

Personalized Cardiac Mechanical Model using a High Resolution Contraction Field

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1. Introduction

Patient-specific cardiac modelling is an area of research that has received much attention in the last years. This is mainly due to better imaging techniques, increasing computational power combined with more efficient numerical methods, but also to the fact that computational models allows us to compute features such as mechanical stress that otherwise are impossible to measure *in-vivo*. This can provide us with an increased understanding of the complex mechanical events occurring during a heartbeat.

In this study we use left ventricular cavity volumes and regional strains obtained from speckle tracking echocardiography, along with invasive blood pressure measurements, to personalize the mechanics of a cardiac computational model. The patients considered suffer from dyssynchronous heart failure in which the timing of mechanical activation varies between the different regions of the heart. These variations are captured by the measured regional strains. However, in order to capture these regional differences in the model, it is essential to allow the contractility to vary in space as well as in time. We introduce the contractility as a high-resolution field and use adjoint optimization techniques to fit the model to the measured data at a cost the does not significantly depend upon the resolution of the contractility field.

2. Materials and Methods

The left ventricular reference geometry is derived from a segmented 4D echocardiographic image. To estimate the position of the myocardial walls throughout the cardiac cycle we employ a quasi-static formulation, which assumes that the cardiac walls are at equilibrium at each measurement point.

The pressure measurements are used as a Neumann boundary condition at the endocardium, and the base is fixed in the longitudinal direction, but is allowed to move in the basal plane subject to a linear spring, which limits rotations.

For the material model, we adapt a transversally isotropic version of the Holzapfel and Ogden strain energy function[1] with four material parameters (a , b , a_f , b_f) as shown Eq. 1. These parameters can be tuned in order to obtain patient specificity:

$$\mathcal{W} = \frac{a}{2b} \left(e^{b(I_1-3)} - 1 \right) + \frac{a_f}{2b_f} \left(e^{b_f(I_{4f}-1)^2_+} - 1 \right) \quad (1)$$

To model the active contraction of the muscle fibres, we apply the active strain framework[2], which assumes a multiplicative decomposition of the deformation gradient into an active and an elastic part. The active part represents the actual distortion of the microstructure and is in our case given by the simple relation in Eq. 2.

$$\mathbf{F}_a = (1 - \gamma) \mathbf{e}_f \otimes \mathbf{e}_f + \frac{1}{\sqrt{1 - \gamma}} (\mathbf{I} - \mathbf{e}_f \otimes \mathbf{e}_f) \quad (2)$$

Here e_f is the fibre orientation, which was assigned using a rule-based algorithm[3], and γ represents the relative shortening in the muscle fibre direction and will be referred to as the contraction field.

Together, the four material parameters and the contraction field are used as control variables in the model-personalization process. The strain and volume measurements are fitted to the model by formulating the problem as a PDE-constrained optimization problem, where the objective functional represents the misfit between simulated and measured strains and volumes as shown in Eq. 3.

$$I_\alpha^i = \alpha I_{vol}^i + (1 - \alpha) I_{strain}^i = \alpha \left(\frac{V^i - \tilde{V}^i}{V^i} \right)^2 + (1 - \alpha) \sum_{j=1}^{17} \sum_{k \in \{c,r,l\}} (\varepsilon_{k,j}^i - \tilde{\varepsilon}_{k,j}^i)^2 \quad (3)$$

Here V^i and \tilde{V}^i represents the measured and simulated volume at measurement point i respectively, and $\varepsilon_{k,j}^i$ and $\tilde{\varepsilon}_{k,j}^i$ are the measured and simulated strain at measurement point i in the direction k averaged over segment number j . For each measurement point there are in total 51 strain measurements (3 components in the circumferential, radial and longitudinal direction, for each of the 17 segments), and 1 volume measurement.

The model personalization-process is divided into two phases: passive material parameter optimization during atrial systole and contraction field optimization during the remaining part of the cardiac cycle. The formulation of the problems is displayed in Eq. 4.

$$\begin{aligned} \underset{a, a_f, b, b_f}{\text{minimize}} \quad & \sum_{i=1}^{N_{ED}} I_{vol}^i & \underset{\gamma^i}{\text{minimize}} \quad & I_\alpha^i + \lambda \|\nabla \gamma^i\|_{L^2(\Omega)}^2 \\ \text{subject to } \delta W = 0 \quad & \forall i, & \text{subject to } \delta W = 0 \quad & \forall i, \\ & & & \gamma^i(\mathbf{X}) \in [0, 1), \quad \mathbf{X} \in \Omega. \end{aligned} \quad (4)$$

Here δW refers to the virtual work of all forces applied to the system, which vanishes in equilibrium, and λ is a regularization parameter.

The method is fully parallelized and the underlying mechanics equations are solved using the finite element framework FEniCS[4]. To solve the optimization problems, we apply a gradient-based sequential quadratic programming algorithm where the gradient is computed by solving an automatically derived adjoint equation[5].

3. Results

The method is verified using a synthetic data test with noise added to the strain and volume input. A prescribed sequence of contraction fields, together with a fixed set of material parameters are used to generate the synthetic strains and volumes which are used as input to the optimization. In both the noise and clean cases the displacement field is reproduced within a maximum error of less than 1.4 mm.

We tested the method on multiple patients selected for cardiac resynchronization therapy(CRT), who suffer from dyssynchronous heart failure. The results of the data matching using clinical data show an excellent fit between measured and simulated data. Plots of a selection of strain curves and the pressure-volume loop for one of the patients are provided in Fig. 2.

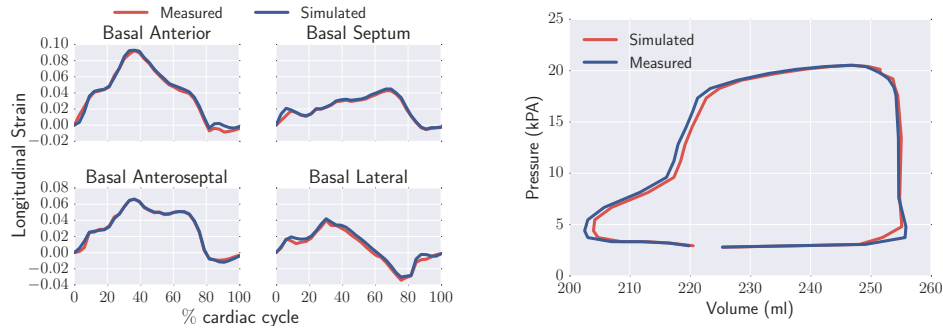


Figure 1: Left figure shows the match between simulated and measured strain for 4 out of the 51 strain measurements. Right figure shows the simulated and measured pressure-volume loop.

We visualize the motion of the left ventricle throughout the cardiac cycle in Fig. 3 with mechanical stress along the muscle fibers as colormap. The simulation shows clear signs of dyssynchrony, which is also observed in the echocardiographic images.

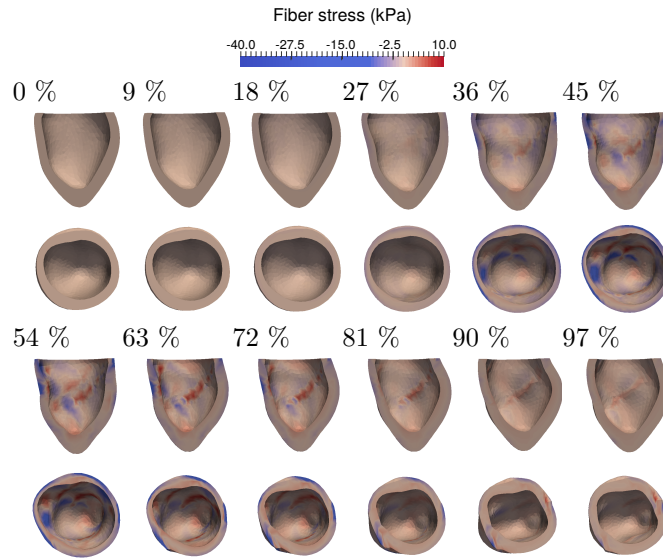


Figure 2: Visualization of the motion of the left ventricle throughout the cardiac cycle with mechanical stress in along the muscle fibers as colormap.

In Fig 4 we compare the motion of the model with the echocardiographic images used to calibrate the model at end-systole. The comparison shows that the model move in the same manner as what is observed in the image. Here we also visualize the contraction field.

4. Discussion and Conclusions

We conducted parameter estimations on synthetic data in order to verify the method under ideal circumstances. Using synthetic strains and volumes with added noise, we were able to reproduce the synthetic displacement field within a maximum error of less than 1.4 mm.

When applied to clinical data, results show an excellent fit with measured strains and volumes, and the model thereby incorporates the basic mechanical motion of a patient's heart. Moreover, since the regional strains measure local mechanical motion, we are able to capture dyssynchrony. The need for validation of features computed from this computational model remains a challenge of their clinical use. One possible route is as mechanical stresses along the muscle fibres should be related to the

metabolic processes in the heart, one could validate the computed mechanical stresses with the use of e.g PET imaging. Further work in this direction is needed.

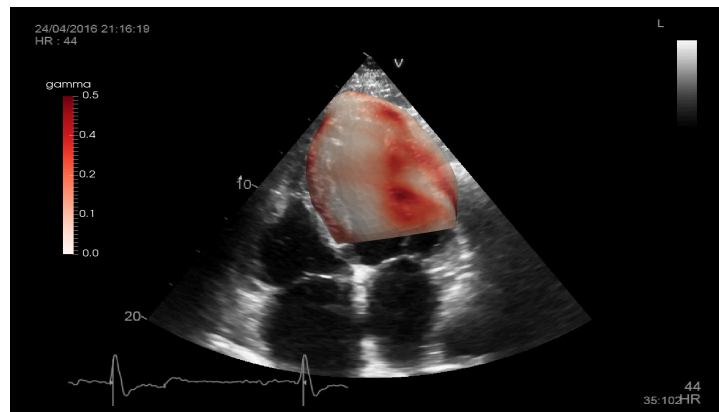


Figure 3: Simulation of the contraction field overlaid on top of echocardiographic image at end-systole.

The adjoint optimization technique does not significantly depend on the number of control variables, and is therefore highly computational advantageous. Gradient-based optimization methods require derivative evaluation of the objective functional with respect to the control parameters. This evaluation typically increases in complexity with the number of control parameters. Using the adjoint method, we only have to solve the adjoint system, which is automatically derived using the symbolic representation inherited by FEniCS.

Reducing the computational cost of personalizing the computational model is a key step towards translating modelling into clinical utility. Using models to compute mechanical features such as stress, which are currently not possible to measure, can provide us with more insight into all the mechanical signals that are likely to be important for remodelling. In terms of CRT this may lead to better patient selection and optimal lead placement.

5. References

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