PATIENT-SPECIFIC MODELING OF CARDIAC MECHANICS USING THE ACTIVE STRAIN FORMULATION

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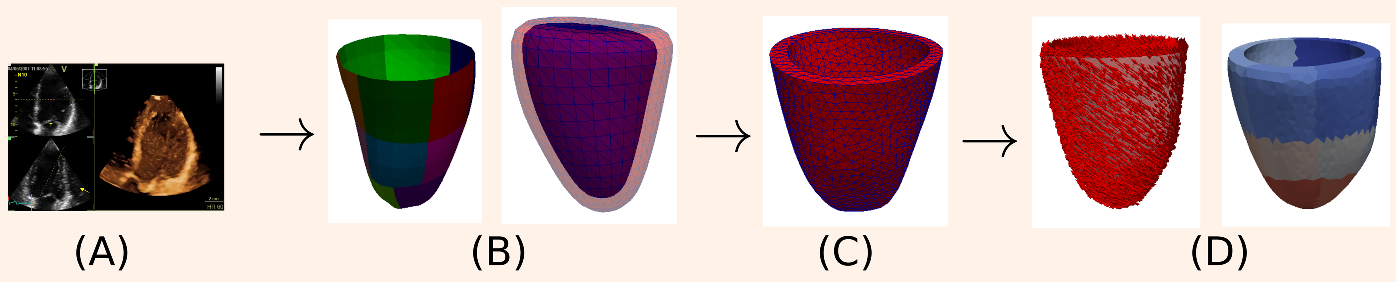
INTRODUCTION

Important features in cardiac mechanics that cannot easily be measured in the clinic, can be computed using a computational model that is calibrated to behave in the same way as a patient's heart. To construct such a model, clinical measurements such as strain, volume and cavity pressure are used to personalize the mechanics of a cardiac computational model. The problem is formulated as a PDE-constrained optimization problem where the minimization functional represents the misfit between the measured and simulated data. The target parameters are material parameters and a spatially varying contraction parameter. The minimization is carried out using a gradient based optimization algorithm and an automatically derived adjoint equation. The method has been tested on synthetic data, and is able to reproduce a prescribed contraction pattern on the left ventricle.

METHODS

PRE-PROCESSING



Triangulated data points for left ventricular endocardial and epicardial surfaces(B) are extracted from 4D echocardiography(A). Together with these surfaces we are also given a strain mesh(B) that can be used to identify the location of particular regions of the LV. We use Gmsh to mesh these surfaces together in order to obtain a linear tetrahedral mesh(C). The strain mesh is used to mark the cells so that each cell is assigned one value according to the AHA-zone representation(D). We assign fiber orientations(D) using the rule based method proposed by Bayer et al.

MECHANICAL MODEL

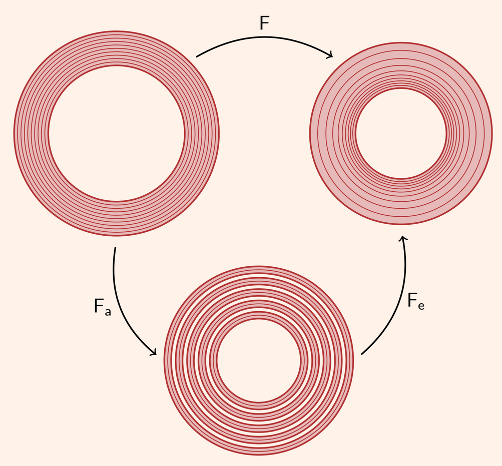
We model the heart as a continuum body with a reference configuration taken at the beginning of the passive filling phase. To model the active contraction of the heart we introduce a single spatially varying parameter $\gamma = \gamma(\mathbf{x}, t)$, and apply the active strain formulation.

- This is based on a multiplicative decomposition of the deformation gradient,

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_a,$$

where $\mathbf{F}$ is the isochoric part of the deformation gradient, $\mathbf{F}_e$ is the elastic part, and

$$\mathbf{F}_a = (1-\gamma)\mathbf{f}_0 \otimes \mathbf{f}_0 + \frac{1}{\sqrt{1-\gamma}}(\mathbf{I} - \mathbf{f}_0 \otimes \mathbf{f}_0).$$



- The myocardium is modeled as an incompressible, hyperelastic material. We use a transversally isotropic version of the strain energy density function proposed by Holzapfel and Ogden,

$$\mathcal{W}(\mathbf{C}_e) = \frac{a}{2b}\left(e^{b(I_1-3)} - 1\right) + \frac{a_f}{2b_f}\left(e^{b_f(I_{4\mathbf{f}_0}-1)_+^2} - 1\right).$$

Here I_1 is the first isotropic invariant of the elastic part of the right Cauchy-Green tensor $\mathbf{C}_e = \mathbf{F}_e^T\mathbf{F}_e$, $I_{4,\mathbf{f}_0} = \mathbf{f}_0 \cdot (\mathbf{C}_e\mathbf{f}_0)$ is the quasi-invariant with a preferred direction along the fibers $\mathbf{f}_0$ and $(\cdot)_+ = \max\{\cdot, 0\}$.

- The force balance equations is found by solving for the minimum elastic energy:

$$\Pi = \int_\Omega \mathcal{W}(\mathbf{C}_e) + p(J-1)dV + \text{boundary conditions}, \quad R(\mathbf{u}, p) = \begin{pmatrix} D_{\delta\mathbf{u}}\Pi \\ D_{\delta p}\Pi \end{pmatrix} = \mathbf{0}$$

PARAMETER ESTIMATION

- We use measured left ventricular cavity volume and 4D regional strain obtained from 4D echo, and invasive left ventricular pressure measurement to personalize the mechanics. Pressure measurements are incorporated as a Neumann boundary condition on the endocardium, while strain and volume measurements are incorporated into an objective misfit functional.
- The average regional strain is measured in the circumferential(e_c), radial(e_r) and longitudinal(e_l) direction relative to the left ventricle. The left ventricle Ω, is partitioned into 17 regions, $\Omega = \bigcup_{k=1}^{17} \Omega_k$. The average regional strain over the region Ω_j in the direction e_k can be approximated as $\tilde{\varepsilon}_{k,j} = G_j(e_k^T \nabla \mathbf{u} \cdot e_k)$ where G_j is the linear functional given by $G_j(f) = \frac{1}{|\Omega_j|}\int_{\Omega_j} f\,dx$,, e_k indicates a unit direction field, and $|\Omega_j|$ the volume of segment j. Let N be the number of discrete measurements during a cardiac cycle, and let N_{ED} be the number of points from the beginning of the passive filling to end diastole. For point $i = 1, \cdots, N$ we define the strain misfit functional as,

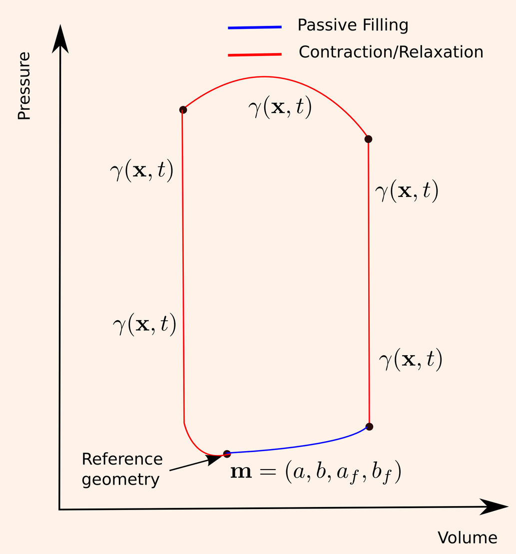
$$I_{\text{strain}}^i = \sum_{j=1}^{17} \sum_{k \in \{c,r,l\}} \omega_{k,j}\left(\varepsilon_{k,j}^i - \tilde{\varepsilon}_{k,j}^i\right)^2.$$

The weights $\omega_{k,j}$ are based on the quality of the strain measurement over the region Ω_j in the direction e_k. The volume misfit functional is defined as

$$I_{\text{vol}}^i = \left(\frac{V^i - \tilde{V}^i}{V^i}\right)^2, \quad \tilde{V}^i = -\frac{1}{3}\int_{\partial\Omega_{\text{endo LV}}} (\mathbf{X} + \mathbf{u}) \cdot J\mathbf{F}^{-T}\mathbf{N}\, dS.$$

where $\partial\Omega_{\text{endo LV}}$ is the surface inside the left ventricular cavity. We combine the mismatch between the strain and volume into one total mismatch functional of the form

$$I_\alpha^i = \alpha I_{\text{vol}}^i + (1-\alpha) I_{\text{strain}}^i.$$



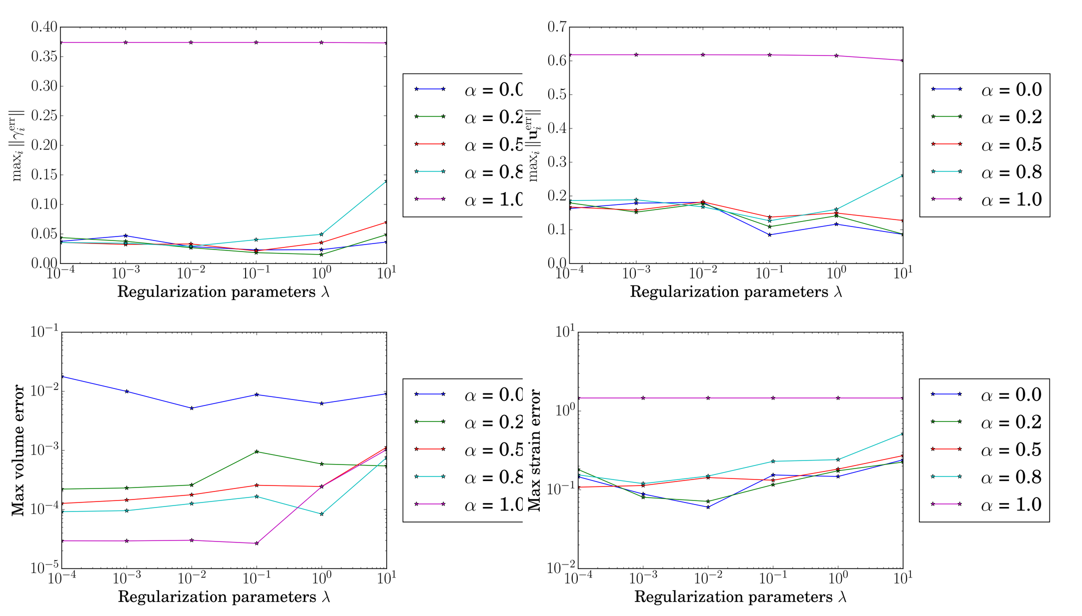
- We use the passive filling phase to determine the material parameter set $\mathbf{m} = (a, b, a_f, b_f)$ and the active contraction/relaxation phase to determine γ by solving the following PDE-constrained optimization problems:

Passive filling phase	Contraction/Relaxation phase
$\underset{\mathbf{m}}{\text{minimize}} \sum_{i=0}^{N_{\text{ED}}} I_\alpha^i$ subject to $R(\mathbf{u}, p) = 0$.	$\underset{\gamma(\mathbf{x},i)}{\text{minimize}}\ I_\alpha^i + \lambda\|\nabla\gamma\|_{L^2(\Omega)}^2$ subject to $R(\mathbf{u}, p) = 0$, $\mathbf{x} \in \Omega, i = N_{\text{ED}} + 1, \cdots, N$.

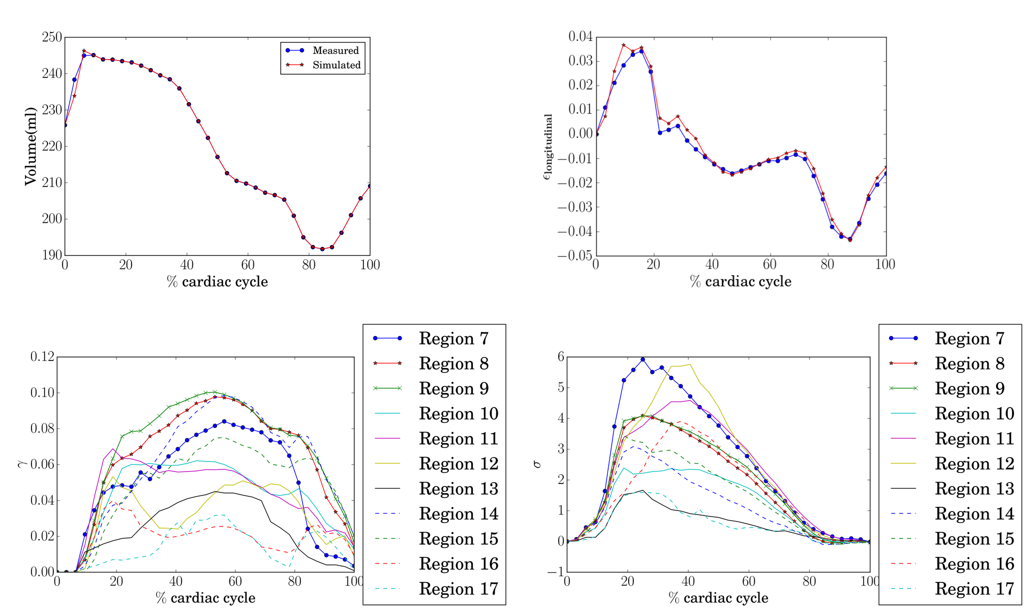
- Here we have also included a regularization of the contraction parameter γ.
- The solver is fully parallelized and based on the opensource finite element framework FEniCS. To solve the PDE-constrained optimization problem we use a gradient based optimization algorithm where the gradient is computed by solving the automatically derived adjoint equation using dolfin-adjoint.

RESULTS

To verify that the method works, we generate synthetic data with prescribed parameters, and use strain and volume curves generated from this data as input to the model. We add noise to the strain data to account for drift. Below a summary of the maximum relative error using synthetic data for γ, displacement, volume and strain is displayed for different parameter sets.



We apply the method to the a patient with left bundle branch block. We are able to match the volume and strain curves very well. Below we see the resulting simulated and measured volume, and the simulated and measured longitudinal strain for the mid septal strain region. This allows us to visualize regional activation and regional fiber stress.



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DOWNLOADS

FEniCS project dolfin-adjoint gmsh