{\partial \zeta} = (\tilde{p} - p_o)\sum_{i=1}^{N}\left(\tilde{d}_i + \zeta\frac{\partial \tilde{d}_i}{\partial \zeta}\right) < 0 \tag{25}$$

which means that the control performance of DRM decreases with an increase of outage.

Appendix C

Concavity of $R(\tau)$

***Lemma** 1:* $R(\tau)$ is concave.

Proof: Assume the cost function C and the gain function G_i are at most quadratic, thus we have $f'' = -2\left(\sum_{i=1}^{N}\left(G_i'\right)^{-1}\right)'\frac{\left(\left(C'\right)^{-1}\right)'\sum_{i=1}^{N}\left(G_i'\right)^{-1} - \left(C'\right)^{-1}\left(\sum_{i=1}^{N}\left(G_i'\right)^{-1}\right)'}{\left(\sum_{i=1}^{N}\left(G_i'\right)^{-1}\right)^3} > 0$. Further, $\frac{\partial^2 \tilde{p}}{\partial \zeta^2} = -\frac{f''}{\left(f'\right)^2} < 0$ and $\frac{\partial^2 \tilde{d}_i}{\partial \zeta^2} = \left(\sum_{i=1}^{N}\left(G_i'\right)^{-1}\right)'\frac{\partial^2 \tilde{p}}{\partial \zeta^2} > 0$.

It can be verified from (25) that

$$\frac{\partial^2 \tilde{\Phi}}{\partial \zeta^2} = \frac{\partial \tilde{p}}{\partial \zeta} \sum_{i=1}^{N} \left(\tilde{d}_i + \zeta \frac{\partial \tilde{d}_i}{\partial \zeta} \right) + (\tilde{p} - p_o) \sum_{i=1}^{N} \left(2 \frac{\partial \tilde{d}_i}{\partial \zeta} + \zeta \frac{\partial^2 \tilde{d}_i}{\partial \zeta^2} \right) < 0$$

and further from (24) we have

$$\frac{\partial^2 R}{\partial \tau^2} = \frac{\partial^2 \tilde{\Phi}}{\partial \zeta^2} \left(-\frac{A}{\sqrt{\tau}} e^{-\frac{z^2}{2}} \right)^2 + \frac{\partial \tilde{\Phi}}{\partial \zeta} \frac{A}{2\tau\sqrt{\tau}} \left(1 + z\gamma\sqrt{W} \right) e^{-\frac{z^2}{2}} - \kappa\varphi'' < 0$$

It says that $\frac{\partial R}{\partial \tau}$ is decreasing, which means $R(\tau)$ is concave. ■

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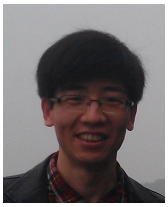

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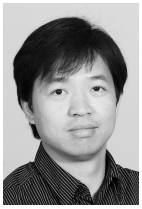

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