Optimization of a Spatially Varying Cardiac Contraction parameter using the Adjoint Method

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1 Introduction

Patient-specific cardiac modeling can be used as a tool in diagnosis and to optimize patient treatment. A key benefit of using modeling is that it allows you to compute mechanical features that otherwise are impossible to measure safely in a human heart, and thereby increase the insight and understanding of the state of the heart. Moreover, modeling enable us to test possible treatments numerically without surgical intervention. However, in order for the model to be reliable and realistic, the underlying cardiac mechanics model has to be adapted to the patient under consideration. This means that measurements throughout the cardiac cycle have to be taken into account in the model personalisation process.

2 Methods

In this study we constrain a cardiac computational model using measurements obtained from the hospital. More specifically we use a patient-specific left ventricular (LV) geometry together with 4D LV regional strain, LV volume and LV pressure measurement as input to the model. The pressure is imposed through a Neumann boundary condition on the LV endocardium, while the volume and strain are fitted to the model by formulating the problem as a PDE-constrained optimization problem. The total mismatch between simulated and measured strain and volume at measurement point i can be combined into a misfit functional of the form

$$I_{\alpha}^{i} = \alpha I_{\mathrm{vol}}^{i} + (1-\alpha) I_{\mathrm{strain}}^{i}, \tag{1}$$

where I_{vol}^{i} and I_{strain}^{i} represents the volume and strain misfit respectively, and α controls the weight on the volume versus strain fit.

The optimization procedure is divided into two phases: passive and active. In the passive phase, the LV is inflated from the assumed stress-free configuration at diastasis up to point of end-diastole(ED). This phase is used to determine passive elastic material properties. We employ a transversally isotropic version of the Holzapfel-Ogden strain energy law [2]

$$\mathcal{W} = \frac{a}{2b}\left(e^{b(I_1-3)} - 1\right) + \frac{a_f}{2b_f}\left(e^{b_f \max\{(I_{4f}-1),0\}^2} - 1\right), \tag{2}$$

where $\mathbf{C}$ is the Right Cauchy Green tensor, $I_1 = \mathrm{tr}\ \mathbf{C}$ and $I_{4f} = \mathbf{e_f} \cdot \mathbf{C}\mathbf{e_f}$ with $\mathbf{e_f}$ being a unit vector field pointing in the direction of the muscle fibers. The parameters a, a_f, b, b_f is made specific to the patient by minimizing the misfit between measured and simulated volume.

To model the active contraction we apply the active strain formulation[1] which is based on a multiplicative decomposition of the deformation gradient $\mathbf{F}$ into an elastic(e) and an active part(a): $\mathbf{F} = \mathbf{F}_e\mathbf{F}_a$. The active deformation gradient is chosen to have the following form:

$$\mathbf{F}_a = (1-\gamma)\mathbf{e_f} \otimes \mathbf{e_f} + \frac{1}{\sqrt{1-\gamma}}(\mathbf{I} - \mathbf{e_f} \otimes \mathbf{e_f}), \tag{3}$$

where $\gamma \in [0,1)$ represents the active muscle fiber shortening and is used as control variable for this phase. In order to capture local properties and to obtain a more realistic contraction pattern we let γ be a piecewise linear function. For this phase we solve

$$\begin{aligned} \underset{\gamma^i}{\text{minimize}} \quad & I_\alpha^i + \lambda \|\nabla \gamma^i\|^2_{L^2(\Omega)} \\ \text{subject to} \quad & \delta W = 0 \\ & \gamma^i(\mathbf{X}) \in [0,1), \;\; \mathbf{X} \in \Omega, \end{aligned} \tag{4}$$

where Ω is the left ventricular wall and δW being the virtual work[6] of all forces applied to the system. Here we have also introduced a total variation regularization for the purpose of numerical stability. The solver is fully parallelized and based on the open-source framework FEniCS[4]. The problem is solved using a gradient-based optimization algorithm[5], where the gradient is computed by solving an automatically derived adjoint equation[3]. Note that since the contraction parameter γ is spatially resolved, the number of parameters to determine at each point in the active phase is equal to the number of vertices in the mesh. This makes adjoint gradient calculations computationally advantageous over standard finite difference approximations.

3 Results

We tested the method on synthetic data and were able to reproduce the synthetic displacement field within a maximum error of 0.06 cm on clean data and 0.13 cm on noisy data. Using $\alpha = 0.2$ and $\lambda = 0.1$ for the active phase we are able to get a relative average regional error in strain of less than 10% and a relative average volume error of less than 0.0045% using real patient data as input.

References

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